Problem Set 2
Due: Tuesday 26 Nov 2013

In this problem set you are asked to solve a dynamic programming problem similar to that (to be) discussed in class. The problem investigates migration decision making with more than two periods and with more than two locations.

Suppose a newly graduated UW–Madison grad (Jodie) is considering whether to remain in Wisconsin or move to one of the four States[1] listed in Table 1. She partitions her lifetime in half with the first–half comprised of three working periods, while the last half represents retirement. Before retirement, Jodie decides where to live. In retirement Jodie makes no decisions; all decisions are made before retirement. There is no discounting, however, utility flow of the retirement (last) period is three times the value of utility in each of the three decision periods. To place the model in a real world context, each of the first three periods represent a 10 year segment of the worker's career, say ages 25 to 55 while retirement is from age 55 to 85.

At time zero (the start of her decision–process), Jodie decides where to live and work for the first period of adulthood (ages 25–34). Since she has yet to start work, Jodie's parents will loan her funds at no interest to cover any moving costs. The parents must be paid back out of first period earnings. Jodie gets to make three subsequent migration decisions. Any moves made in these time periods will be self–financed and paid at the time of the move. Period 3 is the last decision and its purpose is to allow Jodie to move to a location for retirement.

Jodie's preferences are such that during a work period utility is $u = \sqrt{y}+a$, where y is consumption, and a is climate amenity. Consumption equals earnings less any moving costs, $y = w - c$. One period utility is $\sqrt{y} + a$, and in retirement is $\sqrt{3y} + 3a$, reflecting its longer duration.

Jodie realizes her retirement income will depend on her savings, social security (should it continue to exist) and perhaps a pension plan from her employer. But to keep these initial calculations straightforward Jodie assumes that her retirement income is \$1,000,000, *independent* of her retirement location.[2] Jodie recognizes that places may offer different "marriage market" opportunities, but she decides to defer consideration of that complication to another time.

The payoffs (earnings and amenity) for each location are described in Table 1.

Table 1. Earnings and Climate Amenity by Location

		Period Flow during Work		
		Earnings		Climate
Index	Location	Low	High	Amenity
1	California	300,000	500,000	100
2	Florida	100,000	400,000	250
3	Ohio	100,000	700,000	-100
4	Wisconsin	400,000	400,000	0

In constructing Table **??**, Jodie decides that income growth potential is the same across location so she only needs to be concerned with baseline amounts. She has a job offer that lets her remain

[1] I need to distinguish States of the union, and decision states of the dynamic programming representation of Jodie's life cycle residential choice problem. I use the convention that State with an upper case S mean location, while state in all lower case represents a decision process state.

[2] Recall the length of the last period equals the sum of the three work periods. Consequently, at \$1M. her annual income is assumed to be about \$33,000.

in Wisconsin if she so chooses. Consequently she has no uncertainty about her earnings potential in Wisconsin. Jodie however, is uncertain as to how well she will "match" with the other locations. After some reflection, she decides again to keep things simple, and to represent her uncertainty as a binary outcome "low" or "high" with the outcomes having equal probability. Moreover, Jodie considers these outcomes as permanent and represents her earnings whenever she is in the State.[3]

Jodie determines that the first move to a State costs \$15,000 while a return move to a previously visited State is less at \$5,000.[4] The lower cost for a return move reflects Jodie's beliefs that friends and other contacts will enable her to return to an area more cheaply than moving to a new area.

(a) Get a sense of the relative importance of earnings and climate amenities. Take a moment to think how these relative values may influence the optimal sequence of residential choices. [Hint: Think before calculating, as Bellman's curse of dimensionality is ever present.]

(b) At time $t = T = 4$ (retirement, no decision made). Calculate the utility for each State. Importantly, what is the role of savings (or retirement income) in affecting utility during retirement?

(c) Determine the best outcome for $t = 4$; i.e., $V_{t=4} = \max\{U_(c_4, A_j), j = 1, 2, 3, 4\}$.

(d) Now consider $t = 3$ (the last decision period). In part (c) we determined the best retirement location. Assume that Jodie has never worked in this location prior to retirement. Using Bellman's equation determine the payoff from each possible decision from each possible earnings level. Now assume that Jodie has worked in the preferred retirement location (and may be there in $t = 3$). Use Bellman's equation to determine the payoff from each possible decision from each possible earnings level.

(e) The two sets of calculations done in part (d) determine the choice specific payoff from each decision state at time $t = 3$. Find $V_{t=3}(s, d_j), j = 1, 2, 3, 4$ for each state s.

(f) Repeat calculations (c), (d), (e) for $t = 2$.

(g) Repeat calculations (c), (d), (e) for $t = 1$. Remember Jodie has to pay back her parents if she moved at $t = 0$; that is, to start her working career outside of Wisconsin.

(g) Repeat calculations (c), (d), (e) for $t = 0$.

(h) Summary the optimal sequence of residential choices.

Optional: Write a program to solve Jodie's dynamic residential choice problem. Use MATLAB, OCTAVE, Maple, FORTRAN, any variety of C (C++, C#, etc), or Python or Gauss.

[3] To be clear a visit to a State removes all uncertainty. Say Jodie moves to Ohio on the first move and receives a "low" match, 100,000. In *any* subsequent period Jodie's earnings in OH is 100,000.

[4] Say Jodie moves from Wisconsin to the California in the first period. That move will cost \$15,000. If in the second period she moves from California to Florida, New York, or Ohio the cost will be \$15,000 while a return move to Wisconsin costs \$5,000. If in the third period Jodie were to move back to California that move would cost \$5,000. This is to highlight that moving to a previous location is cheaper, not just returning to Wisconsin as "home".