General Equilibrium Model of the Atlantic System by Findlay

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?) presents a general–equilibrium model of the Eighteenth Century Atlantic System that connected Europe, the New World (the Americas and the Caribbean) with Africa. It provides an analytical framework for understanding how changes in one country impacted (via trade) other countries. Findlay's model is deceptively simple: he makes insightful modeling choices that keeps the model manageable without losing key insights.

The purpose of this note is to offer another presentation of the model's graphical solution, Figure 2, in **?**) reproduced in Figure 2. As usual, an understanding of the graphical representation of a model enables straightforward determination of comparative statics (i.e., how the system will move from one equilibrium to another if one of the elements "held constant" were to shift).

Figure 1 presents the basic structure of the model. There are three countries, Europe which produces manufactures, M, (e.g., textiles) using labor, capital (K) and raw materials (R). Europe trades manufactures with the New World and with Africa. The New World uses land (assumed fixed) and slave labor (S) to produce raw material (e.g., unrefined sugar, cotton) that is shipped to Europe. Africa exchanges slaves for manufactures.

Recall that variables determined within the economic model are called *endogenous* while variables determined outside of the model are labeled *exogenous*. The model has four *endogenous* variables: manufactures M, raw materials, R, slaves R, and relative price of raw materials, p (equal to the ratio of the price of raw materials divided by the price of manufactures)[1] A fifth endogenous variable, π the relative price of slaves (π) is determined by S and p. Labor in Europe and land in the New World are exogenous.

Findlay makes a number of assumptions to further simplify the model. First, he assumes that the supply price of slaves is increasing; kidnapping humans takes time and effort so it is costly to kidnap more people (most young males) than fewer. This assumption fully characterizes decisions and behavior in Africa. Second, Findlay assumes that labor (population) in Europe is fixed and land in the New World is fixed. All resources (land and labor) are used in equilibrium as profit maximization requires there to be no idle resources. Third, capital K is supplied perfectly elastically at the interest rate ρ (equal to price consumers must be paid to defer consumption from today to tomorrow). With only a single price of capital implies capital markets are fully integrated and the "the–law–of–one–price" holds. Fixity of land, labor and the perfectly elasticity of K determines the first two quantities outright and implies the (shadow) price of the factors are

[1] The price of manufactures is the nummeraire and equals one. Deciding which good is to be the nummeraire is arbitrary and a matter of convenience. It makes sense to use manufactures as the nummeraire as it is the only commodity that appears in every region.

determined by the model, while the assumption on capital determines its price and the quantity of capital in equilibrium is determined by the endogenous variables.

Notice that production of raw materials depends only on slave labor. He assumes this production function has the standard properties of a positive first derivative (an increase in slaves increases the production of raw materials) and a negative second derivative (the increment in raw material declines as more slaves are employed). Third, Findlay assumes that the production of manufactures follows a particular function form

$$M = \min(F(K, L), R/\alpha) \tag{1}$$

To simplify the exposition let me give a label to $F(K, L)$ and call it Q, $Q = F(K, L)$. Then substituting into (1) we obtain

$$M = \min(Q, R/\alpha) \tag{2}$$

The production function for M is called Leontief after the famous Russian economist who introduced its use. To increase M by one unit requires exactly 1 unit of Q and α units of R. And (importantly) there are no substitution opportunities between Q and R; using 1.1 units of Q and α units of R also produces one unit of M. Hence, the isoquants of a Leontief production function are right angles that emanate along a ray from the origin whose slope is the factor input ratio (1 : α, in $Q - R$ space. Because of the absence of substitution possibilities, factor price changes have only scale effects.[2]

The use of the Leontief production function also means that raw materials and Q vary proportionally with M. That is, if M increases by ΔM units, R must increase by $\alpha \Delta M$ units (and Q must increase by a unit).

Label the northwest quadrant as I and label the others in counterclockwise direction, so quadrant IV is directly below I. Quadrant I is the most complicated so rather than begin the discussion with it, let's consider Quadrant III first. Apply the result that R is proportional to M to draw the ray in Quadrant III. Each unit of M increases R by α units. Quadrant IV is the $R - S$ space (in English the raw materials and slaves quadrant). In quadrant IV we plot the production function mapping slaves (input) to raw materials (output). We have assumed that the production function of raw materials exhibits diminishing marginal returns. Thus, the production function is concave to the origin ($R'' < 0$).

The bottom half of the figure is completed. Consider quadrant II and the curve labeled MM. The curve MM maps out combinations of manufactures and the *relative price of raw materials*. So, MM is not *the* demand curve for M, but it is close. Hold M constant, an increase in the relative price of raw materials increases the cost (and hence supply price) of M. As mentioned above, the Leontief production function implies there is no opportunity to substitute away from the more costly input factor (R). However, as M becomes more expensive consumers will shift their demand away from M and into substitute materials (e.g., from cotton cloth to wool). So, M and p are negatively related as shown in Quadrant II.[3]

[2] In general, a change in the price of an input factor (R) will change the rate of factor utilization as the economic agent will substitute away from the (now) more expensive factor. However, within the Leontief class of production function such substitution is ruled out by the assumed structure of technology. Hence, in equilibrium the capital labor ratio (denoted by Findlay as k) and the capital raw material ratio (K/R) does not vary.

[3] To test your understanding use the "Marshallian Cross" [i.e., the usual Supply and Demand diagram] to represent how MM would appear in the usual partial equilibrium framework of a single commodity, M.

Finally, in Quadrant I, the curve BB represents the increasing supply price of slaves in the production of the raw material. That is, slave traders require a higher price π to supply more slaves. However, slaves are the only (variable) input into the production of R, so a higher (relative) price of slaves translates into a higher (relative) price of raw materials.[4] Thus, the BB curve exhibits a positive relationship between S and p.

The AA curve captures the relationship between the rising supply price of slaves through the derived demand of raw materials and the demand for manufacturers. By exactly the same logic supporting the MM curve, the AA curve is downward sloping as well. Mathematically, this is true, because the production function $R = G(S)$[5] of raw materials has only one variable input. The first derivative is assumed to be strictly positive which implies we can invert the production function, $S = G^{-1}(R)$. And since $dS/dR > 0$ so is its inverse, dR/dS. Hence, the AA curve is downward sloping capturing the intuitive result that the demand for slaves varies inversely with the price of raw materials.

[4]Slavery was abolished in 1807, but persisted in the United States because its slave population *was* self–renewing, as birth rates exceeded death rates. With a sufficiently large stock of slaves it was possible to replace and augment the stock of slaves through reproduction rather than trade. However, with the death rate above the birth rate this is not possible, and the slave trade must occur in every period.

[5]Findlay's notation is $R = R(S)$. The inverse function is confusing, hence my change of notation to distinguish between the quantity R and the function G relating input S to output R.

Figure 1: The Atlantic Trade System

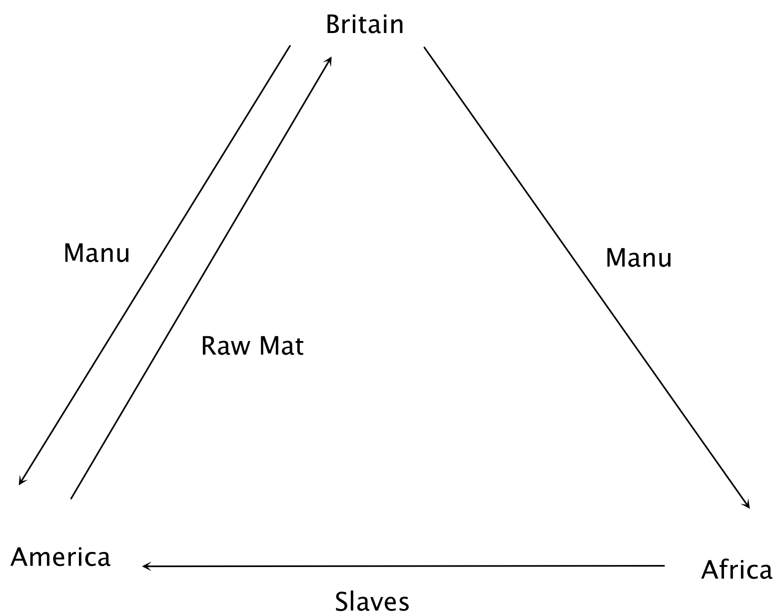

Figure 2: Solution of the System