Cultural Integration and Its Discontents∗

Timur Kuran  
Department of Economics  
Duke University  
213 Social Sciences Building  
Durham, NC 27708  
t.kuran@duke.edu  
http://www.rcf.usc.edu/~kuran

William H. Sandholm  
Department of Economics  
University of Wisconsin  
1180 Observatory Drive  
Madison, WI 53706  
whs@ssc.wisc.edu  
http://www.ssc.wisc.edu/~whs

July 6, 2007

∗ JEL Classification Codes: C7, D7, N4, Z1.
We have benefited from the comments of seminar audiences at the Stockholm School of Economics, the University of California at Berkeley, the University of Southern California, and the University of Tokyo, as well as from those of Bryan Caplan, Dhammika Dharmapala, Steven Durlauf, Sam Fraidin, James Montgomery, Larry Samuelson, Todd Sandler, Dan Simon, Michael Walton, three anonymous referees, and a Managing Editor. Sung Han Tak provided exemplary research assistance. Financial support from NSF Grants SES-0092145 and SES-0617753 is gratefully acknowledged.
Cultural Integration and Its Discontents

Abstract

A community’s culture is defined by the preferences and equilibrium behaviors of its members. Contacts among communities alter individual cultures through two interrelated mechanisms: behavioral adaptations driven by payoffs to coordination and preference changes shaped by socialization and self-persuasion. This paper explores the workings of these mechanisms through a model of cultural integration in which preferences and behaviors vary continuously. It identifies a broad set of conditions under which cross-cultural contacts promote cultural hybridization. The analysis suggests that policies to support social integration serve to homogenize preferences across communities, thereby undermining a key objective of multiculturalism. Yielding fresh insights into strategies pursued to influence cultural trends, it also shows that communities benefit from having other communities adjust their behaviors.
1. Introduction

Diminishing transportation and communication costs facilitate contacts among culturally distinct communities. Through cross-cultural interactions, hybrid cultures emerge from habits and norms once identified with different cultures. This process of “cultural integration” can fuel social tensions, as reflected in campaigns to protect existing cultures and in anti-globalization movements. These and other manifestations of cultural protectionism are often motivated by the perception that cross-cultural influences favor the spread of one particular culture at the expense of others.

Currently the social sciences lack an analytical framework suitable to systematic study of such themes. Even the prevailing definitions of culture are problematic, because they preclude specificity or stress shared attributes. Here we define culture as a pair of distributions that jointly provide a distinct communal identity: a preference distribution and an equilibrium behavior distribution. This focus on distributions accommodates the heterogeneity that societies show in tastes and choices. It also yields insights into the tensions that accompany cultural integration. We explore the hybridization process through an explicitly dynamic theoretical framework based on individual choice and, departing from conventional economic analysis, feedback from cultural outcomes to individual preferences.

A key feature of the model is that the equilibrium behaviors of individuals reflect compromises between respecting their own personal preferences and coordinating with the choices of others. These compromises shape preferences through two distinct mechanisms. First, children’s preferences are influenced by their parents' observed behaviors, so that preferences in each family lineage come to reflect equilibrium choices. Second, individual preferences adjust to lessen the discontents fueled by discrepancies between ideal and actual choices. The preference changes alter equilibrium behaviors, which then induce further preference adaptations. We demonstrate how this circle of influences promotes cultural hybridization and homogenization, and characterize both the ultimate composition of the hybrid culture and the speed of cultural change.

Our analysis relies on techniques from evolutionary game theory to describe in closed form the time paths of each agent's preferences and actions. We posit a two-speed formulation whereby gradual changes in preferences are accompanied by immediate behavioral adjustments that maintain equilibrium play.

The model to be developed speaks to two popular social objectives of our time: “multiculturalism” and “social integration.” Most variants of multiculturalism aim to preserve the multiplicity of existing cultures. For their part, social integration campaigns promote interactions across boundaries of class, ethnicity, religion, and national origin. Daily headlines reveal that in combination, these objectives generate
social conflict. Policies designed to legitimize and strengthen the identities of immigrant groups lead to social frictions and even violence, especially when immigrant lifestyles are perceived to conflict with those of the host population.\footnote{These frictions are currently the subject of vigorous debates in Europe. For accounts pertaining specifically to the Netherlands and France, see Buruma (2006) and Laurence and Vaisse (2006), respectively.}

While recognizing these short-term problems of adjustment, the following analysis points to a distinct and generally overlooked conflict between the two social objectives themselves. Our model describes how cross-cultural contacts generate behavioral adaptations to improve interpersonal coordination. These changes in behavior induce changes in preferences, eroding preference diversity both within and across cultures. By dampening cultural differences, both sets of adjustments undermine multiculturalism.

Although central to the analysis presented here, the cultural homogenization that accompanies integration seldom draws attention in policy discussions. The promoters of civil rights laws, ethnic affirmative action policies, school busing programs, and other such tools of social integration tend also to favor some form of multiculturalism.\footnote{See, for instance, Kymlicka (1995) and Parekh (2000).} Universities in many advanced countries, including the United States, try to integrate students of various backgrounds while also facilitating and even encouraging expressions of social, ethnic, and religious separateness. The supporters of these programs do not recognize that the success of integration policies will undermine the goal of multiculturalism by homogenizing the cultures ostensibly being preserved.\footnote{Glazer (1997) makes complementary observations.}

Rigorous economic analysis can avert such incoherence. In pursuing its substantive agenda, this paper demonstrates how economic reasoning can assist the formulation of realistic policies in contexts commonly considered outside the purview of economic inquiry.

The model also yields insights into other aspects of the political economy of cultural change. Our analysis helps explain the logic behind the assimilationist campaigns that often accompany “nation building”, and illuminates why groups differ in their resistance to cultural integration. Finally, we offer a rationale for campaigns, common all over the world, to make immigrants endure cultural adaptations.

The paper is organized as follows. The next section defines concepts, justifies assumptions, and places the topic in an empirical context. Section 3 describes the motivations of the individual member of a cultural community. Using these building blocks, Section 4 models preferences and equilibrium behavior within an isolated community. Section 5 introduces cross-cultural interactions and describes the dynamics...
of cultural integration. This analysis yields explicit information on the magnitudes of the discontents accompanying integration, thus offering insights into why members of a community may gain from policies that induce conformism by non-members. Our analysis also identifies factors that influence the ultimate composition of preferences in the integrated society. Surprisingly, within-group conformity has no influence at all. Section 6 discusses policy implications, including ones related to the consequences of immigration. Concluding remarks are presented in Section 7, and an appendix contains some proofs and auxiliary results.

2. Culture and Cultural Integration

2.1 Culture

For our purposes here, a culture consists of two distributions that give a community a distinct identity: a preference distribution and an equilibrium behavior distribution. This definition allows for diversity both across and within cultures. It also accommodates the tensions that individuals experience as they try to meet social demands.4

2.2 Coordination and Behavioral Compromises

When an agent interacts with others, his behavior is driven by two competing motives. While he wants his choices to agree with his personal preferences, his gains depend on the degree to which his choices are coordinated with those of the agents with whom he interacts. Commonality of language is the most obvious form of coordination that enhances interactions. Shared symbols, meanings, and communication rules facilitate both economic exchanges and social cooperation.5 Common culinary habits present another source of coordination benefits. Although individuals could eat foods suited to their own particular tastes, a shared menu reduces efforts expended in cultivation and preparation. Age of marriage, the locus of the marriage decision and family size offer another cluster of examples.

---

4 Economists have used the term culture in various other senses. Kreps (1990) views culture as a vehicle for providing generally accepted solutions to problems that can be tackled in different ways. Cremer (1993), following Arrow (1974), defines culture as that portion of a stock of knowledge that is “shared” by a substantial segment of a group, but not by the general population from which that group is drawn. Outside of economics, definitions of “culture” vary much more widely. Well before the explosion in cultural studies Kroëber and Kluckhohn (1952) had identified 161 formal definitions used across the social sciences.

5 Lazear (1999) proposes a simple equilibrium model in which commonality of language facilitates trade. His model’s main prediction, that acquisition of the majority’s language is more likely when the minority group is smaller, is broadly consistent with our results.
The tensions between personal preferences and coordination extend to settings at the heart of modern economics. Consider work norms. While the individual member of a team will have distinct preferences regarding work effort, the benefits resulting from his choice often depend on the efforts made by other members. The interactions thus exhibit the structure of a coordination game, with work norms appearing as equilibrium outcomes. Public goods provision is a source of analogous coordination problems. Even as individuals differ in their civic-mindedness, their optimal contributions to a public good may depend on those made by others. This basic observation dates back to the stag hunt game of Rousseau, and it continues to drive a large body of economic research.

What is critical for our purposes here is that in interacting with others individuals face trade-offs. In principle, a person could contend with his competing needs by adjusting his behaviors at each new interpersonal encounter. For instance, by speaking one language at home and another at work. In a wide variety of contexts, however, such compartmentalization is inordinately costly. Hence, choices of behavior may be invariant to context. To capture this invariance in a simple way, our preliminary model postulates that agents choose a single behavior to be used with all interaction partners. This assumption is partially relaxed in the full model of Section 5.

In an isolated cultural community, incentives to coordinate would originate entirely from within the community itself. Although individuals with unusual preferences would accommodate the preferences prevailing in their own community, they could ignore the preferences of outsiders. By contrast, in the typical cultural community, which interacts with other communities, incentives to coordinate are driven partly by cross-cultural contacts. Consequently, cultures influence each other. In other words, both behaviors and preferences are biased by interactions across group boundaries.

What fuels cultural integration, our focus here, is precisely the need to coordinate with individuals belonging to other cultures. Meeting this need produces gaps between individuals’ preferences and behaviors. In turn, these gaps cause preferences to change.

2.3 Mechanisms of Preference Evolution

One source of preference change is socialization. Most parents attempt to inculcate their own preferences into their children. At the same time, the preferences of children are shaped also by parental behaviors. Thus, socialization influences preferences

---

7 See, e.g., Crawford (1995), Battalio, Samuelson, and Van Huyck (2001), and the references therein. The importance of cultural factors to public good contribution is emphasized in the recent work of Francois and Zabojnik (2005), discussed below.
through both parental training and parental behavior. For reasons already outlined, parents’ behaviors may differ from their preferences. Consequently, children’s preferences resemble those of their parents, but are biased in the direction of parental behaviors.\(^8\)

Our second source of preference change is psychological. In a wide range of experimental settings, psychologists find that discrepancies between attitudes and behaviors can induce preference changes through “self-persuasion”. To give an example, Freedman and Fraser (1966) find that eliciting compliance with a small request (to display a tiny public service slogan on one’s property) vastly increases the likelihood of compliance with a much more costly request (to install a large, ugly billboard). Evidently the act of abiding by the initial request makes subjects perceive themselves as civic-minded, inducing compliance with the larger request.\(^9\) A number of different mechanisms have been proposed to explain such phenomena. The earliest explanation, Festinger’s (1957) theory of cognitive dissonance, invokes a need for self-consistency.\(^10\)

Ordinarily preference changes occur in a largely subconscious manner. Accordingly, psychologists find that people make systematic errors when asked to predict how their preferences will evolve in response to new experiences.\(^11\) In some settings, of course, preference adaptation also has a conscious component, as choices are made both for their direct value and in anticipation of beneficial preference changes.\(^12\) In what follows we abstract from personal self-transformation strategies in order to focus on social processes that transform entire societies. Conscious preference adjustment may complement the subconscious processes of interest here.

The postulated preference adjustment mechanism resembles that of Kuran (1995, Ch. 10-14), where publicly expressed preferences at odds with privately held preferences affect the evolution of the latter by distorting the process of socialization. Our mechanism also shares characteristics with models of Bisin and Verdier (2000, 2001) in which people are endowed with one of two cultural traits. Each person wants his children to inherit his own trait, and actual outcomes depend on the traits of both

---

\(^8\) Of course, insofar as children interact with the wider community, the tastes they develop will show even greater biases toward the norms of the society at large. For evidence, see Douglas (1984).

\(^9\) For a related study of this phenomenon, see Pliner \textit{et. al.} (1974). Literature reviews include Brehm \textit{et. al.} (2002, Ch. 6), Cialdini (2001, Ch. 3) and Aronson (1999, Ch. 4).

\(^10\) For elaborations on this approach, see Cooper and Fazio (1984). Bem’s (1965) self-perception theory argues that preference adaptations occur as people use observations of their own behavior to “discover” their preferences. Other distinct mechanisms have been proposed by Baumeister (1982) and Steele (1988).

\(^11\) These prediction errors are resilient to learning, which confirms that the adaptation mechanisms in question are subconscious. See Kahneman and Snell (1992) and Loewenstein and Schkade (1998).

\(^12\) People become educated partly to meet future coordination needs. The knowledge that they expect to acquire will prove useful in developing friendships and business relationships; however, they also know that the learning will alter their preferences, for example, their literary tastes. Schelling (1984, Ch. 2-3) and Kuran (1998) discuss strategies used to achieve self-transformation.
parents, efforts devoted to socialization, and the distribution of traits in the population. In the presence of frictions in the marriage market both traits persist indefinitely, as parents with the minority trait accept relatively high costs to pass it down. Building on this approach, as well as on the work of Uphoff (2000) on the role of cultural norms in sustaining contributions to public goods, Francois and Zabojnik (2005) analyze the role of trustworthiness in economic development. They show that if firms are able to choose a technology that is efficient only if workers can be trusted, socialization by parents can sustain trustworthiness across generations and promote economic growth.

With respect to these works, our model differs in its emphasis on the integration of heterogeneous cultural communities and on the discontents that accompany it. In invoking sociological and psychological forces as the engines of preference evolution, we deviate also from the game-theoretic literature on preference evolution, in which preference changes are driven by differences in biological fitness or material payoffs.13

2.4 Discontents of Cultural Integration

Researchers who study self-persuasion have long understood that preference changes motivated by self-consistency impose psychic costs.14 Discomfort due to inconsistencies between behaviors and preferences is also a salient theme in studies of cultural adaptation. In analyzing the history of Americanization, Rubin (1995) describes the alienation of immigrants who distanced themselves from their family and ancestral heritage.15 Akerlof and Kranton (2000) capture this same phenomenon through the notion of “identity.” In growing up, they observe, an individual develops a sense of selfhood. When interactions with other societies compel this individual to pursue a different lifestyle, the blurring of his identity causes a psychic loss.

These observations fit naturally into the framework developed here. What Akerlof and Kranton (2000) define as identity corresponds to the individual’s personal preference in relation to the two groups’ preference distributions. Likewise, when an individual adopts a different lifestyle, the consequent identity confusion is represented in our model by behavior at odds with his preference. Feelings of distance from family and heritage can also arise as consequences of preference change. As the daughter of an

---


14 Indeed, Festinger’s (1957) dissonance theory is explicit in linking preference change to psychic discomfort. See Brehm et al. (2002, Ch. 6) and Aronson (1999, Ch. 4) for further discussion.

15 Rubin (1995) also shows that this alienation manifested itself in tensions within families and across generations, which could be viewed as a consequence of the biased socialization described above. For additional insights into the personal stresses and the interpersonal tensions generated by acculturation, see Thomas (1995) and Ahmad (1962/1992).
immigrant makes choices responsive to those of the host society, her behaviors will conflict with those of her differently socialized parents. As this daughter’s behaviors shape her preferences, her interactions with her ancestral community will become poorly coordinated and, hence, strained.

All key components of our model have now been introduced: cultural communities whose members interact with both insiders and outsiders, limits on situation-specific behavioral adaptations, tradeoffs between achieving individual ideals and reaping coordination benefits, and preference changes driven by social and psychological forces. We will show how these elements lead to cultural hybridization.16

3. Coordination Payoffs and Personal Payoffs

In the illustrations given above, individual preferences and actions are drawn from a continuous range. A person’s diet may adjust gradually as he varies the frequency with which he eats particular dishes. Likewise, his willingness to contribute to public goods may change incrementally as well. The continuum assumption thus facilitates a meaningful analysis of the process through which a culture evolves.17

In our model, agents interact in pairs, and their payoffs consist of two components: a coordination payoff and a personal payoff. The first component captures the benefits that agents derive directly from the interaction itself. It is a strictly decreasing function of the difference between the actions chosen by the two individuals.

Were the coordination payoff the only payoff, the agents would face a pure coordination game, and an action pair would constitute a Nash equilibrium if and only if both agents chose the same action. But our agents also care about identity-driven personal ideals.18 Specifically, in any interaction each derives a personal payoff that is a decreasing function of the distance between ideal and chosen actions, where the former is represented by an individual-specific preference parameter \( \pi \). Accordingly, an agent maximizes not just a coordination payoff, but a combination of two distinct payoffs.19

For tractability, we assume that every agent’s utility function has a simple quadratic form. Suppose that an agent with preference parameter \( \pi \) chooses action \( x \), and that the

---

16 For the time being, we ignore the possibility of collective action designed to mold and control cultural evolution through political means. Later we shall show how cultural protectionism, multiculturalism, and anti-globalization movements all aim to restrict the mechanisms described here.

17 The assumption that the choice set is continuous stands in contrast to works on social interactions in which agents with preferences drawn from a continuum make binary choices. See Schelling (1973), Akerlof (1976), Kuran (1989), Bikchandani, Hirshleifer, and Welch (1992) and Brock and Durlauf (2001).

18 That individuals derive utility from following their personal ideals has been recognized in other analyses of social interactions. See, for instance, Kuran (1995) and Akerlof (1997).

19 In Kuran’s (1989) and Bernheim’s (1994) models of conformity, utility is also the sum of a personal and a social loss function. However, their models are otherwise different from ours: their social loss functions measure status costs due to public perception of one’s preferences.
person with whom he is paired chooses action \( x' \). Then our agent obtains the payoff

\[
u(x, x', \pi) = (-w(x - x')^2) + ((-x)^2).
\]

The scalar \( w \), the *conformity parameter*, represents the absolute weight the individual places on his coordination payoff; the weight he places on his personal payoff is normalized to unity. If two agents with preference parameters \( \pi \) and \( \pi' \) play the game described by (1), then by substituting one agent's first order condition into the other's, one finds the unique Nash equilibrium, in which the chosen actions are

\[
\frac{1}{1+2w}\left((1+w)\pi + w\pi'\right) \text{ and } \frac{1}{1+2w}\left((1+w)\pi' + w\pi\right).
\]

The value of \( w \) is culture-specific. It is meant to capture the importance that the agent’s community attaches to coordination, and hence to conformity. Some communities seek to enforce codes of correct behavior relating to marriage, diet, and language; others allow considerable diversity. In our full-blown model with multiple communities, the weight the individual places on coordination depends on his and his interaction partner’s community memberships. But we begin our analysis by studying equilibrium behavior and preference evolution within an isolated community.

4. Cultural Evolution within an Isolated Community

Let \( A = [0, 1] \) be the set of agents in an isolated community. While the term "agent" is used for convenience, each \( \alpha \in A \) actually represents a single multigenerational lineage.

Agent \( \alpha \)'s preference parameter at time \( t \) is denoted by \( \Pi_t(\alpha) \in \mathbb{R} \), and the set of all agents’ preference parameters by the *preference profile* \( \Pi_t: A \rightarrow \mathbb{R} \). Similarly, \( X_t(\alpha) \in \mathbb{R} \) represents agent \( \alpha \)'s action at time \( t \), and \( X_t: A \rightarrow \mathbb{R} \) the corresponding *action profile*. We assume that both preference and action profiles are uniformly bounded over all finite time spans.

To describe the average preference and average action within the community, we use notation from probability theory. In particular, \( E \) denotes the expectation operator for functions defined on \( A \). Accordingly, \( E\Pi_t = \int_A \Pi_t(\alpha) d\alpha \) denotes the average preference parameter at time \( t \), and \( EX_t = \int_A X_t(\alpha) d\alpha \) the average action.

---

20 The formulation of utility in equation (1) posits that coordination payoffs depend only on the distance between the agents’ actions, not on the actions themselves. This assumption is natural in contexts in which the contents of different cultures are economically neutral. It is less appropriate in settings where payoffs depend on the nature of the coordination achieved. We maintain the assumption of action neutrality to focus on our central interest: assimilation and its accompanying discontents.
4.1 The Short Run: Equilibrium Behavior

Suppose that members of the community are repeatedly paired at random, and that payoffs in each interaction are determined by the utility function $u$. As is usual in evolutionary models with random matching, agents condition their behaviors on information about the distribution of actions of potential match partners rather than on information about the particular partner with whom they are matched. As explained above, this assumption is natural in settings where it is costly to adjust continually to the characteristics of different match partners. It is also reasonable in contexts where agents find it hard to anticipate how specific partners will behave.

Given this setup, agent $\alpha$ chooses an action $x$ which maximizes his expected utility,

$$Eu(x, X, \Pi(\alpha)) = E\left(-w (x - X)^2 \right) + \left(-(x - \Pi(\alpha))^2 \right).$$

The action profile $\hat{X}_i$ is a Nash equilibrium of the random matching game if for all $\alpha$,

$$\hat{X}_i(\alpha) = \arg\max_x Eu(x, \hat{X}_i, \Pi(\alpha)).$$

It is easy to show that this game has a unique equilibrium.

**Proposition 1:** Fix the preference profile $\Pi$. The unique Nash equilibrium of the random matching game is

$$(2) \quad \hat{X}_i(\alpha) = \frac{1}{w+1} (w E\Pi_i + \Pi(\alpha)) \text{ for all } \alpha \in A.$$

**Proof:** Taking the first order condition, we see that if $\hat{X}_i$ is a Nash equilibrium, then for each agent $\alpha \in A$, the action $\hat{X}_i(\alpha)$ must satisfy

$$(w + 1) \hat{X}_i(\alpha) = w E\hat{X}_i + \Pi(\alpha).$$

Taking expectations and canceling like terms, we find that $E\hat{X}_i = E\Pi$. Substituting this expression into the previous equation and rearranging establishes the proposition.

In the game’s unique Nash equilibrium, each agent $\alpha$ selects an action that is a weighted average of his own preference $\Pi_i(\alpha)$ and the average preference in the population $E\Pi$, which itself equals the average equilibrium action $E\hat{X}_i$. The weight put on the average preference is an increasing function of the conformity parameter $w$. 


4.2 The Long Run: Preference Evolution

The Nash equilibrium (2) describes the behavior of the community at a single moment in time. Over longer time spans, the distribution of preferences evolves in response to this equilibrium. We capture the two sources of preference change, socialization and self-persuasion, through the dynamic

\[
\frac{d}{dt} \Pi_i(\alpha) = \hat{X}_i(\alpha) - \Pi_i(\alpha) \quad \text{for all } \alpha \in A.
\]

This equation states that the preferences of agent \( \alpha \) move in the direction of that agent’s current equilibrium behavior, at a rate proportional to the distance between them. It implicitly defines a two-speed adjustment process. At each moment, the population follows an equilibrium that is uniquely determined by current preferences via equation (2). Discrepancies between individual agents’ equilibrium behaviors and underlying preferences cause the latter to change according to equation (P).

Equation (P) requires that during preference adaptations, behavioral adjustments to maintain equilibrium play occur instantaneously. The rationale is that while an agent can quickly switch actions, preference change, within or across generations, is a gradual process. By assuming that behavior adjusts an order of magnitude more quickly than preferences change, we capture these relative rates in the simplest possible way. This assumption is standard in the game-theoretic literature on preference evolution. But our model differs from this literature in an important way. Instead of just looking for stable equilibria of the process of preference evolution, we seek to determine explicitly the preference and behavior trajectories associated with any initial preference profile.21

Equation (P) describes a cyclical relationship connecting preferences and behaviors. The trajectory of agent \( \alpha \)’s preferences, \( \Pi_i(\alpha) \), depends on his equilibrium behavior, \( \hat{X}_i(\alpha) \). By definition, this behavior depends on the other agents’ behaviors, which in turn depend on their preferences. The evolution of the preference \( \Pi_i(\alpha) \) thus depends on the entire preference profile \( \Pi \), implying that the evolution of an agent’s preferences cannot be studied in isolation: to solve equation (P), one must describe the preference changes of all agents simultaneously. Proposition 2 characterizes this solution.

Proposition 2: Fix an initial preference profile \( \Pi_0 \). The unique solution to equation (P) from this initial condition is

\[
\Pi_i(\alpha) = E\Pi_0 + (\Pi_0(\alpha) - E\Pi_0) \exp\left(-\frac{w}{w+1} t\right) \quad \text{for all } \alpha.
\]

21 Some models of preference evolution (e.g., Güth and Yaari (1992) and Dekel, Ely and Yilankaya (2007)) assume that players can observe and condition their behavior upon the personal preference of their partner in a match. We assume that such conditioning is impossible.
Proof: Substituting equation (2) into the preference dynamic (P), we obtain

\[
\frac{d}{dt} \Pi_t(\alpha) = \frac{1}{w+1} \left( \Pi_t(\alpha) + w \cdot E \Pi_t \right) - \Pi_t(\alpha) = \frac{w}{w+1} \left( E \Pi_t - \Pi_t(\alpha) \right).
\]

Differentiating under the integral sign and substituting yields \( \frac{d}{dt} E \Pi_t(\alpha) = E \left( \frac{d}{dt} \Pi_t(\alpha) \right) \) = 0, which implies that \( E \Pi_t = E \Pi_0 \) for all \( t \). Substituting this expression into equation (4) yields the ordinary differential equation

\[
\frac{d}{dt} \Pi_t(\alpha) = \frac{w}{w+1} \left( E \Pi_0 - \Pi_t(\alpha) \right),
\]

whose solution is equation (3).  

The solution to (P) is illustrated in Figure 1, which presents preference and behavior distributions at two times, 0 and \( t \). Via equation (2), the initial preference distribution corresponding to the preference profile \( \Pi_0 \) determines the equilibrium behavior profile \( \hat{\chi}_0 \), whose distribution is also shown. Each of the agents \( \alpha, \beta, \gamma \) and \( \delta \) chooses a behavior situated closer than his personal preference to the mean \( E \Pi_0 \). In the aggregate, the behavior distribution is less dispersed about \( E \Pi_0 \) than is the preference distribution.

Over time, each agent’s preference adjusts in the direction of his equilibrium behavior, according to the law of motion (P):

\[
\frac{d}{dt} \Pi_t(\alpha) = \hat{\chi}_t(\alpha) - \Pi_t(\alpha).
\]

The rates of change for agents \( \alpha \) through \( \delta \) are represented by arrows beneath the distributions. Since each agent's preference moves in the direction of \( E \Pi_0 \), the preference distribution is less diffuse at time \( t \) than at time 0. This concentration causes a corresponding change in equilibrium behavior, which leads to further preference adjustments. In the limit, all preferences and behaviors converge to the initial mean preference \( E \Pi_0 \); the cultural community becomes homogeneous in terms of both preferences and behaviors.

5. Interactions across Cultures

To explore the dynamics of cultural integration, we now introduce interactions among agents divided into multiple communities. For the time being, we continue to assume that the sets of agents in each community are fixed. The analysis will subsequently be extended to a setting in which the communities grow over time.

---

22 These distributions should not be confused with the preference and behavior profiles, which are maps from the unit interval to the real line.
5.1 Communities of Fixed Size

Let $A^1 = [0, m^1]$ and $A^2 = [0, m^2]$ be the sets of agents in communities 1 and 2; $m^i$ is thus the total mass of community $i$. The communities’ preference profiles at time $t$ are $\Pi_t^1: A^1 \to \mathbb{R}$ and $\Pi_t^2: A^2 \to \mathbb{R}$, and their action profiles are $X_t^1: A^1 \to \mathbb{R}$ and $X_t^2: A^2 \to \mathbb{R}$. Average preferences are $E\Pi_t^1 = \frac{1}{m^1} \int_{0}^{m^1} \Pi_t^1(\alpha) \, d\alpha$ and $E\Pi_t^2 = \frac{1}{m^2} \int_{0}^{m^2} \Pi_t^2(\alpha) \, d\alpha$, and average actions $EX_t^1 = \frac{1}{m^1} \int_{0}^{m^1} X_t^1(\alpha) \, d\alpha$ and $EX_t^2 = \frac{1}{m^2} \int_{0}^{m^2} X_t^2(\alpha) \, d\alpha$.

As in the single-community case, agents interact in pairs, and person receives two distinct payoffs. But now the importance an agent attaches to coordination depends on both his own communal affiliation and that of his partner. When interacting with an “insider”, the weight that an agent from community $i$ attaches to his coordination payoff is the within-group conformity parameter $w^i$. If this agent has preference parameter $\pi$ and plays action $x$ while his partner plays action $x'$, his total payoff is

$$u^{ii}(x, x', \pi) = \left(-w^i(x-x')^2\right) + \left(-(x-\pi)^2\right).$$
If instead the partner is from community \( j \), the weight our agent attaches to coordination is given by the \textit{across-group conformity parameter} \( a^j \). Accordingly, his total payoff is

\[
u^{ij}(x, x', \pi) = (-a^j(x - x')^2) + (-x - \pi)^2.
\]

It is natural to posit that \( w^i \geq a^j \). Communities are less tolerant of internal differences than of differences across communities. Indeed, the very concept of a community presumes greater commonality among insiders than between them and outsiders; and, as we will see later, members who deviate substantially from the communal norm often endure heavy conformist pressures. By contrast, outsiders are expected to behave differently, so their “deviance” does not necessarily induce retribution.\(^{23}\) The effect of such differentiation is to make agents relatively more eager to coordinate with members of their own community than with those of other communities. We also assume that \( a^j > 0 \). This ensures that coordination across communal boundaries, while pursued less vigorously than internal coordination, is still considered beneficial.

In our single-community model, agents could not tailor their behavior to the specific partner in a random match. Now we modify this assumption, allowing agents to condition behaviors on their partner’s communal affiliation. What makes conditioning more feasible in this context is the existence of settings in which minority members can feel assured of interacting exclusively with insiders. For instance, many immigrants make a point of shopping at stores and eating at restaurants owned by co-ethnics, thus creating spaces relatively free of natives. Indeed, cultural activists eager to limit outside influences try to create segregated spaces that limit temptations to make behavioral compromises. Picnics, parties, festivities, and religious instruction organized by ethnic activists for their fellow ethnics offer examples of such efforts. Even without deliberate segregation campaigns, communities may enjoy culturally exclusive spaces if they are residentially segregated—think of New York’s Chinatown and of Berlin’s heavily Turkish Kreuzberg district.

To capture the existence of segregated spaces, we allow interactions in three distinct locations. Each community lives in a separate neighborhood, where its members have little need to accommodate the preferences of the other group. In addition to these two

\(^{23}\) A co-author of this paper was in Tunisia for a conference, and a number of the participants were invited to dinner at the home of a professor known to shun alcohol for religious reasons. An American guest showed up with a bottle of wine, which prompted the Tunisian guests to chuckle. Had a Tunisian professor brought the same gift, his faux pas would have triggered derision. For evidence on differentiation according to group affiliation, see Anderson (1991, especially Ch. 2-3 and 8), and Prentice and Miller (1999). Elster (1989, Ch. 3) and Akerlof and Kranton (2000) offer complementary observations.
neighborhoods, there is a city center in which the two groups interact. We suppose that each agent has some interactions in his own neighborhood and some in the center, but none in the other neighborhood. We let $c^i$ represent the percentage of interactions that each group $i$ agent has in the center; the remaining percentage $1 - c^i$ of that agent’s interactions occur in his home neighborhood.

Our agents may behave differently, then, depending upon whether they are in their own neighborhood or in the center. At the same time, to respect our premise that it is costly to switch actions from moment to moment, each agent must choose the same action for all interactions occurring at a single location.

5.1.1 Equilibrium Behavior

To analyze this model, we must first compute equilibrium behavior at each location. Since interactions within each neighborhood are homogenous, Proposition 1 may be used to characterize the neighborhood equilibria. Thus,

1. For $i$ agent in Group 1,
   \[
   \hat{X}_1^i(\alpha) = \frac{1}{w^1 + 1} \left( w^1 \Pi_1^i + \Pi_1^i(\alpha) \right) \text{ for all } \alpha \in A^1;
   \]
   \[
   \hat{X}_2^i(\alpha) = \frac{1}{w^2 + 1} \left( w^2 \Pi_2^i + \Pi_2^i(\alpha) \right) \text{ for all } \alpha \in A^2.
   \]

To analyze behavior at the city center, we assume that all matches are made via independent draws from the individuals present. Since community $i$ is of size $m^i$, and since each of its members has fraction $c^i$ of his interactions in the center, its percentage representation in the center is $\phi^i = m^i c^i / (m^i c^i + m^i c')$. Therefore, the expected utility obtained by a population $i$ agent with preference parameter $\pi$ who plays action $x$ is

\[
U^i(x, X_i^i, X_j^j, \pi) = \phi^i E u_{x'}^i(x, x', \pi) + \phi^j E u_{x'}^j(x, X_j^j, \pi)
= -\phi^i w^i E(x - X_i^i)^2 - \phi^j a^i E(x - X_j^j)^2 - (x - \pi)^2.
\]

With this expression in hand, we can determine equilibrium behavior at the city center.

Proposition 3: Fix the preference profiles $\Pi_1^i$ and $\Pi_2^i$. The unique city center equilibrium is

\[
\hat{X}_i^i(\alpha) = (1 - o^i - p^i) E \Pi_1^i + o^i E \Pi_2^i + p^i \Pi_1^i(\alpha)
\]

for $\alpha \in A^i$ and $i \in \{1, 2\}$. The coefficients in this expression are $1 - o^i - p^i > 0$,

\[
o^i = \frac{m^i c^i a^i}{m^i c^i (a^i + 1) + m^i c'(a^i + 1)} > 0, \quad p^i = \frac{m^i c^i + m^i c'}{m^i c^i (a^i + 1) + m^i c'(w^i + 1)} > 0.
\]
Proof: In the Appendix.

In the unique Nash equilibrium at the city center, each agent’s optimal action is a weighted average of his own community’s mean preference, the mean preference in the other community, and his own personal preference. Note that by taking expectations of both sides of equation (7) and rearranging, we obtain

\[ E\tilde{X}_i^i = (1 - \alpha^i) E\Pi_i^i + \alpha^i E\Pi_j^i. \]

In words, mean behavior in each community is a weighted average of mean preferences in the two populations; \( \alpha^i \) gives the weight placed on the other community’s mean preference. Substituting this expression back into equation (7), we find that

\[ \tilde{X}_i(\alpha) - E\tilde{X}_i^i = p^i (\Pi_i(\alpha) - E\Pi_j^i). \]

Hence, the distance between agent \( \alpha \)'s behavior and his population’s average behavior is proportional to the distance between his preference and his population’s average preference; the ratio between these differences is \( p^i \).

5.1.2 Welfare Implications

Before addressing preference evolution, we explore the welfare implications of changes in the across-group conformity parameter \( a^i \).

Proposition 4: For appropriate choices of \( \kappa > 0 \), we have that

\[
\begin{align*}
(i) \quad & \frac{\partial}{\partial a_i^i} E_a \left[ - (\bar{X}_i^i(\alpha) - \Pi_i^i(\alpha))^2 \right] = -\kappa_1 \left( E\Pi_i^i - E\Pi_j^i \right)^2 - \kappa_2 \text{Var}(\Pi_j^i) \leq 0; \\
(ii) \quad & \frac{\partial}{\partial a_i^i} E_a U_i^i(\bar{X}_i^i(\alpha), \bar{X}_j^i, \bar{\Pi}_i^i(\alpha)) = -\kappa_3 \left( E\Pi_i^i - E\Pi_j^i \right)^2 + \kappa_4 \text{Var}(\Pi_j^i); \\
(iii) \quad & \frac{\partial}{\partial a_i^i} E_a U_j^j(\bar{X}_j^j(\alpha), \bar{X}_i^j, \bar{\Pi}_j^j(\alpha)) = \kappa_5 \left( E\Pi_i^i - E\Pi_j^i \right)^2 + \kappa_6 \text{Var}(\Pi_j^i) \geq 0.
\end{align*}
\]

Proof: In the Appendix.

Part (i) of the proposition establishes that raising community \( i \)'s across-group conformity parameter \( a^i \) lowers its aggregate personal payoffs. However, since increasing \( a^i \) improves community \( i \)'s coordination payoffs both within and across groups,\(^{24}\) the overall impact of this parameter change is less clear. By the envelope theorem, the decline in agent \( \alpha \)'s personal payoffs caused by his change in equilibrium

\(^{24}\) Coordination improves within community \( i \) itself because increasing \( a^i \) reduces the variance in the community’s equilibrium actions.
action is exactly offset by the increase in coordination payoffs due to this change in action. Hence, the effect of increasing $a^i$ on agent $\alpha$’s overall equilibrium utility results from changes in the equilibrium choices of other agents.

Part (ii) shows that in aggregate, the effect of an increase in $a^i$ on community $i$’s payoffs is ambiguous. On the one hand, increasing $a^i$ reduces the variance in community $i$’s behaviors, improving within-group coordination; the larger the variance in community $i$’s preferences, the more significant is this effect. On the other, increasing $a^i$ reduces the degree to which community $j$’s equilibrium actions accommodate community $i$’s preferences, reducing cross-group coordination; the further apart are the mean preferences of the two populations, the more significant is this contribution.

Finally, part (iii) of the proposition shows that increasing community $i$’s across-group conformity parameter is certain to improve community $j$’s aggregate payoffs. Thus, while the effects of increasing $a^i$ on the welfare of community $i$ are subtle, the effects on community $j$’s welfare are unambiguously positive. We will elaborate on these points when discussing policy implications.

5.1.3 Preference Evolution

In modeling preference evolution in a single population, we assumed that each agent’s preferences adjust in the direction of his current equilibrium behavior. Now each agent has two equilibrium behaviors: one for his own neighborhood and another for the center. To address this complication, we first let $\lambda \in (0, 1]$, and then define

$$X^i_t(\alpha) = (1 - \lambda c^i) \hat{X}^i_t(\alpha) + \lambda c^i \bar{X}^i_t(\alpha)$$

as a weighted average of agent $\alpha$’s equilibrium behaviors in his own neighborhood and in the city center. If $\lambda = 1$, the weights equal the percentages of interactions occurring in each location. More generally, the weights on city center behavior are proportional to these percentages but scaled down by a factor of $\lambda$. Using this weighted average, we define preference evolution by

$$(P2) \quad \frac{d}{dt} \Pi^i_t(\alpha) = \bar{X}^i_t(\alpha) - \Pi^i_t(\alpha).$$

As before, each agent’s personal preference moves toward his current “target action”; and as preferences evolve, agents adjust their behaviors to maintain equilibrium play.

Under this specification, the target toward which preferences gravitate depends disproportionately on neighborhood behavior. One motivation is that people are more open to influences stemming from groups with which they identify than to ones from outside groups; for instance, children pay more attention to the behaviors of their
parents and classmates than to those of tourists passing through their neighborhood. Another justification applies to self-persuasion. The psychological burden of an inconsistency between an agent’s preference and behavior is likely to be greatest in his home neighborhood, because that is where he expects to fit in. Although we are requiring only that \( \lambda \) not exceed 1, all these points support positing that \( \lambda \) is small.

Proposition 5 describes each agent’s preference trajectory under the dynamic \((P2)\). By substituting these trajectories into the equilibrium equations derived above, one obtains the corresponding behavior trajectories.

**Proposition 5:** Fix the initial preference profiles \( \Pi^1_0 \) and \( \Pi^2_0 \), and suppose that the sets of agents are fixed. The unique solution to the dynamic \((P2)\) is described by

\[
\begin{align*}
\pi^* &= i^1 \ E\Pi^1_0 + i^2 \ E\Pi^2_0; \\
E\Pi^i_t &= \pi^* + i^1 (E\Pi^i_0 - E\Pi^1_0) \exp(-\rho^* t) \text{ for } i \in \{1, 2\}; \\
\Pi^i_\alpha(t) &= E\Pi^i_t + (\Pi^i_\alpha(0) - E\Pi^i_0) \exp(-\rho^i t) \text{ for } \alpha \in A^i \text{ and } i \in \{1, 2\}.
\end{align*}
\]

The influence levels \( i^1 \) and \( i^2 \) and the convergence rates \( \rho^*, \rho^1, \) and \( \rho^2 \) are given by

\[
\begin{align*}
i^i &= \frac{m^i a^i}{m^i a^i + m^i a^j}, \\
\rho^* &= \frac{\lambda c^j c^2 (m^i a^i + m^i a^j)}{c^2 m^i (a^i + 1) + c^j m^i (a^j + 1)} > 0, \text{ and} \\
\rho^i &= \frac{w^i + \lambda c^1}{w^i + 1} - \frac{\lambda c^i (m^i c^i + m^i c^j)}{m^i c^i (w^i + 1) + m^i c^j (a^i + 1)} > 0.
\end{align*}
\]

**Proof:** See the Appendix.

Among other things, Proposition 5 shows that in the long run, all preferences and behaviors converge to a single point \( \pi^* \), which is a weighted average of the initial mean preferences in each population. Each community’s mean preference converges to the limit value of \( \pi^* \) at rate \( \rho^* \); and the preferences of an individual belonging to community \( i \) converge to the population mean \( E\Pi^i_\alpha \) at rate \( \rho^i \).

Figure 2 illustrates this dynamic for the special case where all interactions occur at the city center \((c^1 = c^2 = \lambda = 1)\). The figure presumes that group 1 is larger than group 2, so that the distribution of its preference profile \( \Pi^1_0 \) is larger than the corresponding distribution for population 2. From these distributions we can derive the equilibrium behavior profiles \( \tilde{X}^1_0 \) and \( \tilde{X}^2_0 \) described in equation (7). These profiles induce preference change according to equation (P2); again, arrows beneath the distributions represent the
underlying forces. Each population’s preferences tend to move, on the whole, toward those of the other population; however, there are agents whose preferences move away from those of the other group.

These changes lead to new preference distributions that lie closer together than the initial distributions. The new distributions induce new equilibrium behaviors via equation (7) and, hence, further preference changes. In the limit, the preferences and behaviors of both populations become concentrated at $\pi^*$.\(^{25}\)

5.1.4 The Comparative Statics of Discontents

Proposition 5 explicitly describes all agents’ preference and behavior trajectories (the latter via equations (5)-(7)). With these solutions, we can analyze completely the effects of changes in the exogenous parameters on the evolution of the cultures.

Formulas (S1)-(S3) describe the preference trajectories in terms of five endogenous parameters: $t^1$, $t^2$, $\rho^*$, $\rho^1$, and $\rho^2$. Differentiating the expressions for these parameters

\(^{25}\) Section 5.2 shows that complete homogenization does not occur if the populations grow over time.
with respect to the exogenous parameters leads to the comparative statics listed in Table I. A glance at the table reveals that nearly every potential influence can be signed.

<table>
<thead>
<tr>
<th>Exogenous parameter</th>
<th>Effect on endogenous parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>$\perp$</td>
</tr>
<tr>
<td>$c_i$</td>
<td>$\perp$</td>
</tr>
<tr>
<td>$c_j$</td>
<td>$\uparrow$ $\downarrow$</td>
</tr>
<tr>
<td>$w_i$</td>
<td>$\perp$</td>
</tr>
<tr>
<td>$w_j$</td>
<td>$\perp$</td>
</tr>
<tr>
<td>$a_i$</td>
<td>$\downarrow$ $\uparrow$</td>
</tr>
<tr>
<td>$a_j$</td>
<td>$(\uparrow$ if $c_1 = c_2)$</td>
</tr>
<tr>
<td>$m_i$</td>
<td>$\uparrow$</td>
</tr>
<tr>
<td>$m_j$</td>
<td>$\downarrow$</td>
</tr>
</tbody>
</table>

Table I: Comparative statics under fixed community sizes

These comparative statics allow us to draw qualitative conclusions about the determinants of the discontents that accompany the assimilation process. Recall from equation (P2) that the process of preference change is driven by differences between an individual’s preferences and his behavior. For reasons explained in Section 2, this process is inherently unpleasant, as it produces internal as well as intergenerational conflict. By examining the rates at which preferences change we can evaluate the magnitude of these discontents.

Let us focus on the rate of change of group $i$’s mean preference, $E \Pi_i$. Differentiating equation (S2) with respect to time, we obtain

$$\frac{d}{dt} E \Pi_i = \rho^* t' (E \Pi_0 - E \Pi_i) \exp(-\rho^* t).$$

Thus, the rate of change of $E \Pi_i$ depends on the initial difference between the groups’ mean preferences, as well as on the endogenous parameters $\rho^*$ and $t'$. Focusing for now on the initial time $t = 0$, we find that the rate of change is increasing in $\rho^*$, the rate of convergence of the groups’ mean preferences to the limit preference $\pi^*$, as well as in $t' = 1 - t'$, the influence of group $j$’s initial preferences on $\pi^*$.

Table I reveals how the model’s exogenous parameters influence $\rho^*$ and $t'$.

---

$^{26}$ We could also examine the rates of change of individuals’ preferences $\Pi_i (\alpha)$. But the rate of change in $E \Pi_i$ captures discontents in the community as a whole: agent $\alpha$’s preferences will change more or less quickly than the mean preference depending on which side of it his preference lies.
Increasing $\lambda$, the weight placed on city center interactions, or $c_i$ or $c'_j$, the proportions of each group's interactions occurring in the city center, increases $\rho^*$ without affecting $t^i$. Also, if $c_i$ and $c'_j$ are equal, then increasing the across-group interaction weight $a_i^j$ increases both $\rho^*$ and $t^j$. To summarize, one can lower the discontents of preference change by reducing the frequency and importance of cross-group interactions. We shall return to this point in exploring the effects of segregation on the assimilation of minority groups (Section 6.3), and in interpreting various forms of cultural protectionism (Sections 6.4 and 6.5).

Equation (8) also points to a tradeoff between the discontents of current and future group members. Considering this equation for different values of $t$, we see that reductions in $\rho^*$ lessen the discontents of current group members at the expense of future group members. We will return to this point too, in considering the differential effects of cultural protectionism on current and future generations (Section 6.4).

Typically the costs of preference change are higher for a member of the minority than for a member of the majority. In our model, this is reflected in the dependence of the limiting preference $\pi^*$, which determines the distance that each group's original preferences will travel, on group sizes. In particular, the smaller a group's mass $m_i^j$, the less influence $t^i$ it has on the limiting preference. Group $i$'s influence also depends positively on the cross-group interaction weight $a_i^j$ and negatively on $a_i^j$. As we shall see, these results are consistent with the tendency for incumbent populations to demand that other groups make behavioral adjustments (Section 6.2), and also with the success of certain campaigns to preserve distinctive minority cultures (Section 6.3).

Interestingly, the limiting preference is independent of $w_i$ and $w'_j$, the within-group conformity parameters: a community with strong norms for internal coordination is no more successful at influencing $\pi^*$ than with one with weak norms. Observe that the coefficient $w_i$ determines the degree of behavioral conformity within community $i$. While a high value of $w_i$ keeps the actions of agents in community $i$ close together, it need not prevent them from reflecting the preferences of outsiders. Indeed, Proposition 3 (specifically, the value of the coefficient $o'_i$) shows that the degree to which group $j$ preferences influence the equilibrium actions of group $i$ is independent of $w_i$. Accordingly, increasing $w_i$ has no effect on the rate at which group $i$ preferences approach those of group $j$, or on the preferences that obtain in the limit.

---

27 Equation (7) shows that the dispersion of community $i$'s equilibrium actions at the city center is given by $\text{Var}(\tilde{X}_i) = \left(p_i\right)^2 \text{Var}(\tilde{X}_i)$, where $p_i$ is a decreasing function of $w_i$. 
5.2 Growing Communities

The preceding model oversimplifies reality by fixing the membership of each community. In practice, communities change through births and deaths as well as immigration and emigration. In the contemporary United States, for instance, the Mexican-American population changes over time not only through natural replacement but also through a steady flow of new immigrants from Mexico. At any given time, therefore, the Mexican-American population includes brand new cohorts, along with older ones that have had time to assimilate to the broader American population. In altering community membership, these factors also sustain cultural diversity, allowing hybridization to proceed without ever resulting in complete homogenization.

The effects of changing community size may be captured in a tractable way by introducing population growth. We now assume that the population masses take initial values \( m^1 \) and \( m^2 \) and grow at rate \( r > 0 \), so that the masses of the populations at time \( t \) are \( m^1 e^{rt} \) and \( m^2 e^{rt} \). Also, at each time \( t > 0 \), the preferences of the time \( t \) entrants to community \( i \) follow a fixed distribution that is independent of \( t \).

To formalize these ideas, we let \( A^i_t = [0, m^i_t e^{rt}] \times [0, 1] \) denote the set of agents in community \( i \) at time \( t \). Since the sets \( A^i_t \) are increasing in \( t \), entrants to a community remain in it forever. The preference profiles for the communities at time \( t \) are \( \Pi^1_i : A^i_t \to \mathbb{R} \) and \( \Pi^2_i : A^i_t \to \mathbb{R} \), and the corresponding action profiles are \( X^1_i : A^i_t \to \mathbb{R} \) and \( X^2_i : A^i_t \to \mathbb{R} \). The average preference and average action in community \( i \) are given by

\[
E\Pi^i_i = \frac{1}{\mu(A^i_t)} \int_{A^i_t} \Pi^i_i(\alpha) d\mu(\alpha) \quad \text{and} \quad EX^i_i = \frac{1}{\mu(A^i_t)} \int_{A^i_t} X^i_i(\alpha) d\mu(\alpha),
\]

where \( \mu \) represents Lebesgue measure on \( \mathbb{R}^2 \).

The initial preference profiles \( \Pi^i_0 \) and \( \Pi^2_0 \) are primitives of the model, as are the time \( t \) preferences of the time \( t \) entrants. To define entrant preferences, we introduce random variables \( \tilde{\Pi}^1 : [0, 1] \to \mathbb{R} \) and \( \tilde{\Pi}^2 : [0, 1] \to \mathbb{R} \) to represent the preference distributions within each new cohort. The initial preference of an agent born at time \( t \) is

\[
\Pi^i_t(\alpha) = \tilde{\Pi}^i_t(\omega) \quad \text{when} \quad \alpha = (m^i_t e^{rt}, \omega) \in A^i_t \quad \text{and} \quad t > 0.
\]

Under these definitions, each community grows at rate \( r \), and the newcomers to community \( i \) at each time \( t > 0 \) have the preference distribution \( \tilde{\Pi}^i_t \), as specified above.

Though the sets of agents have been redefined, the rest of the model is unchanged.\(^{28}\) At each instant, equilibrium behaviors at the three locations are described by equations

\(^{28}\) The base model can be recovered as a special case by setting \( r = 0 \).
Given these behaviors, the preferences of agents alive at time $t$ evolve according to equation (P2). To keep the notation manageable, we rewrite equation (P2) as

$$\frac{d}{dt} \Pi_i^j(\alpha) = \bar{X}_i^j(\alpha) - \Pi_i^j(\alpha)$$

$$= (1 - \lambda c^j) \hat{X}_i^j(\alpha) + \lambda c^i \bar{X}_i^j(\alpha) - \Pi_i^j(\alpha)$$

$$= \sigma^i \bar{E} \Pi_i^j + \delta^i \bar{E} \Pi_i^j - (\sigma^i + \delta^i) \Pi_i^j(\alpha),$$

where the coefficients $\sigma^i$ and $\delta^i$ are given by

$$\sigma^i = \frac{w^i + \lambda c^i}{w^i + 1} - \frac{\lambda c^i c^i d^i m^i}{c^i m^i(a^i + 1) + c^i m^i(a^i + 1)} - \frac{\lambda c^i (m^i c^i + m^i c^i)}{m^i c^i(w^i + 1) + m^i c^i(a^i + 1)}$$

and

$$\delta^i = \frac{\lambda c^i c^i d^i m^i}{c^i m^i(a^i + 1) + c^i m^i(a^i + 1)}.$$

Although we can still derive explicit preference trajectories for each agent, the equations that describe them are complicated. We therefore focus on limits.

**Proposition 6:** Suppose that both communities grow at rate $r$, that the initial preference profiles are $\Pi_0^1$ and $\Pi_0^2$, and that the newcomers' preference distributions are $\bar{\Pi}^1$ and $\bar{\Pi}^2$. Then the dynamic (P2) admits a unique solution. This solution satisfies

$$\lim_{t \to \infty} E \Pi_i^j = \frac{(\delta^i + r)E \bar{\Pi}^i + \delta^i E \bar{\Pi}^i}{\delta^i + \delta^i + r} \equiv E \Pi_\infty^i$$

and

$$\lim_{t \to \infty} \Pi_i^j(\alpha) = \frac{(\sigma^i \delta^i + \sigma^i r + \delta^i \delta^i)E \bar{\Pi}^i + \delta^i (\sigma^i + \delta^i + r)E \bar{\Pi}^i}{(\delta^i + \delta^i + r)(\sigma^i + \delta^i)} \equiv \Pi_\infty^i$$

for all $\alpha \in A_i^i \equiv [0, \infty) \times [0, 1]$. If $E \bar{\Pi}^1 < E \bar{\Pi}^2$, we have that

$$E \bar{\Pi}^1 < E \Pi_\infty^1 < \Pi_\infty^1 < \Pi_\infty^2 < E \Pi_\infty^2 < E \bar{\Pi}^2.$$

**Proof:** In the Appendix.

Thus, if the communities are growing and the preferences of newcomers are exogenous, no longer will all agents’ preferences converge to a single limit point. Rather, preferences of agents in populations 1 and 2 converge to distinct limits, $\Pi_\infty^1$ and $\Pi_\infty^2$, which are different weighted averages of the mean preferences of entrants, $E \bar{\Pi}^1$ and $E \bar{\Pi}^2$.

The constant flow of newcomers sustains preference diversity indefinitely. This is evidenced by the fact that $\Pi_\infty^i$, the limiting preference of each individual in community
\( E i \), differs from \( E i_{\infty} \), the limiting average preference of this community. When much time has passed, the average preference \( E i_{\infty} \) incorporates both the preferences of long-time incumbents, which are near \( i_{\infty} \), and those of relative newcomers, which are dispersed and have a mean near \( E i \). Thus, the average preference \( E i_{\infty} \) lies in between. Since \( E i_{\infty} \) and \( i_{\infty} \) are distinct, preferences remain diverse even in the limit.

Diversity is also sustained across populations. Because each community’s incumbents need to coordinate with its newcomers, the equilibrium behaviors of agents from different communities remain distinct. Since preferences follow behaviors, they remain distinct as well, even in the limit: \( \Pi_{1\infty} \) and \( \Pi_{2\infty} \), the limiting preferences for agents in communities 1 and 2, are different, and the limiting average preferences, \( E i_{1\infty} \) and \( E i_{2\infty} \), lie even further apart.

This analysis shows how continued immigration can help to preserve the multiplicity of cultures. As long as the Mexican-American population of the United States keeps receiving new immigrants, the behaviors and preferences of older Mexican-American cohorts will be “pulled” in the direction of those of the newcomers, limiting the extent of assimilation.

Our previous results suggested that in the absence of population growth or immigration, multiculturalism is incompatible with cultural integration. Policies that promote cultural integration undermine multiculturalism by shrinking and ultimately eliminating the cultural distinctness of individual communities. We now see that a steady flow of new immigrants makes it possible to integrate natives and immigrants to a degree—without destroying, that is, society’s multicultural character. Note, however, that preserving the multiplicity of a society’s cultures does not amount to keeping any particular culture intact. Each will continue to change over time.\(^29\)

6. Extensions and Policy Implications

The model offers rich insights into the broad themes presented at the start of the paper. We now return to them with an eye toward drawing policy implications.

6.1 The Melting Pot

Richard Alba (1990) observes that as an “American culture” emerged out of dozens of “immigrant cultures”, the behaviors and preferences of the early immigrant communities converged: today, marriages between Americans of different European origins are rarely considered intermarriages. This assimilation process began to unfold at a time when powerful government and civic leaders actively promoted

\(^{29}\) Jones (2006, Ch. 6) offers complementary observations.
“Americanization”, and counter-policies to preserve ancestral cultures were relatively weak. As our model would predict, interactions among immigrants gave way to a single hybrid culture.\textsuperscript{30} It is relevant, of course, that by the early twentieth century the great waves of European immigration were over. Had they continued, the mingling of cultural traits would have been limited, leaving substantial differences between, say, Italian-American and Irish-American cultures.

6.2 Assimilation Pressures and Collective Responses

In our model, preference changes driven by behavioral compromises produce intrapersonal and interpersonal stresses. The individuals making the compromises would be better off if they could benefit equally from their new interactions \textit{without} having to make behavioral adjustments themselves. Such would be the case were the other group culturally more similar. Because of the influence of behavior on preferences, agents can benefit from forcing other groups to make the requisite behavioral adjustments.

This observation helps explain why immigrants are often pressured to view themselves first and foremost as members of their host society. President Woodrow Wilson legitimized conformist pressures on Americans who clung to their identities as members of ethnic groups: “You cannot become thorough Americans if you think of yourselves in groups. America does not consist of groups. A man who thinks of himself as belonging to a particular national group in America has not yet become an American.”\textsuperscript{31} At the time, the early 20\textsuperscript{th} century, immigrants who opted for assimilation were rewarded with promotions and status. In criticizing immigrants trying to preserve an ethnic identity, Wilson spoke for already-assimilated Americans who wanted immigrants to carry the burdens of the integration process. His criticism amounted to collective action aimed at protecting the host culture.

In stigmatizing behaviors associated with immigrants, the majority imposes costs on individuals who exhibit them during interactions with the majority. Our model can capture such conformist pressures by positing a large value for the across-group conformity parameter $a_i$ of the immigrant group $i$. By the logic of our analysis, the consequent conformist pressures induce immigrants to make behavioral compromises, lowering the incumbent population’s costs from cross-cultural interactions. Shifting the burdens of compromise to immigrants also hastens their assimilation into the host culture, and it reduces their influence on the ultimate hybrid culture.

\textsuperscript{30} One major group that may appear to have been excluded from this “cultural melting pot” is African-Americans. Section 6.3 addresses this important case.

6.3 Cultural Segregation and Policies to Preserve Cultural Distinctness

Just as groups that stand to gain from making others bear the burdens of cultural convergence can engage in collective action, groups who would carry disproportionate burdens can try to block the convergence process. Thus, there are African-American leaders who encourage their followers to differentiate themselves from other Americans. Akerlof and Kranton (2000) suggest that their campaigns resonate particularly with African-Americans lacking the resources to succeed according to “mainstream” ideals. Such individuals defiantly mark themselves as “different”, creating separate cultures in which their skills are more valued and their consumption patterns, linguistic particularities, and lifestyles enjoy greater acceptance.32 In the context of our model, disadvantaged African-Americans may be viewed as a small minority expected to conform more or less fully to the majority’s norms in schools, workplaces, and other public settings. As our comparative statics indicate, their dissonance from majority-minority interactions is likely to be particularly strong. Accordingly, they form a constituency that is unusually responsive to movements of resistance to cultural convergence.33

In our model, such resistance is reflected in low or even negative values of the across-group conformity parameter $a'$. As we showed in Section 5.1.4, lowering this parameter limits the accommodations made by minority group members to the majority culture. It thus slows the process of assimilation and limits the discontents generated by this process.

African-Americans are themselves highly diverse, and in general African-American behaviors and tastes lie much closer to those of other Americans than to those of members of other societies. The differences that exist are rooted partly in forced racial segregation. Segregation is represented in our model by low values of the parameter $c'$, the proportion of minority group interactions occurring in the city center. As Section 5.1.4 demonstrates, lowering $c'$ reduces the rate of minority assimilation. Forced integration, represented by higher values of $c'$, has the opposite effect. These predictions are testable by comparing preferences across racial groups for the military and civilian populations separately: the armed forces maintain a strict policy of non-segregation, in contrast with substantial racial segregation in civilian life. In line with our analysis, the preferences of African-Americans in uniform lie significantly closer to those of the

32 See Montgomery (1994) for a formal model of this phenomenon.
33 Portes and Rumbaut (2001) document the diversity of immigrant experiences in the United States. They find that Americans have assimilated and continue to assimilate most immigrant groups, even as others remain persistently “different” because of “reactive identity formation”.
military as a whole than the preferences of non-military African-Americans do to those of the overall non-military population.\textsuperscript{34}

6.4 Multiculturalism and Integration

Our analysis speaks to two of the ideals that define the politics of our age: multiculturalism and integration. Most variants of multiculturalism find virtue not only in tolerating cultural diversity but also in preserving its existing manifestations. Others go further, in that they consider cultural diversity per se a basic source of prosperity.\textsuperscript{35} For their part, promoters of integration believe that when interactions occur without regard to ethnicity or creed, society reaps benefits, including economic gains. Civil rights laws and anti-discrimination statutes are motivated not only by considerations of fairness, but also by the belief that social integration promotes economic efficiency (Frederickson (1999)).

It is often taken for granted that multiculturalism and integration are mutually compatible. Our model provides a reason to be skeptical. If integration proceeds naturally, decentralized attempts at interpersonal coordination will result in cultural hybridization. Marriage norms, linguistic conventions, and other social patterns of the communities will become increasingly similar. Therefore, pre-existing cultures will fade away, and cultural diversity will diminish, except insofar as natural population growth and immigration augment heterogeneity. Conversely, forced multiculturalism is feasible only if cross-cultural accommodations are somehow blocked. The necessary restrictions may be achieved by segregating communities, or by dampening individual drives to coordinate with outsiders. Without such barriers, cross-cultural interactions fuel cultural integration.

Why, then, does multiculturalism enjoy a constituency? Our model suggests that cultural integration is an asymmetric process that makes minorities shoulder disproportionate adjustment costs. Viewed in this light, cultural protectionism appears as a vehicle for limiting these immediate costs. By reducing the rate their own community’s assimilation, minority group members can lower their own adjustment costs, shifting the burden partly onto future generations who, because they will have been socialized differently, can adjust at lower cost.

6.5 Cultural Globalization

The process of cultural integration is not limited to geographically circumscribed

\textsuperscript{34} See Moskos and Butler (1997). Self-selection into the military contributes to this pattern.

\textsuperscript{35} Goldberg (1994) offers a spectrum of such arguments that vary in their positions regarding the preservation of existing cultures. See also Barber (1995) and Rao and Walton (2004).
regions. National cultures develop common traits through mutual exposure, and coordination benefits form one of the underlying motives: the spread of English football as a standardized spectator sport and the standardization of the world’s hospitality industry are manifestations of consumption coordination on a global scale.

Cultural globalization fuels tensions akin to those that accompany cultural integration within nation-states. A common complaint is that the influences of national cultures are asymmetric. Many French politicians charge that France is turning into “McFrance”—an allusion to the crushing influence of American popular culture, as symbolized by McDonald’s franchises that are altering eating habits and the food service industry. The efforts of these politicians are akin to those aimed at increasing the cultural assertiveness of American ethnic groups. Just as separatist ethnic leaders represent constituencies with high coordination costs, so certain opponents of globalization speak for people who suffer from economic or cultural dislocation.

7. Concluding Remarks

Our model predicts that cultural integration will continue both within and across political boundaries. As a practical matter, this means that in coming decades today’s cultures will undergo major transformations; efforts to protect existing cultures from foreign influences seem doomed to fail. At the same time, cultural integration will induce conflicts within and among countries. Indeed, today’s political instabilities are rooted partly in tensions fueled by cultural integration. These tensions have a rational basis, as do competing movements that attempt to shape cross-cultural influences.

Whether a given process of cultural integration is economically beneficial depends on the traits themselves. Some outcomes are simply matters of taste, with limited implications for wealth creation or distribution. Others have enormous economic implications. The homogenization of culinary practices, marriage and family size patterns, work norms, and civic mindedness have consequences for health, productivity, and distributions of income. There may exist, then, strictly economic justifications for policies that control the pace or nature of cultural integration. Nevertheless, the process itself is unstoppable.

36 As Wolf (2000) notes, the term globalization has many meanings. Some convey technological interdependence, others economic interdependence, still others the limited cultural significance of national borders. These phenomena are mutually supportive.

37 Rodrik (1997) stresses complementary tensions that arise through increased economic competition and diminished political control.

38 Cowen (2002) demonstrates that cultural globalization has been under way since time immemorial, and that cultures that cultural protectionists depict as pure are in fact hybrid cultures formed through previous cross-cultural interactions.
Appendix

Proof of Proposition 3: Proposition 3 follows by applying the next result with

$$ s^i = \frac{m'c^i}{m'c^i + m'c^j} w^i \quad \text{and} \quad d^i = \frac{m'c^j}{m'c^i + m'c^j} a^i. $$

Proposition A1: Fix $\Pi^i_1$ and $\Pi^i_j$, and consider a single location game in which player $\alpha$'s expected utility from choosing action $x$ is

$$ -s^i E(x - X^i_1)^2 - d^i E(x - X^i_2)^2 - (x - \Pi^i_1(\alpha))^2. $$

The unique equilibrium of this game is

$$ \hat{X}^i_1(\alpha) = (1 - \frac{d^i}{d^i + d^j + 1} - \frac{1}{s^i + d^i + 1}) E\Pi^i_1 + \frac{d^i}{d^i + d^j + 1} E\Pi^i_j + \frac{1}{s^j + d^j + 1} \Pi^j_1(\alpha). $$

Proof: If $(\hat{X}^i_1, \hat{X}^j_2)$ is an equilibrium of this game, each equilibrium action $\hat{X}^i_1(\alpha)$ of each agent $\alpha \in A^i$ must satisfy the first order condition

$$ -2 \left[ s^i (\hat{X}^i_1(\alpha) - E\hat{X}^i_1) + d^i (\hat{X}^j_1(\alpha) - E\hat{X}^j_1) + (\hat{X}^j_1(\alpha) - \Pi^i_1(\alpha)) \right] = 0. $$

Rearranging this equation yields

(A1) \hspace{1cm} (s^i + d^i + 1) \hat{X}^i_1(\alpha) = s^i E\hat{X}^i_1 + d^i E\hat{X}^j_1 + \Pi^i_1(\alpha).

Taking expectations and rearranging again yields

$$ E\hat{X}^i_1 = \frac{1}{d^i + 1} (d^i E\hat{X}^j_1 + E\Pi^i_1(\alpha)). $$

Performing the same computation for population $j$ and substituting gives us

$$ E\hat{X}^j_1 = \frac{1}{d^j + d^i + 1} ((d^j + 1)E\Pi^i_1 + d^i E\Pi^j_1), $$

which implies that

$$ E\hat{X}^i_1 = \frac{1}{d^i + d^j + 1} ((d^j + 1)E\Pi^i_1 + d^i E\Pi^j_1). $$

Substituting this expression for $E\hat{X}^i_1$ and the corresponding one for $E\hat{X}^j_1$ into equation (A1) and rearranging the result proves the proposition. ■
Proof of Proposition 4: To begin, we compute the signs of certain derivatives of the parameters used in equation (7) to describe equilibrium behavior:

\[ \frac{d\varphi}{da} = \frac{m_c e^c [m_c e^c (a^c + 1) + m_c e^c (a^c + 1)]}{[m_c e^c (a^c + 1) + m_c e^c (a^c + 1)]^2} > 0; \quad \frac{d\varphi}{d\alpha} = -\frac{m_c e^c m_c e^c}{[m_c e^c (a^c + 1) + m_c e^c (a^c + 1)]^2} < 0; \]

\[ \frac{d\varphi'}{da} = -\frac{m_c e^c [m_c e^c (a^c + 1) + m_c e^c (a^c + 1)]}{[m_c e^c (a^c + 1) + m_c e^c (a^c + 1)]^2} < 0; \quad \frac{d\varphi'}{d\alpha} = 0. \]

To prove part (i), first observe that

\[ \frac{d}{da} (\bar{X}_i^j(\alpha) - \Pi_i^j(\alpha))^2 = 2(\bar{X}_i^j(\alpha) - \Pi_i^j(\alpha)) \frac{d}{da} \bar{X}_i^j(\alpha) \]

\[ = 2 \left( (1 - o^j - p^j)\Pi_i^j + o^j \Pi_i^j + (p^j - 1)\Pi_i^j(\alpha) \right) \left( \frac{d\varphi'}{da} (\Pi_i^j(\alpha) - \Pi_i^j) + \frac{d\varphi}{da} (E\Pi_i^j - E\Pi_i^j) \right). \]

Then differentiating under the integral sign yields

\[ \frac{d}{da} E \left[ -(\bar{X}_i^j(\alpha) - \Pi_i^j(\alpha))^2 \right] = -E \left[ \frac{d}{da} (\bar{X}_i^j(\alpha) - \Pi_i^j(\alpha))^2 \right] \]

\[ = -2 \frac{d\varphi'}{da} (1 - o^j - p^j)\Pi_i^j + o^j \Pi_i^j + (p^j - 1)\Pi_i^j(\alpha) \]

\[ + 2 \frac{d\varphi}{da} (p^j - 1) \left( (\Pi_i^j)^2 - (E\Pi_i^j)^2 \right) \]

\[ = -2 \frac{d\varphi'}{da} o^j \Pi_i^j + o^j \Pi_i^j(\alpha)^2 - 2o^j \Pi_i^j(\alpha) \Pi_i^j \]

\[ + 2 \frac{d\varphi}{da} (p^j - 1) \left( (\Pi_i^j)^2 - (E\Pi_i^j)^2 \right) \]

\[ = -2o^j \frac{d\varphi}{da} \Pi_i^j + \Pi_i^j(\alpha)^2 \]

\[ = -\kappa (\Pi_i^j - E\Pi_i^j)^2 - \kappa_2 Var(\Pi_i^j) \leq 0. \]

To begin the proof of part (ii) we compute \( \frac{d}{da} U_i^j(\bar{X}_i^j(\alpha), \bar{X}_i^j, \bar{X}_i^j, \Pi_i^j(\alpha)) \). Since agent \( \alpha \)'s equilibrium action \( \bar{X}_i^j(\alpha) \) is chosen to maximize this expression, the partial derivative of \( U_i \) with respect to its first component is zero when evaluated at \( \bar{X}_i(\alpha) \). Thus, to compute \( \frac{d}{da} U_i \) we need only consider the effect that a change in \( a^i \) has on \( U_i \) through its influence on opponents' equilibrium behaviors \( \bar{X}_i \) and \( \bar{X}_j \). (This reasoning is simply the envelope theorem.) Thus, using \( \beta \) and \( \gamma \) to denote arbitrary members of groups \( i \) and \( j \), respectively, we find that

\[ \frac{d}{da} U_i^j(\bar{X}_i^j(\alpha), \bar{X}_i^j, \bar{X}_i^j, \Pi_i^j(\alpha)) \]

\[ = 2\phi^i \varphi^i E_{\beta} (\bar{X}_i^j(\alpha) - \bar{X}_i^j(\beta)) \frac{d}{da} \bar{X}_i^j(\beta) + 2\phi^i \varphi^i E_{\gamma} (\bar{X}_i^j(\alpha) - \bar{X}_i^j(\gamma)) \frac{d}{da} \bar{X}_i^j(\gamma) \]

\[ = 2\phi^i \varphi^i p^j E_{\beta} (\Pi_i^j(\alpha) - \Pi_i^j(\beta)) \left[ \frac{d\varphi'}{da} \Pi_i^j(\beta) - E\Pi_i^j \right] + \frac{d\varphi}{da} (E\Pi_i^j - E\Pi_i^j) \]

\[ + 2\phi^i \varphi^i p^j E_{\gamma} (\bar{X}_i^j(\alpha) - \bar{X}_i^j(\gamma)) \frac{d\varphi'}{da} (E\Pi_i^j - E\Pi_i^j) \]
\[ = 2\phi'w^i p^i \left[ -\frac{d\alpha}{da^i} \text{Var}(\Pi^i) + \frac{d\alpha}{da^i} \left( \Pi^i(\alpha) - E\Pi^i \right) \right] \]
\[ + 2\phi' a^i \frac{d\alpha}{da^i} \left( \tilde{X}^i(\alpha) - E\tilde{X}^i \right) \left( E\Pi^i - E\Pi^i \right). \]

Averaging over all agents \( \alpha \in A^i \), we conclude that
\[ \frac{d}{da^i} E_a U^i(\tilde{X}^i(\alpha), \tilde{X}^i, \tilde{X}^i, \tilde{\Pi}^i) = E_a \left[ \frac{d}{da^i} U^i(\tilde{X}^i(\alpha), \tilde{X}^i, \tilde{X}^i, \tilde{\Pi}^i) \right] \]
\[ = 2\phi' a^i \left( E\tilde{X}^i - E\tilde{X}^i \right) \left( E\Pi^i - E\Pi^i \right) - 2\phi' \omega p^i \frac{d\alpha}{da^i} \text{Var}(\Pi^i) \]
\[ = 2\phi' a^i \left( 1 - o^i - o^i \right) \left( E\Pi^i - E\Pi^i \right)^2 - 2\phi' \omega p^i \frac{d\alpha}{da^i} \text{Var}(\Pi^i) \]
\[ = -\kappa_3 \left( E\Pi^i - E\Pi^i \right)^2 + \kappa_3 \text{Var}(\Pi^i). \]

To prove part (iii), we use the envelope theorem as in the proof of part (ii):
\[ \frac{d}{da^i} U^i(\tilde{X}^i(\alpha), \tilde{X}^i, \tilde{X}^i, \tilde{\Pi}^i) \]
\[ = 2\phi' w^i E_y(\tilde{X}^i(\alpha) - \tilde{X}^i(\gamma)) \frac{d}{da^i} \tilde{X}^i(\gamma) + 2\phi' a^i E_{\beta}(\tilde{X}^i(\alpha) - \tilde{X}^i(\beta)) \frac{d}{da^i} \tilde{X}^i(\beta) \]
\[ = 2\phi' w^i p^i \frac{d}{da^i} \left( \Pi^i(\alpha) - E\Pi^i \right) \left( E\Pi^i - E\Pi^i \right) \]
\[ + 2\phi' a^i E_{\beta} \left[ \left( \tilde{X}^i(\alpha) - \tilde{X}^i(\beta) \right) \frac{d}{da^i} \left( \Pi^i(\beta) - E\Pi^i \right) + \frac{d}{da^i} \left( E\Pi^i - E\Pi^i \right) \right] \]

Averaging over all agents \( \alpha \) in population \( i \), and reversing the order of integration to obtain the third equality, we conclude that
\[ \frac{d}{da^i} E_a U^i(\tilde{X}^i(\alpha), \tilde{X}^i, \tilde{X}^i, \tilde{\Pi}^i) = E_a \left[ \frac{d}{da^i} U^i(\tilde{X}^i(\alpha), \tilde{X}^i, \tilde{X}^i, \tilde{\Pi}^i) \right] \]
\[ = 2\phi' a^i E_y E_{\beta} \left[ \left( \tilde{X}^i(\alpha) - \tilde{X}^i(\beta) \right) \frac{d}{da^i} \left( \Pi^i(\beta) - E\Pi^i \right) + \frac{d}{da^i} \left( E\Pi^i - E\Pi^i \right) \right] \]
\[ = 2\phi' a^i E_{\beta} \left[ \left( 1 - o^i - o^i \right) E\Pi^i + \left( o^i - (1 - o^i - p^i) \right) E\Pi^i - p^i E\Pi^i(\beta) \left( \Pi^i(\beta) - E\Pi^i \right) \right] \]
\[ + 2\phi' a^i \frac{d}{da^i} \left( 1 - o^i - o^i \right) \left( E\Pi^i - E\Pi^i \right)^2 \]
\[ = 2\phi' a^i p^i \frac{d}{da^i} \left( E\Pi^i \right)^2 + 2\phi' a^i \frac{d}{da^i} \left( 1 - o^i - o^i \right) \left( E\Pi^i - E\Pi^i \right)^2 \]
\[ = \kappa_3 \left( E\Pi^i - E\Pi^i \right)^2 + \kappa_3 \text{Var}(\Pi^i) \geq 0. \]

The proofs of Propositions 5 and 6 utilize a well-known formula for the solutions of linear differential equations with a forcing term. Let \( \frac{d}{dt} x_t = Ax_t + b(t) \) be a differential equation on \( \mathbb{R}^n \). The solution to this equation with initial condition \( x_0 \) is
where \( \exp(At) \) and \( \exp(-As) \) are matrix exponentials (Hirsch and Smale (1974, p. 100)).

**Proof of Proposition 5:** To prove Proposition 5, one applies the following result, substituting in the appropriate expressions for \( \sigma^i \) and \( \delta^i \) from equations (9) and (10) into equations (S1’), (S2’), and (S3’) and simplifying the outcome.

**Proposition A2:** Fix \( \Pi^i_0 \) and \( \Pi^j_0 \), and suppose that

\[
\frac{d}{dt} \Pi^i_0(\alpha) = \sigma^i \ E \Pi^j_0 + \delta^i \ E \Pi^j_0 - (\sigma^i + \delta^i) \Pi^i_0(\alpha)
\]

for \( \alpha \in A^i \) and \( i \in \{1, 2\} \), with \( \sigma^i, \delta^i > 0 \). Then the unique solution to this equation satisfies

\[
\text{(S1')} \quad \pi^* = \frac{\delta^2}{\delta^1 + \delta^2} \ E \Pi^1_0 + \frac{\delta^1}{\delta^1 + \delta^2} \ E \Pi^2_0;
\]

\[
\text{(S2')} \quad E \Pi^i_t = \pi^* + \frac{\delta^i}{\delta^1 + \delta^2} (E \Pi^i_0 - E \Pi^i_0) \exp(-\delta^i t);
\]

\[
\text{(S3')} \quad \Pi^i_t(\alpha) = E \Pi^i_t + (\Pi^i_0(\alpha) - E \Pi^i_0) \exp(-\sigma^i t).
\]

**Proof:** We first determine the trajectories of the average preferences in each population by differentiating under the integral sign and substituting in equation (A3):

\[
\frac{d}{dt} E \Pi^1_t = E \left( \frac{d}{dt} \Pi^1_t \right) = \delta^1 (E \Pi^2_t - E \Pi^1_t);
\]

\[
\frac{d}{dt} E \Pi^2_t = E \left( \frac{d}{dt} \Pi^2_t \right) = \delta^2 (E \Pi^1_t - E \Pi^2_t).
\]

This is a linear ODE in \( E \Pi^1_t \) and \( E \Pi^2_t \). Its solution is

\[
E \Pi^1_t = \frac{1}{\delta^1 + \delta^2} [(\delta^2 E \Pi^1_0 + \delta^1 E \Pi^2_0) + \delta^1 (E \Pi^1_0 - E \Pi^2_0) \exp(-\delta^1 + \delta^2 t)];
\]

\[
E \Pi^2_t = \frac{1}{\delta^1 + \delta^2} [(\delta^2 E \Pi^1_0 + \delta^1 E \Pi^2_0) + \delta^2 (E \Pi^2_0 - E \Pi^1_0) \exp(-\delta^1 + \delta^2 t)].
\]

This is an alternate form of equations (S1’) and (S2’).

Now that the trajectories \( \{E \Pi^1_t\}_{t=0} \) and \( \{E \Pi^2_t\}_{t=0} \) are known, we can take them as exogenous when analyzing equation (A3). Doing so makes (A3) a one-dimensional, nonhomogenous ODE for each fixed \( \alpha \). By equation (A2), the solution to this ODE is

\[
\Pi^i_t(\alpha) = \exp(-\sigma^i t) [\Pi^i_0(\alpha) + \int_0^t \exp((\sigma^i + \delta^i) s) (\sigma^i E \Pi^1_s + \delta^i E \Pi^2_s) ds].
\]
Now, equation (S2') implies that
\[
\sigma' E\Pi^i_s + \delta^i E\Pi^j_s = \frac{1}{\delta + \delta'} \left[ (\sigma^i + \delta^i)(\delta^j E\Pi^i_0 + \delta^j E\Pi^j_0) \\ + \delta^i (\sigma^i - \delta^i)(E\Pi^i_0 - E\Pi^j_0) \exp(-(\delta^i + \delta^i)t) \right].
\]
Substituting this expression into equation (A4), we obtain

\[
\Pi^i_t (\alpha) = \exp(-(\sigma^i + \delta^i)t) \Pi^i_0(\alpha) \\ + \frac{1}{\delta + \delta'} \exp(-(\sigma^i + \delta^i)t) \left[ (\sigma^i + \delta^i)(\delta^j E\Pi^i_0 + \delta^j E\Pi^j_0) \int_0^t \exp((\sigma^i + \delta^i)s)ds \\ + \delta^i (\sigma^i + \delta^i)(E\Pi^i_0 - E\Pi^j_0) \int_0^t \exp((\sigma^i - \delta^i)s)ds \right] \\ = \exp(-(\sigma^i + \delta^i)t) \Pi^i_0(\alpha) + \frac{1}{\delta + \delta'} \left[ (\delta^i E\Pi^i_0 + \delta^j E\Pi^j_0)(1 - \exp(-(\sigma^i + \delta^i)t) \\ + \delta^i (E\Pi^i_0 - E\Pi^j_0)(\exp(-(\delta^i + \delta^i)t) - \exp(-(\sigma^i + \delta^i)t)) \right] \\ = (\frac{\delta^i}{\delta + \delta'} E\Pi^i_0 + \frac{\delta^j}{\delta + \delta'} E\Pi^j_0) + \frac{\delta^i}{\delta + \delta'} (E\Pi^i_0 - E\Pi^j_0) \exp(-(\delta^i + \delta^i)t) \\ + (\Pi^i_0(\alpha) - E\Pi^i_0) \exp(-(\sigma^i + \delta^i)t). \\ = E\Pi^i_t + (\Pi^i_0(\alpha) - E\Pi^i_0) \exp(-(\sigma^i + \delta^i)t).
\]
This is equation (S3').

**Proof of Proposition 6:** We seek solutions to

(A5) \[ \frac{d}{dt} \Pi^i_t (\alpha) = \sigma^i E\Pi^i_t + \delta^i E\Pi^j_t - (\sigma^i + \delta^i) \Pi^i_t (\alpha). \]

for all \( \alpha \in A^i_0 \) and \( i \in \{1, 2\} \); the initial conditions \( \Pi^i_0(\alpha) \) for \( \alpha \in A^i_0 \) are given, as are the initial conditions \( \Pi^i_t(\alpha) = \tilde{\Pi}^i_0(\omega) \) for \( \alpha = (m^i e^{rt}, \omega), \ t > 0 \). Once again, we begin by finding the trajectories of the average preference

\[
E\Pi^i_t = \frac{1}{\mu(A^i_0)} \int_{A^i_0} \Pi^i_t(\alpha) \, d\mu(\alpha) = \frac{1}{m^i e^{rt}} \int_0^{m^i e^{rt}} \int_0^1 \Pi^i_t(m^i e^{\omega'}, \omega) \, d\omega \, ds.
\]

To do so, we must derive the law of motion for \( E\Pi^i_t \). Define

\[
z_t^i = \int_0^{m^i e^{rt}} \int_0^1 \Pi^i_t(m^i e^{\omega'}, \omega) \, d\omega \, ds = m^i e^{rt} E\Pi^i_t.
\]

By differentiating under the integral sign and applying Leibniz’ rule, we find that
\[
\frac{d}{dt} z^i_t = \frac{d}{dt} \int_0^{m^i e^{\alpha i}} \int_0^{m^i e^{\alpha i}} \Pi^i_t(m^i e^{\alpha i}, \omega) d\omega ds \\
= \int_0^{m^i e^{\alpha i}} \int_0^{m^i e^{\alpha i}} \frac{d}{dt} \Pi^i_t(m^i e^{\alpha i}, \omega) d\omega ds + r m^i e^{\alpha i} \int_0^{m^i e^{\alpha i}} \Pi^i_t(m^i e^{\alpha i}, \omega) d\omega \\
= \int_0^{m^i e^{\alpha i}} \int_0^{m^i e^{\alpha i}} (\sigma^i E \Pi^i_t + \delta^i E \Pi^j_t - (\sigma^i + \delta^i) \Pi^i_t(m^i e^{\alpha i}, \omega) d\omega ds + r m^i e^{\alpha i} E \tilde{\Pi}^i_t \\
= m^i e^{\alpha i} \left( \sigma^i E \Pi^i_t + \delta^i E \Pi^j_t - (\sigma^i + \delta^i) E \Pi^i_t(\alpha) \right) + r m^i e^{\alpha i} E \tilde{\Pi}^i_t \\
= m^i e^{\alpha i} \left( \delta^i \left( E \Pi^i_t - E \Pi^j_t \right) + r E \tilde{\Pi}^i_t \right); \text{ and so} \\
\frac{d}{dt} E \Pi^i_t = \frac{d}{dt} \left( \frac{1}{m^i e^{\alpha i} z^i_t} \right) \\
= -\frac{r z^i_t}{m^i e^{\alpha i}} + \frac{1}{m^i e^{\alpha i}} \left( m^i e^{\alpha i} \left( \delta^i \left( E \Pi^i_t - E \Pi^j_t \right) + r E \tilde{\Pi}^i_t \right) \right) \\
= -r E \Pi^i_t + \delta^i \left( E \Pi^i_t - E \Pi^j_t \right) + r E \tilde{\Pi}^i_t \\
= \delta^i \left( E \Pi^i_t - E \Pi^j_t \right) + r \left( E \tilde{\Pi}^i_t - E \Pi^j_t \right).
\]

Writing this expression for both \( i = 1 \) and \( i = 2 \) gives us a two-dimensional linear differential equation with a forcing term. Applying equation (A2) and manipulating the result, we obtain the trajectory of \( E \Pi^i_t \):

\[
E \Pi^i_t = \frac{1}{(d^i + d^j + d^i + d^j + r)} \left( d^i \left( E \Pi^i_0 - E \Pi^j_0 \right) + d^j \left( E \tilde{\Pi}^i_t - E \tilde{\Pi}^j_t \right) \right) e^{-(d^i + d^j + r)t} + \frac{1}{(d^i + d^j + d^i + d^j + r)} \left( d^i \left( E \Pi^i_0 - E \tilde{\Pi}^i_t \right) + d^j \left( E \Pi^j_0 - E \tilde{\Pi}^j_t \right) \right) e^{-rt} + \frac{1}{(d^i + d^j + r)} \left( d^i + d^j \right) E \tilde{\Pi}^i_t + d^i E \tilde{\Pi}^j_t.
\]

Taking \( t \) to infinity leaves only the final term of this expression, which is the value of \( E \Pi^i_\infty \) stated in the proposition.

To determine the limiting behavior of each individual agent, one can substitute this expression for \( E \Pi^i_t \) and \( E \Pi^j_t \) back in to the dynamic (A5), use (A2) to solve the resulting forced equation, and then take the limit as \( t \) goes to infinity of this solution; the resulting computation is quite involved, but yields the expression for \( \Pi^i_\infty \) stated in the proposition. One can also find the value of this expression by substituting the limits \( E \Pi^i_\infty \) and \( E \Pi^j_\infty \) into equation (A5); the zero of the resulting expression is the global attractor of (A5). This method yields the value of \( \Pi^i_\infty \) stated in the text as well.
References


Michigan Press.


