Prof. William H. Sandholm
Department of Economics
University of Wisconsin
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Syllabus: Economics 805, Part 1
Evolution and Learning in Games

Course Description

The aim of this course is to introduce a variety of topics from evolutionary game theory (= myopic disequilibrium dynamics in games played by large populations) and the theory of learning in games (= disequilibrium dynamics in games played by small groups of players) We will start with an extended introduction to the theory population games and evolutionary dynamics using both (some) simulations and (mostly) formal analyses. After this we will more briefly consider a variety of topics, with candidates including (i) evolution in extensive form games; (ii) geometric game dynamics; (iii) stochastic evolution in cooperative games; (iv) learning via calibrated forecasts.

All of the topics we study in this course require some knowledge about dynamical systems and Markov chains. Some topics require more advanced knowledge of probability theory and an assortment of other areas of mathematics. We will cover the essential mathematics in lecture, but we will likely sacrifice mathematical diligence in favor of covering more game theory models less rigorously.

Course requirements

In the first portion of the course (about 8 lectures), the basic reading material will be some combination of my chapter from the *Handbook of Game Theory* and my book. There will be a few problem sets that will be collected and graded. The aim here is to equip you with a basic command of evolutionary game theory and some of the relevant mathematics.

The second portion of the course will focus on a few individual papers and book chapters. Some of the aims here are (i) to expose you to a variety of current research topics; (ii) to help you develop the habit of reading theory papers closely; and (iii) to acquaint you with the mathematics you would need to learn to pursue research in each of these areas.

The homework for the second portion of the course will be to closely read a paper, either one from a list of suggestions or one that you find on your own and I approve. You will write a report summarizing the paper and explaining the main ideas of the proofs. Part of the way one learns to do theory is by reading papers closely until you have mastered part of the literature, and the aim here is to help you develop this habit.

References

General references

W. H. Sandholm (2010). *Population Games and Evolutionary Dynamics*. MIT.

W. H. Sandholm (2015). "Population games and deterministic evolutionary dynamics." In H. P. Young and S. Zamir, eds., *Handbook of Game Theory*, Vol. 4, North Holland, 703–775.

Evolution in extensive form games

W. H. Sandholm, S. S. Izquierdo, and L. R. Izquierdo (2017). "Best experienced payoff dynamics and cooperation in the Centipede game". Working paper.

M. van Veelen and J. García (2016). "In and out of equilibrium I: evolution of strategies in repeated games with discounting". *Journal of Economic Theory* 161, 161–189.

M. van Veelen, J. García, D. G. Rand, and M. A. Nowak (2012). "Direct reciprocity in structured populations". *Proceedings of the National Academy of Sciences* 109, 9929–9934.

Z. Xu (2016). "Convergence of best response dynamics in extensive form games," *Journal of Economic Theory* 162, 21–54.

Geometric and higher-order game dynamics

R. Laraki and P. Mertikopoulos (2013). "Higher order learning and evolution in games." *Journal of Economic Theory* 148, 2666–2695.

R. Laraki and P. Mertikopoulos (2015). "Inertial game dynamics and applications to constrained optimization." *SIAM Journal on Control and Optimization* 53, 3141–3170.

P. Mertikopoulos and W. H. Sandholm (2016). "Learning in games via reinforcement and regularization". *Mathematics of Operations Research* 41, 1297–1324.

P. Mertikopoulos and W. H. Sandholm (2017). "Riemannian game dynamics". Working paper.

More deterministic game dynamics

T. N. Cason, D. Friedman, and E. Hopkins (2014). "Cycles and instability in a Rock-Paper-Scissors population game: a continuous time experiment." *Review of Economic Studies* 81, 112–136.

R. Lahkar (2011). "The dynamic instability of dispersed price equilibria." *Journal of Economic Theory* 146, 1796–1827.

J. Hofbauer, S. Sorin, and Y. Viossat (2011). "Time average replicator and best-reply dynamics." *Mathematics of Operations Research* 345, 263–269.

D. Oyama, W. H. Sandholm, and O. Tercieux (2015). "Sampling Best Response Dynamics and Deterministic Equilibrium Selection". *Theoretical Economics* 10 (2015), 243-281.

Stochastic evolutionary dynamics

I. Arieli and P. Young (2016). "Stochastic learning dynamics and speed of convergence in population games." *Econometrica* 84, 627–676.

D. P. Myatt and C. C. Wallace (2008). "When does one bad apple spoil the barrel? An evolutionary analysis of collective action." *Review of Economic Studies* 75, 499–527.

W. H. Sandholm and M. Staudigl (2016). "Large deviations and stochastic stability in the small noise double limit." *Theoretical Economics* (2016), 279–355.

W. H. Sandholm and M. Staudigl (2017). "Sample path large deviations for stochastic evolutionary game dynamics." Working paper.

Stochastic stability in cooperative games

H. Nax and B. Pradelski (2015). "Evolutionary dynamics and equitable core selection in assignment games." *International Journal of Game Theory* 44, 903–932.

J. Newton and R. Sawa (2015). "A one-shot deviation principle for stability in matching problems." *Journal of Economic Theory* 157, 1–27.

R. Sawa (2014). "Coalitional stochastic stability in games, networks, and markets." *Games and Economic Behavior* 88, 90–111.

Learning via calibrated forecasts

D. Blackwell (1956). "An analog of the minimax theorem for vector payoffs". *Pacific Journal of Mathematics* 6: 1–8.

N. Cesa-Bianchi and G. Lugosi (2006). *Prediction, Learning, and Games*. Cambridge

D. P. Foster and S. Hart (2017). "Smooth calibration, leaky forecasts, finite recall, and Nash dynamics". Working paper.

S. Hart and A. Mas-Colell (2000). "A Simple Adaptive Procedure Leading to Correlated Equilibrium". *Econometrica* 68, 1127–1150.

W. Olszewski (2015). "Calibration and expert testing". In H. P. Young and S. Zamir, eds., *Handbook of Game Theory*, Vol. 4, North Holland, 703–775.

H. P. Young (2004). *Strategic Learning and Its Limits.* Oxford.

Two introductory-level math references

J. R. Norris (1997). *Markov Chains*. Cambridge.

L. Perko (2006). *Differential Equations and Dynamical Systems*, third edition. Springer.

Software and guides

W. H. Sandholm, E. Dokmaci, and F. Franchetti (2014). *Dynamo*: *Diagrams for Evolutionary Game Dynamics*. Software.

F. Franchetti and W. H. Sandholm (2017). "An introduction to *Dynamo*: *Diagrams for Evolutionary Game Dynamics*". *Biological Theory* 8, 167–178.

L. R. Izquierdo, S. S. Izquierdo, and W. H. Sandholm (2017). *Abed*: *Agent-Based Evolutionary Dynamics*. Software.

L. R. Izquierdo, S. S. Izquierdo, and W. H. Sandholm (2017). "An introduction to *Abed*: *Agent-Based Evolutionary Dynamics*". Working paper.