

Study Problems for Midterm Exam 1

As promised important formulas from the textbook are reproduced below, with their textbook equation number and page number listed to the right.

$$\frac{s}{\theta} = (1 + g^*)(1 + n) - (1 - \delta). \quad (3.6, \text{ p.55})$$

$$\frac{s}{\theta} \approx g^* + n + \delta. \quad (3.7, \text{ p.56})$$

$$(1 + n)k(t + 1) = (1 - \delta)k(t) + sy(t). \quad (3.9, \text{ p. 64})$$

$$\frac{k^*}{y^*} = \frac{s}{n + \delta}. \quad (3.10, \text{ p. 65})$$

$$(1 + n)(1 + \pi)\hat{k}(t + 1) = (1 - \delta)\hat{k}(t) + s\hat{y}(t). \quad (3.12, \text{ p.72})$$

$$\frac{\hat{k}^*}{\hat{y}^*} \approx \frac{s}{n + \pi + \delta} \quad (3.13, \text{ p. 84})$$

$$Y(t) = E(t)^\gamma K(t)^\alpha [uH]^{1-\alpha}. \quad (4.5, \text{ p. 110})$$

$$\frac{E(t+1) - E(t)}{E(t)} = a(1 - u)H. \quad (4.6, \text{ p. 110})$$

$$Y(t) = aK^*(t)^\beta K(t)^\alpha P(t)^{1-\alpha}. \quad (4.10, \text{ p. 113})$$

$$\frac{\Delta Y(t)}{Y(t)} = \sigma_K(t) \frac{\Delta K(t)}{K(t)} + \sigma_P(t) \frac{\Delta P(t)}{P(t)} + \text{TFPG}(t). \quad (4.12, \text{ p. 118})$$

Short problems (5–40 minutes). I attempt to scale problems so that the number of points per minute is approximately constant across questions. Some of these questions (counter to what I said in class) do require a calculator.

In questions asking you to calculate, you must show your work. For discussion and definitional questions be brief; explanations of a sentence or two will be sufficient. (Writing longer incorrect explanations are still wrong and not point-worthy.)

1. What is the human development index? How is it used in development economics?
2. A country is described by the Solow model, with a production function of $y = k^{1/2}$. Suppose that $k = 400$. The fraction of output that is invested is 50%. The depreciation rate is 5%. Is the country at its steady-state level of output per worker, above the steady-state, or below the steady-state? Show how you reached your conclusion.
3. In Country 1 the rate of investment is 5%, and in Country 2 it is 20%. The two countries have the same level of productivity, A , and the same rate of depreciation, δ . Assuming that the value of α is $1/3$ (share of capital) what is the ratio of steady-state output per worker in Country 1 to the steady-state output per worker in Country 2? What would the ratio be if the value of α were $2/3$?
4. Consider an economy in which the amount of investment is equal to the amount of savings (that is, the economy is closed to international flows of capital [as we have always tacitly assumed in class]). Any output that is not saved is consumed. The production function is $y = Ak^\alpha$. Find the value of γ , the fraction of income that is invested that will maximize the steady-state level of consumption per worker. (This is called the golden rule of investment.)

5. **Revised** Suppose that there are no investment flows among countries, so that the fraction of output invested in a country is the same as the fraction of output saved. Saving in the economy is determined as follows: If income per capita is less than \bar{y} the savings rate is $s_1 > 0$, if income per capita is greater than \bar{y} the savings rate is s_2 , and $s_2 > s_1$. Find the steady-states of this economy. Is it possible for there to be a single (stable) steady-state capital stock per capita?
6. How fast would a country have to be growing in order to double its output in nine years?
7. Suppose that in a particular country, GDP per capita was \$1,000 in 1900 and \$4,000 in 1948. Without using a calculator, approximate the annual growth rate of GDP per capita.
8. In 2005, GDP per capita in the United States was \$35,806 while GDP per capita in Sri Lanka was \$4,650. Suppose that income per capita in the United States has been growing at a constant rate of 1.9% per year. Calculate the year in which income per capita in the United State was equal to year 2005 income per capita in Sri Lanka.
9. Economic historians calculate the highest to the lowest per capita income in 1776 was about 2. As discussed in class, in today's economy the ratio of highest to lowest per capita incomes is about 25. Why was the income ratio an order of magnitude smaller in 1776 than today?
10. In the models we've studied in class all income is paid to households, households spend money on consumer goods, where does the market for capital goods come from?
11. The number of people worldwide living on less than a dollar per day can be calculated using either market exchange rates or purchasing power exchange rates. Which will be larger? Explain why.
12. Suppose there are only two goods produced in the world computers, which are traded internationally, and ice cream, which is not. The following table shows information on the production and prices of computers and ice cream in two countries.

Table 1: Richland and Poorland

Country	Production per capita		Prices in Local Currency	
	Computers	Ice Cream	Computers	Ice Cream
Richland	12	4	2	4
Poorland	3	1	1	1

- (a) Calculate the level of GDP per capita in each country, measured in its own currency.
 - (b) Calculate the market exchange rate between the currencies of the two countries.
 - (c) What is the ratio of GDP per capita in Richland to GDP per capita in Poorland, using the market exchange rate?
 - (d) Calculate the purchasing power parity (PPP) exchange rate between the two currencies.
 - (e) What is the ratio of GDP per capita in Richland to GDP per capita in Poorland using the PPP exchange rate?
13. Suppose that two countries, A and B, have the same rates of investment and depreciation, the same levels of productivity, and the same levels of output per worker. They differ, however, in their rates of population growth. The growth rate of population in Country A exceeds that

of Country B. According to the Solow model, which country should have the higher growth rate of output per worker?

14. What is *growth accounting*?
15. What is *development accounting*?
16. Consider the following data on the fictional countries of Sylvania and Freedonia. The production function is $y = Ak^\alpha h^{1-\alpha}$, $\alpha = .5$.

Table 2: Sylvania and Freedonia

Measure	Sylvania	Freedonia
Output per worker y	100	200
Physical capital k	100	100
Human Capital per worker h	25	64

- (a) Calculate the level of productivity A in each country.
 - (b) Calculate the countries' relative levels of output if all differences in output were due to productivity.
 - (c) Calculate the countries' relative levels of output if all differences in output were due to factor accumulation.
17. Data on years of schooling for the working age population are commonly used to proxy human capital in many studies in development accounting. However, countries differ in the number of days of school that make up a school year. Rich countries tend to have more days in their school years than do poor countries. Suppose that instead of data on school years, we used data on school days. How would these new data change our assessment of the role of productivity in explaining variations in output per worker among countries? (Be brief.)
 18. Consider the following data, which apply to countries X and Z in the years 1970 and 2005. In both countries, the production function (in per worker terms) is $y = Ak^\alpha h^{1-\alpha}$, where $\alpha = 1/3$, where as per usual, y is output per capita, k is capital per capita and h is human capita per capita.

Table 3: Countries X and Z

Country	Year	y	k	h
X	1970	100	1	1
	2005	1,200	27	8
Z	1970	50	2	2
	2005	1,200	54	16

Question: Which country had higher productivity growth between 1970 and 2005? Show your work. (You do not need to use a calculator to answer the question.)

19. The following table provides data on output per worker, physical capital per worker, and human capital per worker, all relative to the United States.
 - (a) For each of the three countries listed, calculate factor accumulation and productivity relative to the United States.

Table 4: Sweden, Mauritius and Jordan Relative to U.S.
Per Capita

Country	Output	Physical Capital	Human Capital
Sweden	0.76	0.80	0.93
Mauritius	0.55	0.32	0.73
Jordan	0.18	0.13	0.86

- (b) In which country does factor accumulation play the largest role in explaining income relative to the United States?
- (c) In which which country does productivity play the largest role?
20. The technology for cutting hair has changed very little in the last 100 year, while the technology for growing wheat has improved dramatically. What do you think has happened to the relative incomes of barbers and farmers? Explain how your reasoning.
21. Please briefly discuss the difference between *exogenous* and *endogenous* models of economic growth.
22. Please briefly discuss the difference between economic growth and economic development.
23. From the class lectures and the course textbook give a short description of the history of economic growth. In your answer please be sure to identify countries considered to be rich, those considered to be poor, and some middle income countries. Are there commonalities within each group of countries?

True, False or Uncertain. Please label the following statements as True, False, or Uncertain (i.e., there is insufficient information to determine whether true or false). **You must support your answer; no credit for an unsupported label. Be brief.**

1. According to unconditional convergence, if two countries have the same rate of investment but different levels of income, the country with the lower income will have the higher growth rate.
2. According to unconditional convergence, if two countries have the same level of income but different rates of investment then the country with a higher rate of investment will have lower growth.
3. According to unconditional convergence, a country that raises its savings rate will experience an increase in the rate of income growth, but this growth will only be transitory.
4. According to the Solow model if two countries differ in their level of spending on education but are similar in other respects (and have equal levels of income), then the country with higher educational spending will grow more quickly.
5. In the Solow model with exogenous population growth, per capita income grows at the rate of population growth.
6. In the Solow model with exogenous population growth, the stationary income per capita does **not** depend on the savings rate.
7. In the Harrod Domar an increase in savings increases the growth rate of income per capita.

8. If the population rate is assumed to be **endogenous** then the per capita level of consumption will equal the subsistence level.
9. In an endogenous growth model complementarities say in production may give rise to multiple equilibrium so that two countries with the same initial condition may follow different growth paths.
10. An economy with a higher level of human capita per capita will grow faster than an economy with a lower level of human capita per capita.
11. Growth accounting is useful to identify the proximate determinants of growth.
12. Development accounting is favorable to (now) rich countries.
13. If the production function is $Y = F(K, L)$ and is constant returns to scale and thus homogeneous of degree one, then the per capita production function $y = f(k)$, with $y = Y/L, k = K/L$ is homogeneous of degree zero (i.e., an increase of capital and labor by the same proportion λ has no effect on y).
14. A production function written as $Y = AK^\alpha P^\beta$ exhibits constant returns to scale.
15. In a competitive economy with many firms and consumers it is not possible for the production function of each individual firm to exhibit constant returns to scale while the aggregate production (the sum over all firms) exhibits increasing returns to scale.
16. In the Solow growth model a higher depreciation rate has no effect on the rate of per capita income growth.
17. In the Solow growth model a higher depreciation rate lowers the steady-state per capita income.
18. The human capital model presented in class is an example of an endogenous growth model.
19. In a model with directed technical change it is necessary to assume the innovative firm gains some market power and secures a profit from the innovation.
20. The notion of unconditional convergence implies all countries will converge in the long run to the same level of steady state income.
21. In the Solow model with exogenous population change, an exogenous increase in savings generates transitory growth in per capita income (and consumption) but does **not** generate long run growth.
22. If we condition on the initial level of capital stock and the growth rate of population then conditional convergence means that countries will exhibit a similar level of economic growth, as measured say by per capita income.
23. Per capita income offers a biased measure of well-being because it does not consider such rights as political freedom.
24. The human development index is an alternative measure of economic well-being and when the data are available, a preferred measure of well-being.
25. In a model with endogenous technical change it is possible that an economy may get trapped in a no-growth equilibrium.
26. In a model with endogenous technical change and diffusion of technology the convergence to the equilibrium will be rapid.

27. The ratio of income per capita for the richest country to that of the poorest country today (say circa 2000) is several times larger the ratio of max to min average incomes in say 1750.
28. There is no evidence in favor of unconditional convergence.
29. As discussed in class, there is modest empirical support for conditional convergence.
30. A Cobb–Douglas production function can exhibit constant returns to scale, increasing returns to scale or decreasing returns to scale.
31. In the Harrod–Domar model a higher rate of population growth implies a *lower* rate of economic growth.
32. A *decrease* in the capital–output ratio implies improved efficiency.