IRAs and Household Saving

By William G. Gale and John Karl Scholz*

This paper examines the effects of Individual Retirement Accounts (IRAs) on private and national saving. We construct a formal model of dynamic utility maximization that generates closed-form equations for IRA and other saving. Our empirical estimates indicate that raising the annual IRA contribution limit between 1983 and 1986 would have resulted in little, if any, increase in national saving. Results from sensitivity analysis imply substantially smaller effects on national saving than most previous researchers have estimated. Our results are consistent with new evidence we present indicating considerable potential among IRA holders to shift taxable forms of saving into IRAs. (JEL H20, D12)

In recent years, chronically low levels of U.S. private and public saving have generated considerable concern among academics and policymakers. One frequently suggested method for raising national (public plus private) saving is to expand Individual Retirement Accounts (IRAs). First established in 1974 to help workers without pension plans save for retirement, IRAs featured tax-deductible contributions up to an annual limit, tax-free accrual of interest, and substantial penalties for withdrawal before the account-holder reached the age of 59.5. In 1981, eligibility was extended to all taxpayers, and the contribution limits were raised. In 1986, the tax-deductibility of contributions was curtailed. Congress and the Administration have recently considered several new proposals to expand IRAs and to create similar programs aimed at saving for educational, health, and housing expenses.¹

Although IRAs have been very popular,² the important economic issue is not their use per se, but whether IRAs raise national saving. The effect on national saving is the sum of the effects on public and private saving. Because some contributions are tax-deductible, IRAs will typically reduce public saving. Their effect on private saving is less certain. IRA contributions funded by

¹Currently, the annual contribution limit is the smaller of (i) earned income or (ii) $4,000 for a married couple with two workers, $2,250 for a married couple with one worker, and $2,000 for a single person. The early-withdrawal penalty is 10 percent. If either the account holder or spouse is already covered by a private pension plan, deductibility of contributions is phased out for couples with income in the range of $40,000–$50,000 and for singles with income in the range of $25,000–$35,000.

borrowing or by transferring already existing taxable assets into IRAs will not increase private saving. Similarly, IRA contributions will not raise private saving if they are funded by current-period saving that would have been undertaken even in the absence of IRAs. Conventional economic theory suggests that asset transfers or shifts in the composition of saving that would have been done anyway may occur for households who view IRAs and other saving as good substitutes. For these households, IRAs may actually reduce private saving because they provide a higher rate of return on a limited amount of saving. If the household would save large amounts without an IRA program, the IRA will provide no marginal incentive to save; instead, the private-wealth effects induced by the higher return on IRAs relative to other saving will reduce private saving as long as current consumption is a normal good.

In contrast, contributions funded from resources that would otherwise have been consumed will increase private saving. This will occur for households who view IRAs as poor substitutes for other assets. Thus, whether IRAs raise private saving hinges on the source of IRA contributions, which in turn depends on the substitutability of IRAs and other forms of saving.

Previous research using aggregate data has generated mixed results. Chris Carroll and Lawrence H. Summers (1987) point out that the Canadian personal saving rate rose relative to the American rate in the 1970's when Canada liberalized its tax-deferred retirement saving plan. Jonathan Skinner and Daniel Feenberg (1990) show that in the United States, the relation between aggregate IRA contributions and other saving depends critically on the definition of saving.

Previous research using microeconomic data, including R. Glenn Hubbard (1984),

Steven F. Venti and David A. Wise (1986, 1987, 1988, 1990, 1991, 1992), and Feenberg and Skinner (1989) has established an important empirical regularity: households that contribute to IRAs also tend to save more overall, holding observable factors constant. One interpretation of this result is that IRAs raise the overall level of saving.

However, an alternative interpretation is that there are groups in the population with different preferences for saving, again holding observable factors constant. Peter A. Diamond and Jerry A. Hausman (1984), for example, find large variations in saving propensities across households. These differences complicate the task of measuring the effect of IRAs on saving. To see this, suppose there exist two groups, "large savers" and "small savers," due, say, to differences in subjective discount rates. Holding observable factors constant, we would expect to see that IRA holders (where "large savers" were overrepresented) would save more than households without IRAs (where "small savers" were overrepresented). That is, we would observe the empirical regularity documented in the literature. However, for those households who hold IRAs, IRAs could still be very good substitutes for other assets. In this case IRAs would not raise national saving.

This paper examines interactions between IRAs and other saving. Section I presents new evidence on IRA contributions and contributors. We show that most contributors are either older than 59 or have large amounts of non-IRA financial assets. These households may well find IRAs and other saving to be very good substitutes. In addition, we find sizable differences in the asset holdings of IRA holders and households without IRAs. These results indicate that portfolio reshuffling or shifts in the composition of saving that would have been done anyway could be important phenomena. We also review the previous literature on IRAs.

In Section II, we develop a dynamic model of utility maximization that incorporates several important institutional features of IRAs and leads to closed-form equations for IRA and other saving. The model is used to formalize several insights concern-
ing the effects of IRAs on other saving and to provide a framework for the empirical analysis.

In Section III, we present and estimate an empirical version of the model. We follow the previous formal econometric literature, Venti and Wise (1986, 1987, 1990, 1991), by estimating and simulating the effects of increasing the annual contribution limits from their levels during 1983–1985. Our basic findings indicate that of the increased IRA contributions that would have resulted from increases in contribution limits, roughly 2 percent would represent net additions to national saving, if the accompanying tax cut were entirely saved. If one half of the tax cut were consumed in the first year, this estimate falls to −14 percent. Sensitivity analysis shows some variation, but after controlling for how the tax cut is allocated, our results suggest a significantly smaller effect of IRA limit changes on saving than previous researchers estimated. Section IV provides concluding remarks.

I. Background

A. Data

We use data from the 1983–1986 Survey of Consumer Finances (SCF). The survey contains interviews from a random sample of 3,824 U.S. households in 1983, along with a supplemental survey of 438 high-income households. In 1986, 2,822 of these households were reinterviewed. The SCF was designed specifically to collect data on household balance sheets. It also contains detailed information on demographic characteristics, income, and other variables.4

The SCF provides data on IRA and Keogh balances separately in 1983 and together in 1986. After determining IRA balances in 1983 and 1986, we calculate IRA contributions assuming that annual contributions are constant over 1983–1985 and that IRA accounts earn a return of 14 percent.5 If the calculated annual contribution exceeds the contribution limit (L), we set the contribution equal to L. This can occur because individuals may roll over pension funds into IRAs under certain circumstances, or because our assumed interest rate is too low. Appendix A provides more detail on these calculations.

Due to the importance of the IRA variable to our analysis, we compared IRA contributions constructed from the SCF with data taken from a three-year panel of tax returns covering 1983–1986.6 Summary comparisons are shown in Table 1.7 Average contributions over the three-year period for IRA contributors are $4,814 in the SCF, and $5,161 in the IRS micro data. We match the percentage of households with IRAs

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**Table 1—Comparison of IRS and SCF Data on IRA Contributors, 1983–1985**

<table>
<thead>
<tr>
<th>Variable</th>
<th>IRS data</th>
<th>SCF data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage of population contributing to an IRA at least once</td>
<td>24.0</td>
<td>23.2</td>
</tr>
<tr>
<td>Percentage of IRA contributors who contribute to the limit in all three years</td>
<td>30.8</td>
<td>26.8</td>
</tr>
<tr>
<td>Three-year average contribution (for contributors)</td>
<td>$5,161</td>
<td>$4,814</td>
</tr>
</tbody>
</table>

*Source: Authors’ calculations using 1983–1985 data from the Survey of Consumer Finances and the University of Michigan/Ernst and Young Tax Research Data Base.*

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4The SCF is described in Robert B. Avery and Gregory E. Elliehausen (1988), Avery and Arthur B. Kennickell (1988), and Avery et al. (1988).

5The rate is based on stock and bond yields, reported in the *Economic Report of the President* (Council of Economic Advisers, 1990 tables C-71 and C-93).

6For a more complete description of the University of Michigan/Ernst and Young Tax Research Data Base, see Joel Slemrod (1988).

7The SCF data in Tables 1–3 exclude households if (i) the head is younger than 25 in 1986, because the data are suspected not to be representative of the national sample of such households (Avery and Kennickell, 1988); (ii) the household changed marital status during 1983–1986, because we cannot calculate the IRA contribution limit; or (iii) either the head or spouse is self-employed, because there is insufficient information to disentangle Keogh and IRA contributions accurately (see Appendix A).
very accurately and the percentage of IRA contributors that are constrained by the limit in all three years fairly closely. On the basis of these results, we believe the SCF provides accurate data on IRAs and that our assumptions used to construct IRA saving are reasonable.

B. Asset Substitutability, Characteristics of IRA Holders, and the Reshuffling Hypothesis

An important factor that distinguishes IRAs from other forms of saving is the penalty for early withdrawal. In the absence of transactions costs, any household with a positive marginal tax rate whose members believed with certainty that they would not have to cash in their savings until the age of 59.5 would find IRAs to be a perfect substitute for other saving. Each period, the household would first place all saving into an IRA until the limit was reached; only then would it save in other forms. If the household’s saving exceeded the IRA limit, IRAs would not provide any marginal incentive to save.

One preliminary step toward determining the effects of IRAs is to focus on the characteristics of IRA contributors. If contributors are mainly those for whom IRAs and other saving appear to be very good substitutes, the effects on saving of raising the annual contribution limit should be small or negative. We identify two such groups: those who are already older than 59.5 (and therefore face no early-withdrawal penalty) and those with large amounts of non-IRA financial assets (who therefore may have a reduced need to use all of these assets as a cushion against adverse events). We define non-IRA financial assets (NIFA) to include checking, saving, and money-market accounts, certificates of deposit, stocks, bonds, mutual funds, the cash value of life insurance, and other financial assets. These assets represent relatively liquid funds available for transfer into IRAs.

Table 2 presents data on IRA contributions by various groups during 1983–1985. Almost 70 percent of positive contributions were made by households with heads older than 59 or with NIFA in excess of $20,000. These households, accounted for 78 percent of limit contributors. Almost 50 percent of positive contributions were made by households with heads older than 59 or with NIFA above $40,000. These households accounted for 57 percent of limit contributors.

These facts have important implications. A commonly made argument is that most households cannot easily “reshuffle” other saving into IRAs because the financial assets held by a typical household are very small. For example, Martin Feldstein and Feenberg (1983) show that a majority of the population held less than the annual IRA contribution limit in non-IRA financial assets in their sample. Venti and Wise (1991 p. 124) report that

...financial asset saving of a very large proportion of families is close to zero... The model prediction of little

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8Three-year limit contributors made 68.5 percent of limit contributions. Limit changes could also affect households constrained by the limit in only one or two years; that is, households may move into or out of a constrained state. A limitation of our data is that we must treat households that might have been constrained by the contribution limit for a one- or two-year period as unconstrained households.

9Transactions costs, such as the costs of opening and keeping separate accounts, closing accounts, and understanding IRA rules also distinguish IRAs from other forms of saving.

10Even if the household faces a higher tax rate upon retirement, there will be some threshold holding period, beyond which IRAs will be preferred to other assets due to the tax-free accumulation that IRAs offer.

11Assets do not need to be “cashed in” to be placed into IRAs, so investors need not incur capital-gains taxes to move stocks to IRAs. In addition, assets already in an IRA can be sold, as long as the proceeds stay in the account. Some limited short-term borrowing from IRAs is also allowed without penalty. These considerations enhance the substitutability of IRAs and other assets.

### Table 2—1983–1985 IRA Contributions by Selected Groups

<table>
<thead>
<tr>
<th>Group</th>
<th>Percentage of IRA accounts</th>
<th>Percentage of positive contributions</th>
<th>Percentage of accounts that contributed to the limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Household head &gt; 59 years old&lt;sup&gt;a&lt;/sup&gt;</td>
<td>21.8</td>
<td>16.0</td>
<td>22.5</td>
</tr>
<tr>
<td>(b) 1985 non-IRA financial assets $\geq$ $20,000&lt;sup&gt;b&lt;/sup&gt;$</td>
<td>51.9</td>
<td>62.8</td>
<td>72.9</td>
</tr>
<tr>
<td>(c) 1985 non-IRA financial assets $\geq$ $40,000&lt;sup&gt;b&lt;/sup&gt;$</td>
<td>34.8</td>
<td>42.5</td>
<td>50.2</td>
</tr>
<tr>
<td>(a) or (b)</td>
<td>60.2</td>
<td>68.2</td>
<td>78.5</td>
</tr>
<tr>
<td>(a) or (c)</td>
<td>45.9</td>
<td>49.7</td>
<td>57.3</td>
</tr>
</tbody>
</table>

*Source:* Authors' calculations using the 1983–1986 Survey of Consumer Finances. Data are weighted to represent a cross section of the 1986 population.

<sup>a</sup>Calculated as an average of the statistics for those who were older than 59 in 1982 and those who were older than 59 in 1985.

<sup>b</sup>Non-IRA financial assets are defined as checking, saving, and money-market accounts, certificates of deposit, stocks, bonds, mutual funds, cash value of life insurance, and other financial assets.

The substitution [between IRAs and other saving] is consistent with the descriptive data that show very little non-IRA financial asset saving; there is little to substitute away from.

However, the relevant concern is not with the asset holdings of the average household, but with the asset holdings of IRA holders. Moreover, when the focus of the research is on the effects of IRA limit changes, as in the econometric work of Venti and Wise and this paper, the asset holdings of IRA limit contributors are particularly relevant.

Table 3 presents data on demographic and financial characteristics for various groups in 1986. The data show that there are large differences in incomes, assets, and net worth between households with and without IRAs. Over the three-year period the median IRA holder received over twice as much income and had over eight times the increase in net worth as the median household without an IRA, and in 1986 had over four times as much net worth. The median household with an IRA also held $21,695 in NIFA, seven times the NIFA held by the median household without an IRA. The median household constrained by the IRA limit (and therefore most directly affected by limit changes) held $41,269 in NIFA. Therefore, while it is clear that the median household in the population cannot “reshuffle” assets into an IRA, it is equally clear that the median limit contributor can.<sup>13</sup>

### C. Previous Research

Although some of the differences shown in Table 3 can be explained by differences in observable factors, it is also possible that households with and without IRAs have different determinants of saving. Nevertheless, most research in this area relies on explicit or implicit comparisons of the saving behavior of households with IRAs to that of observationally equivalent households without IRAs.

Using cross-section data, Hubbard (1984) finds that IRA contributors have higher ratios of net worth to income than otherwise identical noncontributors. Feenberg and

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<sup>13</sup>If households borrow to purchase an IRA, the existence of significant amounts of non-IRA financial assets is even less important to the reshuffling argument. There appear to be few restrictions on borrowing to finance an IRA. The 1992 Tax Guide for College Teachers (Allen Bernstein, 1992), for example, devotes a full page to a section, “What If You’re Short of Cash To Fund Your IRA?” (pp. 229–30) that describes an IRS private-letter ruling allowing households to finance their IRAs by borrowing.
TABLE 3—CHARACTERISTICS OF HOUSEHOLDS WITH AND WITHOUT IRAS, 1986

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>All households</th>
<th>Households without IRAs</th>
<th>Households with IRAs</th>
<th>Households that contributed to the limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median age</td>
<td>49</td>
<td>49</td>
<td>50</td>
<td>51</td>
</tr>
<tr>
<td>Percentage married</td>
<td>63.9</td>
<td>58.3</td>
<td>77.9</td>
<td>74.5</td>
</tr>
<tr>
<td>Percentage with pension</td>
<td>46.5</td>
<td>36.9</td>
<td>71.2</td>
<td>73.3</td>
</tr>
<tr>
<td>Average (years) education</td>
<td>12.2</td>
<td>11.6</td>
<td>13.8</td>
<td>14.3</td>
</tr>
<tr>
<td>Average family size</td>
<td>2.7</td>
<td>2.6</td>
<td>2.7</td>
<td>2.6</td>
</tr>
<tr>
<td>Median three-year income, 1983–1985</td>
<td>$63,962</td>
<td>$47,000</td>
<td>$105,000</td>
<td>$133,500</td>
</tr>
<tr>
<td>Median non-IRA financial assets</td>
<td>$6,000</td>
<td>$3,000</td>
<td>$21,695</td>
<td>$41,269</td>
</tr>
<tr>
<td>Median net worth</td>
<td>$42,710</td>
<td>25,470</td>
<td>107,946</td>
<td>188,943</td>
</tr>
<tr>
<td>Median change in net worth, 1983–1986</td>
<td>$6,129</td>
<td>$2,884</td>
<td>$23,500</td>
<td>$60,691</td>
</tr>
<tr>
<td>Number of households</td>
<td>62,824,167</td>
<td>45,099,288</td>
<td>17,724,879</td>
<td>3,871,887</td>
</tr>
</tbody>
</table>

Source: Authors' calculations using the 1983–1986 Survey of Consumer Finances. Data are weighted to represent a cross section of the 1986 population.

aNon-IRA financial assets are defined as checking, saving, and money-market accounts, certificates of deposit, stocks, bonds, mutual funds, cash value of life insurance, and other financial assets.

bNet worth is composed of IRA and KEOGH balances, non-IRA financial assets, and the current market value of homes, other properties, businesses, and vehicles, minus mortgages on homes and other properties, minus credit-card debt, consumer debt, and other debt.

Skinner (1989) use the panel of IRS tax returns and infer from interest and dividend income that, in each of several ranges of initial taxable asset holdings, taxpayers with IRAs raised their taxable financial assets (from 1980–1984) by more than did taxpayers without IRAs.

These results must be interpreted with caution. In particular, unobservable characteristics such as subjective discount rates and risk aversion may differ across households with identical observable characteristics, and these unobservables may be positively correlated with both IRA choices and other saving. Thus, these findings may merely reflect heterogeneity in saving behavior, rather than indicating that IRAs raise the overall level of saving.14

Douglas H. Joines and James G. Manegold (1991) use the IRS panel to compare interest and dividend income of new contributors during 1982–1984 with interest and dividend income of the same households during 1979–1981, before they were eligible to contribute. Joines and Manegold suggest that if the expansion of IRA eligibility stimulated saving, new contributors should have increased their saving more than a “control” group of taxpayers who contributed both before and after the 1981 reform. They find that the comparison provides no evidence that IRA eligibility affects saving. They also present regression evidence from the pooled panel which suggests that the fraction of IRA contributions financed by reductions in consumption is roughly 0.3 percent for the full sample and 30.5 percent if they exclude those households with imputed taxable assets greater than $25,000. Although Joines and Manegold (1991) and Feenberg and

14Feenberg and Skinner (1989) use initial taxable financial income in an attempt to control for the household’s taste for saving. However, holdings of taxable financial assets depend on many other factors, including age, income, previous earnings history, earnings prospects, inheritances, willingness to take risks, marginal tax rates, health, and family size and hence may not adequately reflect tastes for saving.
Skinner (1989) provide important evidence on IRAs, the tax panel has limitations when studying saving issues in that it lacks direct data on wealth and detailed information on household characteristics.

Venti and Wise (1986, 1987, 1990, 1991) estimate that increasing the annual contribution limit would raise IRA saving and that only 3–20 percent of the increased IRA contributions would be financed by reductions in other saving. Roughly 35 percent would be financed by reductions in taxes, and the remaining 45–66 percent would be financed by reductions in consumption. They conclude that little substitution of IRAs and other saving occurs, and that “contributions to IRAs represent substantial net saving increases” (Venti and Wise, 1986 p. 594).

Because Venti and Wise present the only formal model of IRAs in the previous literature, it is worthwhile to highlight several features of their approach. First, consumers are assumed to allocate income to consumption, IRA saving, and non-IRA saving according to the following function:

\[
V = \left[ Y - (1 - t)S_1 - S_2 \right]^{1 - \beta} \\
\times \left\{ \alpha (S_1 - a_1)^k + (1 - \alpha) (S_2 - a_2)^{k^{1/k}} \right\}^\beta
\]

where \( Y \) is disposable income, \( t \) is the marginal tax rate, \( S_1 \) is IRA saving, \( S_2 \) is non-IRA saving, and \( \alpha, \beta, k, a_1, \) and \( a_2 \) are parameters. Although this “allocation-function” approach yields closed-form solutions for IRA and other saving, it is not clear what underlying utility function would be consistent with maximization of (1).

Second, as Angus Deaton (1987) notes, individual attributes that are presumably important determinants of saving behavior (e.g., assets or age) do not have first-order effects in the saving equations. Attributes enter only through interactions (via \( \alpha \) and \( \beta \)) with income and thus affect only the marginal propensity to save rather than the level of saving directly.

Third, the effects of changes in the annual contribution limit depend on parameters (\( k, \alpha, \beta, \) and correlations among the error terms) that are estimated on the whole sample. These effects therefore rest on comparisons, which in this case are complicated and nonlinear, between IRA contributors and otherwise identical households who do not contribute. Consequently, the work is subject to the same problem as that of Hubbard (1984) and Feenberg and Skinner (1989): unless there is a way to ensure that unobservable tastes for saving are not correlated with the decision to buy an IRA, their results may underestimate the degree of substitutability between IRAs and other saving and hence overstate the net saving effect of IRA limit changes.

The importance of this problem can best be seen by examining the argument that for IRAs to be a perfect substitute for other saving, every saver would have to hold an IRA (Venti and Wise, 1990 p. 676). Since only about 20 percent of their sample held IRAs, it is not surprising that they reject the hypothesis of perfect substitutability. However, Tables 2 and 3 show that it is clearly

\[15\] Venti and Wise (1990, 1991) find that about 3 percent of the increase in IRA contributions due to a limit change would come from other assets. These papers are most comparable to the current one in that they use data on the level of non-IRA saving. Venti and Wise (1986, 1987) place the figure at 7–20 percent, but use data on only the sign of non-IRA saving rather than the level.

\[16\] Venti and Wise (1986, 1987, 1990, 1991) assume that the entire reduction in taxes is consumed. In the simulations we report in this paper, we allow households to save all or a portion of the tax reduction.

\[17\] The first term equals consumption and is derived from the budget constraint.

\[18\] Alternatively, if \( V \) is meant to represent preferences explicitly, (1) implies that utility is obtained directly from the level of saving rather than from the quantity of goods consumed.

\[19\] Individual characteristics, \( X \), enter the model through \( \alpha \) and \( \beta \), which in most specifications are restricted to lie between 0 and 1 using the normal distribution: \( \alpha = \Phi(Xa) \) and \( \beta = \Phi(Xb) \).
possible that a large portion of IRA holders (e.g., those with large amounts of NIFA or who are older than 59) find IRAs to be perfect substitutes for other saving, while other households do not. In the next sections, we develop and estimate a model of IRAs and saving that addresses these concerns.  

II. Modeling IRAs and Saving

In this section we present a simple model of IRAs and other saving, where decisions are derived from maximization of a utility function that depends only on current and expected future consumption. The model captures several important features of IRAs: the annual contribution limit, the early-withdrawal penalty, tax-free accrual of interest, and tax-deductible contributions. We use the model to illustrate several analytical issues and to motivate the empirical framework used in Section III.

Consider an individual who lives for three periods. In each period $j$, she chooses consumption ($C_j$), and makes contributions to IRA saving ($S_j$) and other saving ($S'_j$). Non-IRA saving earns $R_o$ (gross) per period. Contributions to IRAs earn $R_1$ (gross) per period if they are held until she reaches the third period and 1 per period if cashed in earlier. Let $R_1 > R_o > 1$. IRA contributions made in the first period yield a lower return than ordinary saving if withdrawn in the second period but yield a higher return than other saving if held until the third period. The maximum IRA contribution allowed per period is $L$.

At the beginning of each period, the consumer receives labor income $Y_j$. We set $Y_3$ to zero for simplicity and assume that $Y_2$ is uncertain and the consumer cannot borrow. Thus, if the realization of $Y_2$ is sufficiently low, the consumer will have to cash in first-period IRA saving early and incur a penalty. There is no other uncertainty in the model; in particular, rates of return are certain. We assume that no transactions costs are associated with IRAs; some implications of relaxing this assumption are noted below.

Define the consumer’s available resources at the beginning of each period as $W_j$. The consumer’s problem in period $j$, $j = 1, 2, 3$, is

$$\max_{(S_j, S'_j)} V_j(W_j) = U(C_j) + \frac{1}{1+\rho} E_j[V_{j+1}(W_{j+1})]$$

subject to

$$S'_o \geq 0 \quad S'_1 \geq 0 \quad S'_2 \geq -S'_o R_o$$
$$S'_2 \geq -S'_1$$

(4a) $Y_j = C_j + S'_o + S'_1(1-t)$ if $S'_1 \geq 0$

(4b) $Y_j = C_j + S'_o + S'_1$ if $S'_1 < 0$

(5a) $W_1 = Y_1$

(5b) $W_2 = Y_2 + S'_o R_o + S'_1$

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The model can be generalized to include borrowing. Suppose the household could borrow at gross rate $R_b$. If $R_b < R_1$, the household would borrow in the first period to finance IRA purchases, and IRAs would not raise saving. If $R_b > (R_1)^2$, the substitutability of IRAs and other saving is the same as if the household could not borrow. The household would prefer to withdraw IRA funds in period 2 rather than borrow. If $R_1 < R_b < (R_1)^2$, the cost of a bad draw of $Y_2$ would be reduced relative to the no-borrowing case, so IRAs would be better substitutes for other saving than predicted by the model.

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21 The penalty for early withdrawal in this framework is the interest earned from period 1 to period 2. Nothing of substance would change if the penalty were a percentage of the early withdrawal, as is actually done.
(5c) \[ W_3 = S_{i_0} (R_0)^2 + S_{o_0} R_O + S_{i_1} (R_1)^2 + S_{i_2} R_1 \quad \text{if} \quad S_{i_2} \geq 0 \]

(5d) \[ W_3 = S_{i_0} (R_0)^2 + S_{o_0} R_O + S_{i_1} (R_1)^2 + S_{i_2} (R_1)^2 \quad \text{if} \quad S_{i_2} < 0 \]

(5e) \[ W_4 = 0 \]

where \( V \) is the value function, \( \rho \) is the subjective rate of time preference, \( E_j \) is the expectation operator conditional on all information available at the beginning of period \( j \), and \( t \) is the tax rate, which is assumed to be constant over time. Borrowing constraints are captured in (3). Equations (4a) and (4b) are the per-period budget constraints: income must be allocated either to consumption or to net contributions to saving. These constraints allow already existing assets to be shifted from one form of saving to another. Equations (5a)-(5e) describe the evolution of available resources for consumption.

To generate closed-form solutions, we assume that utility is given by \( U(C_j) = -(k - C_j)^2 \). The quadratic specification has been widely used, particularly in the macroeconomics literature. It is the only commonly used functional form that generates closed-form solutions to the problem above. Nevertheless, it imposes important limitations; for example, it precludes the existence of precautionary saving. Thus, in the current model the expected level of future income affects current saving, but the variability of that expectation does not. Quadratic utility also implies that wealthier households insure more heavily than the poor against the same risk and that the wealthy hold more riskless assets and fewer risky assets than the poor. It is difficult to assess the importance of this implication for IRAs, because neither IRAs nor other saving represents safe assets, and either can be made the more risky.24

Appendix B provides the formal solution to this problem. Intuitively, the solution takes the following form. In the third period, the consumer consumes all remaining resources. At the beginning of period 2, \( Y_2 \) is revealed; the consumer then chooses consumption and contributions to IRAs and other saving. The portfolio choice in this period is simple: the first \( SL \) of new saving or of previously existing non-IRA financial assets \( (S_{i_0} R_O) \) should be placed in an IRA, with the remainder going to other saving. IRA contributions and other saving are perfect substitutes in the second period because there is no prospect of having to make an early withdrawal.25 If low \( Y_2 \) forces the consumer to dissave, all dissaving should occur through non-IRA assets until they are fully depleted.

In period 1, the consumer makes consumption and portfolio choices. Although attracted by the potentially high rate of return on IRAs, she may be deterred by the prospect of having to withdraw funds in period 2 and incur a penalty. First-order conditions for saving are of the following form:

\[
(6) \quad S_i = \beta_1 k + \beta_2 W_1 + \beta_3 - \beta_4 S_O
\]

\[
S_O = \alpha_1 k + \alpha_2 W_1 + \alpha_3 - \alpha_4 S_i
\]

where the \( \alpha \)'s and \( \beta \)'s are functions of the parameters of the model and expectations.26

23No existing empirical model addresses how the variability of income affects IRA choices. For a simulation model that incorporates this effect, see Eric M. Engen and Gale (1993).

24In structural estimation with quadratic utility, some households may be in a position of having negative marginal utility of consumption. This issue is discussed in Section III. Stephen P. Zeldes (1989) and Deaton and John Muellbauer (1980) provide additional comments on quadratic utility.

25With transactions costs of opening an IRA, it would no longer necessarily be true that the first \$L \ of saving should go into IRAs. Once an IRA had been opened, however, IRAs would remain perfect substitutes for other saving in the second period.

26Superscripts on the saving variables have been omitted for simplicity.
Solving for $S_1$ and $S_O$ gives equations of the following form:

\begin{align}
S_1 &= \delta_1 k + \delta_2 W_1 + \delta_3 \\
S_O &= \delta_4 k + \delta_5 W_1 + \delta_6
\end{align}

where the $\delta$'s depend on the $\alpha$'s and $\beta$'s.

Equation (7) represents the $S_1$ and $S_O$ that would occur in the absence of any contribution limits on $S_1$. If actual $S_1$ is constrained to lie between zero and $L$, the corresponding $S_O$ equation when the $S_1$ constraint is binding is determined by substituting the constrained value of $S_1$ into (6). Let $S_1^* = \delta_1 k + \delta_2 W_1 + \delta_3$ denote desired IRA saving. Then,

\begin{align}
S_1 &= 0 \\
S_O &= \alpha_1 k + \alpha_2 W_1 + \alpha_3 \quad \text{if } S_1^* \leq 0 \\
S_1 &= S_1^* = \delta_1 k + \delta_2 W_1 + \delta_3 \\
S_O &= \delta_4 k + \delta_5 W_1 + \delta_6 \quad \text{if } 0 < S_1^* \leq L \\
S_1 &= L \\
S_O &= \alpha_1 k + \alpha_2 W_1 + \alpha_3 - \alpha_4 L \quad \text{if } S_1^* \geq L.
\end{align}

Equations (8)–(10) jointly determine IRA and other saving, explicitly incorporating the effects of the contribution limit and implicitly incorporating (through maximization) the effects of the early-withdrawal penalty and other features of IRAs. These equations can be used to illustrate several points.

First, only individuals who already contribute the maximum amount are affected by changes in $L$. Second, comparative-static effects will in general depend on $\rho$, which is typically unobserved. More generally, differences in unobserved characteristics across households may affect the coefficients in the saving equations. To allow for the possible existence of such differences between households with IRAs and those without IRAs, we show in Appendix B that (10) can be rewritten as

\begin{align}
S_1 &= L \\
S_O &= \delta_4 k + \delta_5 W_1 + \delta_6 + \alpha_4 (S_1^* - L) \quad \text{if } S_1^* \geq L.
\end{align}

The model is given by (8), (9), and (11). Equations (11) and (9) describe IRA holders who contribute the maximum and less than the maximum, respectively. For both groups, $S_O$ is the same linear function of $k$ and $W_1$, with an added term for limit contributors, reflecting the spillover of excess desired IRA saving $(S_1^* - L)$ into non-IRA saving. Equation (8) provides a different function of $k$ and $W_1$ for the $S_O$ of noncontributors. Thus, the model explicitly allows for differences in saving behavior between households with IRAs and observationally equivalent households without IRAs.

The model indicates that the substitutability of IRA contributions and other saving should rise with age (IRAs are perfect substitutes in period 2 and imperfect change in contribution limits would fall as the likelihood of a bad draw on uncertain future income increases (because there would be more periods in which IRAs are illiquid and income is risky). However, the model would still yield equations of the form given by (8)–(10), although the precise formulas for each $\alpha$ and $\beta$ would differ from the three-period model.

27 Incorporating transactions costs would allow the possibility of discontinuous choices, whereby a very small increase in IRA limits could cause households to change their contributions from zero to $SL$. We have no way of assessing empirically the importance of this possibility.

28 For example, in (8) $dS_O/dW_1 = \alpha_2$, where $\alpha_2$ is a function of $\rho$ and other parameters of the model. Thus, two households with different subjective discount rates would have different savings responses to exogenous increases in wealth.
substitutes in period 1) and with non-IRA assets (higher assets imply less likelihood of having a second-period income draw so low as to have to withdraw IRA funds early). Substitutability is measured by $\alpha_4$. For example, if $\alpha_4 = 0$, none of the excess desired IRA saving would be placed in non-IRA financial assets. If $\alpha_4 = 1$, all of the excess desired IRA saving would be placed in non-IRA financial assets. Due to private wealth effects of IRAs, $\alpha_4$ can exceed 1 (see also Venti and Wise, 1991 fig. 1).  

III. Empirical Analysis

A. Specification

An empirical specification consistent with the model above is

$$S^*_t = X\beta + u$$

(12)

$$S_t = \begin{cases} 
0 & \text{if } S^*_t \leq 0 \\
X\beta + u & \text{if } 0 < S^*_t < L \\
L & \text{if } S^*_t \geq L 
\end{cases}$$

(13)

$$S_O = \begin{cases} 
X\gamma_1 + \varepsilon_1 & \text{if } S^*_t \leq 0 \\
X\gamma_2 + \varepsilon_2 & \text{if } 0 < S^*_t < L \\
X\gamma_2 + \eta(S^*_t - L) + \varepsilon_2 & \text{if } S^*_t \geq L 
\end{cases}$$

(14)

where $S^*_t$ is desired IRA saving, $S_t$ ($S_O$) is actual IRA (other) saving, $X$ is a vector of household characteristics, $\eta = \delta X$ measures the spillover of excess desired IRA saving into other saving, $\beta$, $\gamma_1$, $\gamma_2$, and $\delta$ are parameter vectors to be estimated, $L$ is the upper limit on IRA contributions, and $u$, $\varepsilon_1$, and $\varepsilon_2$ are errors.  

The specification corresponds closely to (8), (9), and (11), with $k$ interpreted as a vector of household characteristics.

The model distinguishes sharply between IRA contributors and noncontributors. Unlike previous researchers, we do not impose equal coefficients and error variances on the two groups. Instead, we measure substitutability by comparing limit contributors to nonlimit ("interior") contributors. The only difference between these two groups is that the former has an excess demand for IRAs that is allowed to influence non-IRA saving. Comparing limit contributors to interior contributors, rather than to noncontributors, controls for a specific type of heterogeneity where households with IRAs and households without IRAs have different determinants of saving. We test the validity of this distinction below.

Errors are assumed to be additive and distributed bivariate normal. There are three possible error terms, but only two will correspond to any particular household. The likelihood function has three branches corresponding to the probabilities that the observed value at $S_O$ occurs and (i) $S^*_t \leq 0$, (ii) $0 < S^*_t < L$, and (iii) $S^*_t \geq L$. Appendix C provides further details.

Non-IRA saving, $S_O$, consists of saving in non-IRA financial assets (see Table 2) minus increases in credit-card debt and in loans for items other than mortgages, home repairs, automobiles, or real estate. As with IRA saving, we calculate an assumed constant flow of saving in each category of asset or liability needed to achieve the observed balance in 1986, given the observed balance in 1983 and an average rate of

---

30 We follow the previous literature in setting the lower limit on IRA contributions equal to zero, so the model distinguishes between contributors and noncontributors, rather than between households with and without IRAs.

31 IRAs would seem to be most substitutable with these categories of assets and liabilities, rather than, for example, housing or land. Venti and Wise (1986, 1987, 1990, 1991) use a similar definition for similar reasons.

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29 More generally, a dollar of forgone consumption placed into an IRA is worth $(1 + r)^T$ after $T$ years. A dollar placed into a taxable account will be worth $[1 + r(1-t)]^T$. The maximum value of $\alpha_4$ occurs for a "target saver" and is the ratio of the value of a dollar of IRA saving divided by the value of a dollar of conventional saving. If $r = 0.10$, $t = 0.3$, and $T = 15$, for example, the maximum value of $\alpha_4$ is 1.51. If $T$ is changed to 30, the maximum value is 2.29.
retum on each category of asset and liability. Appendix A provides details. The resulting data set is a cross section on IRA saving, non-IRA saving, and household demographic and financial characteristics.\textsuperscript{32}

The independent variables are chosen to be consistent with previous research. The life-cycle model shows the importance of age, income, and wealth in determining saving. We include age and age squared to capture possible hump-shaped saving profiles, current income, and disaggregated components of wealth (NIFA, illiquid assets, and debt). For a particular household, higher asset levels encourage increased consumption and, thus, reduced saving, holding other factors constant. Across households, however, differences in initial asset holdings may be correlated with tastes for saving, controlling for other factors. Thus, the predicted effect of assets on saving is ambiguous. Debt also has a theoretically ambiguous relationship to IRA purchases. To the extent that households are financing IRA purchases by borrowing, there should be a positive relationship; to the extent that increased debt payments make IRAs less attractive, due to the latter's illiquidity, there should be a negative relationship. Households with large amounts of nonliquid assets, all else being equal, may find IRAs, another nonliquid asset, less attractive.

We follow previous empirical studies by including education of the household head, family size, and a dummy variable for pension status. More educated people may be more financially sophisticated and thereby find the transactions costs of IRAs less burdensome than do others. Differences in education may also be correlated with differences in discount rates, and thus with tastes for saving. In either case, more educated households would be expected to hold higher IRA balances. Other things equal, those with larger families may face higher current expenditures and thus have a lower propensity to tie up funds in an illiquid IRA.

We include a dummy variable for whether the household had an IRA in 1983 as an additional proxy for a household's past saving behavior and financial sophistication. This variable also indicates whether the household has previously incurred the transactions costs associated with starting an IRA.

In some sensitivity runs we include indicator variables for whether the household reports facing borrowing constraints and for home-ownership status.\textsuperscript{33} Borrowing-constrained households should be less likely to invest in illiquid assets like IRAs and may have low propensities to save (high propensities to borrow). Home-ownership may be important because households may not want to tie up funds in an IRA until they have purchased a home.

Holding other variables constant, the spillover effect of excess desired IRA saving into non-IRA saving is measured by \( \eta \). To account for heterogeneity in the model, \( \eta \) is modeled as a linear function of the same covariates as the IRA and non-IRA saving equations.\textsuperscript{34} Parameters are identified in the model through the functional form, which is suggested by the theoretical framework, and by the variation in IRA limits due to variation in marital status and employment across households.

In estimating the model, we exclude households from the sample if the head is 24 or younger, the head changed marital status during 1983–1986, or the head or

\textsuperscript{32}Hence, we cannot estimate structural parameters, such as subjective discount rates and \( k \)'s, which may vary across individuals. Instead, the structural parameters are embedded in the estimated \( \beta \)'s, \( \gamma \)'s, and \( \delta \)'s. Without the structural parameters of the theoretical model, it is impossible to address the issue of whether households are estimated to have negative marginal utility of consumption.

\textsuperscript{33}A household is considered to be borrowing-constrained if it reported that it had been turned down for credit in the previous few years or that it had not applied for credit because it thought it would be turned down. See Tullio Jappelli (1990) for an analysis using this information.

\textsuperscript{34}Thus, unlike the empirical model of Gale and Scholz (1990), we impose no ad hoc exclusion restrictions in the parameterization of \( \beta, \gamma_1, \) and \( \gamma_2 \) and drop only the constant term on \( \eta \).
spouse was self-employed (see footnote 7). We also exclude households in which the head was older than 65 in 1983 to maintain comparability with previous studies and to avoid the added complexities of modeling saving by the elderly. For this group, IRAs should be very good substitutes for other saving since there is no early-withdrawal penalty. Therefore, excluding the elderly presumably biases our results away from finding strong substitutability.

Finally, to reduce the extreme range of saving in the sample, we exclude households if the absolute value of non-IRA saving or dissaving exceeded $100,000 over the three-year period. This restriction is potentially the most troubling, due to problems arising from selecting on an endogenous variable. However, for households saving more than $100,000, limit changes are likely to be inframarginal. Results for other thresholds are presented in sensitivity tests, described below. Appendix D shows the number of households excluded by each selection criterion.

B. Coefficient Estimates

Table 4 presents the number of households and means and standard deviations of the dependent variable for each saving category. The total saving of households constrained by the IRA limit exceeds the saving of interior contributors, which in turn, exceeds the saving of noncontributors. Sample statistics for the independent variables (measured in 1983, except for education, which is an average of 1983 and 1986 levels, and age, which is measured in 1986) are given in Appendix E.

Table 5 presents estimates of (12)–(14) with a saving threshold of $100,000. Esti-

mates in the IRA equation (the $\beta$'s) are generally consistent with previous studies and a priori theorizing. IRA purchases are positively and significantly correlated with income and education. As in Venti and Wise (1988), we find that households with pensions contribute more to IRAs. The terms on age indicate that, conditioning on other attributes, IRA contributions peak at age 47. Households with more debt contribute more to IRAs, consistent with the possibility that households borrow to finance IRAs. As expected, households with an IRA in 1983 contribute significantly more to IRAs between 1983 and 1986. These households already absorbed the transactions costs of establishing an IRA and demonstrated a preference for saving through IRAs. Family size, NIFA balances, and nonliquid asset balances are negatively correlated with IRA purchases but are not statistically significant.

In the non-IRA financial-saving equations (the $\gamma_1$'s for noncontributors and the $\gamma_2$'s for contributors), non-IRA saving is positively and significantly correlated with current income. The estimates imply that households not contributing to IRAs save $0.06 out of every additional dollar of current income; the corresponding figure for interior IRA contributors is $0.08. These figures are consistent with the low rates of household saving observed in the United States.

For noncontributors, nonliquid-assets balances in 1983 are negatively and significantly related to non-IRA financial saving. This may be due to home-ownership. Renters may tend to save in financial forms to accumulate a downpayment. After purchasing a home, households will have a relatively large nonliquid asset and relatively low levels of financial saving. The age coef-

35 For example, see B. Douglas Bernheim (1991) or Michael D. Hurd (1987, 1989).
36 The change in real wealth ranged from less than − $30 million to more than $30 million over the sample (Avery and Kennickell, 1988).
37 Results were estimated using a wide variety of different starting values to ensure, to the best of our ability, that the reported results reflect global maxima.
38 Also see Kotlikoff (1990 p. 240) for a discussion of this point.
39 A single-equation, two-limit Tobit estimate of IRA contributions yields similar estimates. An ordered probit for IRA contributor status yielded the same pattern of coefficients and very similar $t$ statistics.
TABLE 4—Unweighted Saving Statistics, by IRA Contributor Status

<table>
<thead>
<tr>
<th>Saving category</th>
<th>Number of households</th>
<th>Non-IRA saving</th>
<th>IRA saving</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>Standard deviation</td>
<td>Mean</td>
<td>Standard deviation</td>
</tr>
<tr>
<td>Noncontributors</td>
<td>1,035</td>
<td>2,377</td>
<td>17,828</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Interior contributors</td>
<td>331</td>
<td>1,511</td>
<td>29,864</td>
<td>3,522</td>
<td>2,740</td>
</tr>
<tr>
<td>Limit contributors</td>
<td>117</td>
<td>4,088</td>
<td>34,728</td>
<td>8,276</td>
<td>2,611</td>
</tr>
</tbody>
</table>

Source: Authors' calculations using the 1983–1986 Survey of Consumer Finances.

Coefficients are consistent with saving rising at an increasing rate during one's working life but are not statistically significant.

For contributors, non-IRA saving is negatively and significantly related to 1983 NIFA balances and positively and significantly related to 1983 debt. The latter result is consistent with households exploiting the tax- deductibility of interest during this period. The remaining coefficients are statistically insignificant, though for both contributors and noncontributors the 1983 IRA dummy is positive, and larger families saved less than otherwise equivalent households.

Coefficients on $\eta$, along with the distribution of the errors, determine the substitutability of IRA and non-IRA saving. Any factor that raises the substitutability reduces the net saving effect of IRAs. As expected, the estimates show that higher 1983 NIFA balances significantly raise substitutability. Households with higher NIFA balances are in a better position to reschedule existing assets or to redirect new saving that would have been done anyway. Several other effects have point estimates of the expected sign, but only two are statistically significant. Substitutability is estimated to rise with age (once a household reaches age 28). It also rises with education, consistent with more-educated people finding transactions costs less burdensome or having lower discount rates. Substitutability is reduced by higher debt levels, having a pension, or previous IRA contributions. The last effect is consistent with IRAs being funded from transferring assets or redirecting saving. Higher values of current income negatively and significantly affect substitutability. Income variability is positively correlated with income level. Conditioning on NIFA, income variability and the associated need for liquid saving may reduce the substitutability of IRAs and other saving.

For purposes of comparison, single-equation, ordinary least-squares (OLS) estimates of IRA and non-IRA saving are also presented in Table 5. OLS estimates of IRA saving follow roughly the same relative sign and significance pattern as the maximum-likelihood (ML) estimates, but the OLS constant is larger, and the other coefficients are smaller than the ML estimates. This is precisely the expected pattern when constraints on contributions are binding (Venti and Wise, 1986 fig. 1.1). Separate non-IRA saving equations were estimated for IRA contributors and noncontributors. For noncontributors, the OLS and ML estimates are very close. This is to be expected be-

40 Recall that the sample omits households older than 65 in 1983.
41 The youngest limit contributor in the sample is 29. In the weighted sample used for Tables 1–3, 2.6 percent of the IRA contributors are 27 or younger.
42 The SCF provides separate observations on 1983, 1984, and 1985 income. The standard deviation of annual income over the three years increases monotonically from $1,279, for households with average incomes between $0 and $10,000, to $51,469 for households with average incomes that exceed $100,000.
43 The restriction $\eta = 0$ can be rejected at very high confidence levels. The likelihood-ratio test statistic is 68.4. The 99-percent critical value for imposing ten restrictions is 23.2. Estimating the model with $\eta$ as a constant yields a statistically significant estimate of 4.72. However, for reasons discussed above, $\eta$ should vary with household characteristics.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Maximum-likelihood estimates</th>
<th>Ordinary least-squares estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>Standard error</td>
</tr>
<tr>
<td>$\beta$:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-33.233</td>
<td>4.926</td>
</tr>
<tr>
<td>Age</td>
<td>0.791</td>
<td>0.206</td>
</tr>
<tr>
<td>Age squared</td>
<td>-8.359</td>
<td>2.206</td>
</tr>
<tr>
<td>Income</td>
<td>0.771</td>
<td>0.259</td>
</tr>
<tr>
<td>Pension dummy</td>
<td>2.278</td>
<td>0.679</td>
</tr>
<tr>
<td>Education</td>
<td>0.676</td>
<td>0.113</td>
</tr>
<tr>
<td>Family size</td>
<td>-0.331</td>
<td>0.198</td>
</tr>
<tr>
<td>Non-IRA financial assets</td>
<td>-0.027</td>
<td>0.034</td>
</tr>
<tr>
<td>Debt</td>
<td>0.118</td>
<td>0.059</td>
</tr>
<tr>
<td>Nonliquid assets</td>
<td>-0.016</td>
<td>0.135</td>
</tr>
<tr>
<td>IRA dummy</td>
<td>7.745</td>
<td>0.651</td>
</tr>
<tr>
<td>$\gamma$:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>5.800</td>
<td>8.601</td>
</tr>
<tr>
<td>Age</td>
<td>-0.353</td>
<td>0.372</td>
</tr>
<tr>
<td>Age squared</td>
<td>4.960</td>
<td>4.006</td>
</tr>
<tr>
<td>Income</td>
<td>6.012</td>
<td>1.026</td>
</tr>
<tr>
<td>Pension dummy</td>
<td>-1.520</td>
<td>1.430</td>
</tr>
<tr>
<td>Education</td>
<td>-0.111</td>
<td>0.227</td>
</tr>
<tr>
<td>Family size</td>
<td>-0.110</td>
<td>0.395</td>
</tr>
<tr>
<td>Non-IRA financial assets</td>
<td>0.077</td>
<td>0.059</td>
</tr>
<tr>
<td>Debt</td>
<td>1.056</td>
<td>0.248</td>
</tr>
<tr>
<td>Nonliquid assets</td>
<td>-3.998</td>
<td>0.492</td>
</tr>
<tr>
<td>IRA dummy</td>
<td>0.610</td>
<td>2.248</td>
</tr>
<tr>
<td>$\eta$:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>-0.063</td>
<td>0.248</td>
</tr>
<tr>
<td>Age squared</td>
<td>1.133</td>
<td>3.259</td>
</tr>
<tr>
<td>Income</td>
<td>-3.170</td>
<td>0.836</td>
</tr>
<tr>
<td>Pension dummy</td>
<td>-1.376</td>
<td>1.714</td>
</tr>
<tr>
<td>Education</td>
<td>0.222</td>
<td>0.311</td>
</tr>
<tr>
<td>Family size</td>
<td>0.949</td>
<td>0.599</td>
</tr>
</tbody>
</table>
Table 5—Continued.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Maximum-likelihood estimates</th>
<th>Ordinary least-squares estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>Standard error</td>
</tr>
<tr>
<td>( \eta ) (continued): Non-IRA financial assets</td>
<td>0.985</td>
<td>0.235</td>
</tr>
<tr>
<td>Debt</td>
<td>-0.466</td>
<td>0.192</td>
</tr>
<tr>
<td>Nonliquid assets</td>
<td>0.471</td>
<td>0.541</td>
</tr>
<tr>
<td>IRA dummy</td>
<td>-0.869</td>
<td>1.677</td>
</tr>
<tr>
<td>( \sigma(\sigma) )</td>
<td>7.405</td>
<td>0.350</td>
</tr>
<tr>
<td>( \sigma(\epsilon_1) )</td>
<td>17.109</td>
<td>0.376</td>
</tr>
<tr>
<td>( \sigma(\epsilon_2) )</td>
<td>26.515</td>
<td>0.936</td>
</tr>
<tr>
<td>( \rho(\sigma, \epsilon_1) )</td>
<td>0.007</td>
<td>0.106</td>
</tr>
<tr>
<td>( \rho(\sigma, \epsilon_2) )</td>
<td>-0.026</td>
<td>0.122</td>
</tr>
</tbody>
</table>

(Mean log-likelihood = -5.51397, sample size = 1,483)

Notes: Variable definitions are as follows: age = age of head of household in 1986, in years; age squared = age of head of household in 1986 squared, in 1,000 years; income = sum of household’s reported income in 1983, 1984, and 1985, in $100,000’s; pension = 1 if household owns a pension in 1983, 0 otherwise; education = average of 1983 and 1986 values of years of education of household head; family size = number of people in the family in 1983; non-IRA financial assets = 1983 level of checking, savings, and money-market accounts, certificates of deposit, stocks, bonds, mutual funds, cash value of life insurance, and other financial assets, in $10,000’s debt = 1983 debt on credit cards, mortgages, property, and other business or consumer loans, in $10,000’s; nonliquid assets = 1983 value of home and gross value of property and business assets, in $100,000’s; 1983 IRA dummy = 1 if household reported having an IRA in 1983. Ordinary least-squares samples and estimation are described in the text.

because the estimated correlation between \( u \) and \( \epsilon_1 \) in the ML estimates is very small.

The OLS estimates of non-IRA saving for contributors are estimated using a simple two-step procedure. The first step constructs a fitted value for \( S_1^* \) using the OLS estimates of \( \beta \). The second step uses this constructed variable to estimate the following equation by OLS:

\[
S_O = X\hgamma_2 + \lambda \heta (X\hbeta_{OLS} - L) + \upsilon
\]

(15)

where \( \lambda \) takes the value 1 for limit contributors and 0 otherwise and \( \upsilon \) is a residual. The OLS coefficients \( \hgamma_2 \) and \( \heta \) are reported in Table 5 and vary relative to the ML estimates, in some cases by wide margins.

While the OLS estimates are useful for purposes of comparison and easy replication, they are neither unbiased nor consistent. Intuitively, this occurs because (15) does not incorporate the error term from the first-stage estimates of \( S_1^* \). For limit contributors the expected value of \( u \) is positive (i.e., \( E(u|u \geq L - X\beta) > 0 \)) and is negatively correlated with the value of \( X\beta \). Hence, the OLS estimates of (15) suffer from omitted-variable bias, and the OLS estimate of \( \heta \) is biased and inconsistent.

\[\text{To see this, note that the fitted value of } S_1^* = X\hbeta + \hat{\upsilon}. \text{ Substituting for } S_1^* \text{ in (14) yields }
\]

\[
S_O = X\gamma_2 + \eta (X\beta - L) + \eta \hat{\upsilon} + \epsilon_2.
\]

Because (15) omits the \( \eta \hat{\upsilon} \) term in this equation, standard omitted-variable techniques (William H. Greene, 1990 pp. 259–61) imply that estimating (15) yields \( E(\heta) = \eta + \eta \delta_3 \), where \( \eta \) is the true value of the parameter and \( \delta_3 \) is determined by the following regression:

\[
\hat{\upsilon} = d_1 + d_2 X + d_3 (X\beta - L) + \text{residual}.
\]

In general the direction of the bias is impossible to determine, because \( d_3 \) depends on multiple regression coefficients, which have the sign of partial correlations. In practice, comparing the two-step estimates with the consistent maximum-likelihood estimates suggests that the two-step estimates are downward-biased.
An alternative simple approach would be to estimate a two-limit Tobit specification for IRA saving in the first stage, use the Tobit estimates of $\beta$ to construct a fitted value of $S^*_f$, and reestimate (15) by OLS. Unfortunately, this faces a similar problem as the estimates that use OLS in the first stage: for limit contributors the expected value of $u$ is positive and negatively correlated with $X\beta$.

Table 6 reports the implied mean and median values of $\eta$ for alternative specifications. These figures are generated by applying estimates of $\delta$ (where $\eta = X\delta$) to a set of $X$ values for limit contributors, and taking the mean and median of the distribution of $\eta$’s. Using the actual values of $X$ and our central estimates of $\delta$ from the ML model generates a mean $\eta$ of 1.85 and a median of 1.04. These estimates suggest that IRAs do not raise national saving (a fuller test is provided in the next section, where we provide stochastic simulation of IRA limit changes) but are well within the theoretically admissible range for $\eta$. As discussed in footnote 29, the effects of IRAs on saving for a target saver depend on rates of return, tax rates, and holding periods. Using returns of 14 percent (the approximate stock and bond rate in the mid 1980’s) and the tax rates and age distribution of limit contributors in the sample, assuming that IRAs are cashed in lump sum at age 65 implies that the mean value of $\eta$ could be as high as 2.23.

Rows 2 and 3 of Table 6 show that, holding other factors constant, the substitutability of IRAs and saving rises as household age. Rows 4 and 5 report the estimates for “typical” 35- and 55-year-olds. For each household, the $X$ variables reflect the idea that younger limit contributors have high income and high levels of (mortgage) debt relative to the population, but fairly low levels of financial assets, and they are less likely to be covered by a pension or to have an IRA. Older limit contributors are given high income, high wealth, low debt, a pension, and previous IRA participation. Details are given in the table. Under the specifications chosen, $\eta$ is 0.68 for a typical 35-year-old limit contributor; that is, lower assets, younger age, and other factors for these households reduce the substitutability of IRAs relative to the overall average. For the typical 55-year-old limit contributor, higher assets, higher age, and the net effects of other factors raise the substitutability of IRAs and other saving. While these findings are plausible, it should be clear that one can
obtain large negative or positive values for \( \eta \) by judiciously choosing values for the \( X \) variables.

The last row reports the estimates of \( \eta \) from the two-step OLS estimator described above. These estimates are negative, but as noted above, they are biased.\(^{45}\)

### C. Sensitivity Analysis

We use the estimated coefficients and random draws on the error terms to simulate the effects of a hypothetical $1,000 increase in the annual contribution limit on changes in IRA saving, non-IRA saving, consumption, and tax payments.\(^{46}\) Simulations based on the maximum-likelihood estimates in Table 5 show that, holding taxes constant, about 98 percent of the increased IRA contributions due to a $1,000 limit change would be financed by a decrease in non-IRA private saving. Tax reductions (public dissaving) represent about 31 percent of the increased IRA contributions.\(^{47}\) If households saved the whole tax reduction in the first year, reduced non-IRA private saving would finance about 67 percent (98 percent - 31 percent), and reduced private consumption would finance the remaining 2 percent of the increased IRA contributions. Thus, as shown in row 1 of Table 7, public and private dissaving would account for 98 percent of the increased IRA contributions, and net increases in national saving would account for only 2 percent. If households consumed half of the tax cut in the first year, non-IRA saving would fall by 83 percent (98 percent - 15 percent), tax cuts would again finance 31 percent, and consumption would rise (national saving would fall) by 14 percent of the increased contributions.

Rows 2 and 3 show results for thresholds of $\pm 75,000$ and $\pm 200,000$. Using the first threshold reduces the net saving effect. The second threshold leads to implausibly high estimates of substitution, which make it clear that we do not adequately capture the saving behavior of the extremely rich.\(^{48}\) This may be due to different saving responses to observed characteristics among the wealthy. Paul L. Menchik and Burton A. Weisbrod (1987) report similar difficulties in modeling the charitable behavior of the extremely wealthy relative to the rest of the population. Menchik and Martin David (1983) estimate that bequest elasticities are very different for households with very high income than for others. Also, the more complicated portfolio arrangements of extremely wealthy

---

\(^{45}\) Estimating (15) using OLS in both stages and assuming \( \eta \) is not a function of \( X \) yields an estimate of \(-0.546\) with a standard error of 0.545.

\(^{46}\) Simulations were done 25 times for each household. We use a tax calculator that simulates households' tax returns, using income and demographic characteristics reported in the SCF (see Scholz, 1992). Full results for each specification not presented in the paper are available from the authors upon request.

\(^{47}\) These calculations are partial-equilibrium and consider only the effects of the individual income tax. Feldstein (1992), starting from the assumption that IRAs significantly stimulate private saving, suggests that capital deepening, resulting from increased national saving, will increase corporate tax receipts.

\(^{48}\) Increasingly large and untenable amounts of substitution are obtained as the saving threshold is increased to $\pm 250,000$ and $\pm 300,000$. 

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**Table 7—Sensitivity Analysis: Raising the Contribution Limit by $1,000**

<table>
<thead>
<tr>
<th>Model</th>
<th>Percentage of additional IRA contributions representing increases in national saving</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tax cut is saved</td>
</tr>
<tr>
<td>Saving threshold = $\pm100,000$</td>
<td>2.1</td>
</tr>
<tr>
<td>Saving threshold = $\pm75,000$</td>
<td>-17.5</td>
</tr>
<tr>
<td>Saving threshold = $\pm200,000$</td>
<td>-352.2</td>
</tr>
<tr>
<td>No IRA dummy</td>
<td>-6.9</td>
</tr>
<tr>
<td>Borrowing constrained and home</td>
<td>-13.5</td>
</tr>
<tr>
<td>False constraints:</td>
<td></td>
</tr>
<tr>
<td>Regime 1</td>
<td>-4.5</td>
</tr>
<tr>
<td>Regime 2</td>
<td>35.1</td>
</tr>
</tbody>
</table>

**Notes:** For each simulation, the change in IRA saving and non-IRA saving is calculated after increasing the contribution by $1,000, holding other variables constant. The average of the changes is reported. The model specifications are described in the text.
households may not be adequately captured by the conventional asset and liability categories that we use.

Dropping the 1983 IRA dummy variable and adding indicators for home-ownership and being liquidity-constrained had little effect on the remaining coefficients. As expected, owning a home is positively and significantly associated with IRA purchases, while being borrowing-constrained is negatively and significantly correlated with IRA saving. Rows 4 and 5 show that these runs generate smaller net saving effects of limit changes than the specification shown in row 1.49

Feenberg and Skinner (1989) report that a large number of married couples, with legal contribution limits of $2,250 or $4,000, contribute exactly $2,000 to IRAs. They question whether households are being affected by “false” or nonstatutory constraints. If so, households that are not at the statutory limit may well respond to limit changes. Whether they would respond by increasing overall saving or reallocating their saving is an open question. In any case, our model as currently structured would not capture these responses, which could bias our estimates.

Because our IRA variable is constructed from data on asset balances, we do not observe the bunching that Feenberg and Skinner (1989) find. To allow for falsely constrained households, we estimate the model under two regimes. In regime 1, we move the limit to $2,000 for any couple with a statutory limit of $2,250. This has little effect on our results (row 6).

In regime 2, we take two additional steps. Recognizing that we may measure saving with error, we classify one-earner couples with average IRA contributions above $1,900 and two-earner couples with average contributions between $1,900 and $2,350 as limit contributors. This undoubtedly misclassifies some households that know they are not constrained by the limit, but nevertheless it serves as a useful check of model robustness in light of the bunching observed in tax return data. The saving effect in this case rises (row 7) but remains considerably smaller than previous results in the literature.

If rates of return vary across households (see e.g., Shlomo Yitzhaki, 1987), IRA and non-IRA saving will be measured with error, which may bias the estimates for two reasons. First, errors in calculating IRA contributions may cause households’ contributor status to be misclassified. Second, if households that earn higher returns on one asset also earn higher returns on other assets, the errors in (12)–(14) may be spuriously correlated.

We provide evidence on the direction and magnitude of any measurement-error bias by using artificial data to simulate exactly the type of measurement error that we impose on the data. We first estimate the model with data in which all households receive the same rate of return on each asset and liability category. We then generate artificial data that contain household-specific returns on assets and liabilities and compare the results.

These comparisons suggest that our estimates of the effects of limit changes are biased downward. The key parameter in assessing the magnitude of the bias is the standard deviation of the household-specific rate of return for each asset and liability. With a standard deviation of 0.01 (0.05), the recalculated estimates of the reduction in non-IRA private saving are roughly 8 (18) percentage points higher than the benchmark. Applying the latter result to the first row of Table 7, the percentage of new IRA contributions that would represent increases in national saving (if the entire tax cut is saved) would rise from 2 percent to 20 percent. If half of the tax reduction were consumed, national saving would rise by $0.04 for every additional dollar contributed to IRAs.

D. Assessing the Model

In order to gauge the reliability of the underlying model, Table 8 presents data on

49 This and all other results below are based on the $100,000 threshold.
### Table 8—Model Fit by Income Class, Unweighted

<table>
<thead>
<tr>
<th>Income class (three years, $'000's)</th>
<th>Number</th>
<th>Percentage with IRA at limit</th>
<th>Percentage with IRA at limit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Predicted</td>
<td>Actual</td>
</tr>
<tr>
<td>0–50</td>
<td>404</td>
<td>15.1</td>
<td>7.4</td>
</tr>
<tr>
<td>50–75</td>
<td>273</td>
<td>22.7</td>
<td>19.8</td>
</tr>
<tr>
<td>75–100</td>
<td>276</td>
<td>24.6</td>
<td>31.1</td>
</tr>
<tr>
<td>100–150</td>
<td>297</td>
<td>30.3</td>
<td>39.1</td>
</tr>
<tr>
<td>150–200</td>
<td>115</td>
<td>43.5</td>
<td>64.3</td>
</tr>
<tr>
<td>200–300</td>
<td>66</td>
<td>63.6</td>
<td>75.8</td>
</tr>
<tr>
<td>300+</td>
<td>52</td>
<td>73.1</td>
<td>73.1</td>
</tr>
<tr>
<td>Total:</td>
<td>1,483</td>
<td>27.7</td>
<td>30.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Income class (three years, $'000's)</th>
<th>Percentage with $S_0 &gt; 0</th>
<th>Percentage of IRA holders with $S_0 &gt; 0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Predicted</td>
<td>Actual</td>
</tr>
<tr>
<td>0–50</td>
<td>50.2</td>
<td>49.8</td>
</tr>
<tr>
<td>50–75</td>
<td>51.3</td>
<td>58.6</td>
</tr>
<tr>
<td>75–100</td>
<td>59.4</td>
<td>63.4</td>
</tr>
<tr>
<td>100–150</td>
<td>55.6</td>
<td>59.9</td>
</tr>
<tr>
<td>150–200</td>
<td>57.4</td>
<td>67.8</td>
</tr>
<tr>
<td>200–300</td>
<td>68.2</td>
<td>63.6</td>
</tr>
<tr>
<td>300+</td>
<td>50.0</td>
<td>46.2</td>
</tr>
<tr>
<td>Total:</td>
<td>54.6</td>
<td>57.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Income class (three years, $'000's)</th>
<th>Non-IRA saving</th>
<th>Non-IRA saving of IRA holders</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Predicted</td>
<td>Actual</td>
</tr>
<tr>
<td>0–50</td>
<td>−$566</td>
<td>$506</td>
</tr>
<tr>
<td>50–75</td>
<td>$421</td>
<td>$1,864</td>
</tr>
<tr>
<td>75–100</td>
<td>$3,027</td>
<td>$2,751</td>
</tr>
<tr>
<td>100–150</td>
<td>$2,605</td>
<td>$2,432</td>
</tr>
<tr>
<td>150–200</td>
<td>$2,631</td>
<td>$7,028</td>
</tr>
<tr>
<td>200–300</td>
<td>$11,552</td>
<td>$4,774</td>
</tr>
<tr>
<td>300+</td>
<td>−$1,285</td>
<td>$2,320</td>
</tr>
<tr>
<td>Total:</td>
<td>$1,681</td>
<td>$2,319</td>
</tr>
</tbody>
</table>

how well the model fits saving patterns by income category. Panel A shows that the model predicts the percentages of IRA holders and limit contributors fairly well, both overall and by income class. Panel B shows there is a close correspondence for the percentage of savers in the sample and the percentage of IRA holders that save positive amounts. Similar data with similar results are reported by Venti and Wise (1986, 1987, 1990, 1991).

Panel C shows model fit for the level of saving. Not surprisingly, the model fit here is not as precise as in the first two panels. It is difficult, however, to know how accurate the fit is relative to other models, since data on model fit by level of saving have not been reported in previous IRA studies.  

---

50 At the suggestion of Jonathan Skinner, we compared the results to OLS regressions of $S_0$ on the X's. Model estimates are closer to the actual means than OLS estimates in four of the top five income classes for all households and in three of the top five income classes for households with IRAs. These classes account for 81 percent (unweighted) of IRA holders.
Previous researchers estimate substitutability parameters using the entire sample of IRA holders and non IRA holders. That model arises as a special case of our model, with $\gamma_1 = \gamma_2$ and $\sigma(\xi) = \sigma(\xi)$. Thus, the restriction that the two groups have the same response to observed determinants can be tested. A likelihood-ratio test rejected this restriction at all usual levels of confidence.\(^{51}\)

Thus, we find evidence that saving equations for IRA and non-IRA households are different. This is consistent with the idea that IRA and non-IRA households differ systematically in unobserved determinants of saving. This in turn implies that tests of the effects of IRAs on the level of saving that depend on comparisons between IRA households and otherwise equivalent non-contributors are not valid, at least in the context of our model.

E. Policy Implications

Our results indicate that increasing IRA limits during 1983–1985 would have induced little if any increase in national saving. This need not imply, however, that all variants of IRAs are equally ineffective.\(^{52}\) As noted above, households older than 59 make a large share of IRA contributions and may find IRAs to be very good substitutes for other forms of saving. Eliminating IRA eligibility for this group should raise the proportion of increased IRA saving due to limit changes that would represent increases in national saving.

To test this idea, we excluded household heads older than 59 in 1983 and reestimated the model. If all (half) of the tax cut were saved, 35 percent (20 percent) of the increase in IRA contributions would represent new saving. These figures relate only to the proportion of increased IRA contributions that represent increases in national saving. The actual level of IRA contributions would also change in an a priori unknown manner. Nevertheless, the results are consistent with the notion that the substitutability of IRAs and other saving is very high for households aged 60 and over.\(^{53}\)

The Tax Reform Act of 1986 eliminated deductibility of contributions for married taxpayers who have pensions and adjusted gross income above $50,000 and for singles who have pensions and adjusted gross income above $35,000. The policy should reduce contributions by higher-income households relative to lower-income households. In the SCF, households with average 1983–1985 income above these thresholds had much higher NIFA and were substantially older than the rest of the sample.\(^{54}\) This suggests that these households may also have found IRAs and other saving to be very good substitutes and that the Tax Reform Act of 1986 may have raised the proportion of IRA contributions that represent net increases in national saving.

IV. Conclusion

Our principal findings indicate that increasing IRA contribution limits between 1983 and 1986 would have resulted in little if any net increase in national saving. These findings are consistent with the fact that most limit contributors either hold large amounts of non-IRA financial assets, and thus may be able to shift other saving into IRAs fairly readily, or are older than 59,

\(^{51}\) The test statistic was 310. The 1-percent critical value for a likelihood-ratio test with 12 degrees of freedom is 26.2. Much of the difference is due to constraining $\sigma(\xi) = \sigma(\xi)$. However, the statistic for testing $\gamma_1 = \gamma_2$ was 116, which still rejects the null hypothesis of equal coefficients at all usual confidence levels.

\(^{52}\) For example, Bernheim and Scholz (1993) describe a modified system of tax-favored saving accounts that establishes floors and ceilings for eligible contributions and allows these limits to vary with adjusted gross income. Their proposal addresses a number of shortcomings of IRA-type saving plans.

\(^{53}\) Engen and Gale (1993) obtain similar results in a stochastic life-cycle simulation that incorporates IRAs.

\(^{54}\) For married households with average 1983–1985 income above $50,000 and single households with average 1983–1985 income above $35,000; median 1983 NIFA was $66,000, and the median age was 55 years. These compare to $5,200 and 49 years for the rest of the sample.
and thus face no early-withdrawal penalty on IRA contributions.

We have also tested the sensitivity and reliability of our results. Our constructed IRA-contributions variable closely matches IRS data in several important dimensions. Sensitivity tests generally support the principal finding, though, as expected, some variation exists. One caveat to our results is that, at a general level, household saving is difficult to predict using micro data. This caveat, of course, applies equally to other micro-econometric studies.

After controlling for how the tax cut would be allocated, our results suggest a significantly smaller impact of IRAs on saving than previous researchers have estimated. Relative to previous researchers, we use a different theoretical framework, which leads to different functional forms, and employ a different data set. In addition, because of our data, our results are based on the behavior of households constrained by the limit in each of three years, rather than in just one year. Although it is beyond the scope of the current paper to explain the relative importance of each of these factors in generating the differences in results, further research into this area is clearly warranted.

IRAs could affect saving through channels that we have not considered. For example, IRAs may serve as a means of "self control" (Richard Thaler, 1990), whereby individuals who otherwise find it difficult to save place funds in IRAs to restrict themselves from consuming those funds in the (near) future, due to the penalty. As another example, Summers (1986) and others suggest that the heavy promotion of IRAs may have served to raise saving by educating the populace about the opportunities for, and benefits of, saving. While each of these effects may be important, we reiterate our earlier result that IRA contributors, and in particular limit contributors, hold substantial amounts of non-IRA net worth and thus at some level appear to understand the opportunities for and benefits of saving. An alternative tradition argues that households may save to reach specific target levels of wealth. To the extent that house-

holds behave like target savers, IRAs will unambiguously reduce saving because of the higher interest rates provided.

**Appendix A**

This Appendix describes the calculation of IRA contributions and other saving. The SCF reports asset and liability totals for various categories in 1983 and 1986. The general strategy is as follows: to determine saving in an asset $X$, note that

$$\text{(A1)} \quad \text{BALX}_{86} - \text{BALX}_{83}(1 + r_x)^3$$

$$= \text{CONX}_{83}(1 + r_x)^2 + \text{CONX}_{84}(1 + r_x)$$

$$+ \text{CONX}_{85}$$

where $\text{BALX}_t$ is the balance of asset $X$ in year $t$, $r_x$ is the (assumed constant) after-tax rate of return on $X$, and $\text{CONX}_t$ is net additions to $X$ (i.e., saving in the form of asset $X$) in year $t$. We assume $\text{CONX}_t = \text{CONX}$, for all $t$. Rearranging (A1) yields

$$\text{CONX} = \frac{\text{BALX}_{86} - \text{BALX}_{83}(1 + r_x)^3}{(1 + r_x)^2 + 2 + r_x}.$$ 

Saving in asset $X$ is then calculated as $3(\text{CONX})$.

Non-IRA saving is calculated as the net contributions to assets less new debt incurred. To determine asset contributions, we use 1983 and 1986 balances of assets and liabilities and interest-rate assumptions taken from the *Annual Statistical Digest*, published by the Federal Reserve Board, the *Saving Institution Sourcebook*, and the 1992 *Economic Report of the President*. The asset and liability categories, with interest rates given in parentheses, are: checking and saving accounts (6.7 percent); money-market accounts and certificates of deposit (9.02 percent); stocks and mutual funds (16.1 percent); bonds (12.04 percent); profit-sharing and thrift accounts (9.02 percent); the cash value of whole-life policies (6.7 percent); and other financial assets (12 percent). Changes in debt are determined using the 1982 and 1985 balances of credit-card
debt (16.36 percent) and other nonmortgage debt (16.36 percent). Annual saving is then calculated as the annual contributions to assets less liabilities. In the text we discuss measurement-error issues that arise from assuming that all households receive the average return on each category of asset and liability.

Calculating the IRA contribution is slightly more complicated, because IRA and KEOGH balances are reported separately in 1983 but are combined in 1986. Recall that the sample excludes all households where either the respondent or spouse is self-employed. Thus, the following rules were used to calculate the IRA balance in 1985 from the IRA and KEOGH balance: if in 1983 the household had an IRA (KEOGH) but no KEOGH (IRA), impute the 1986 balance to an IRA (KEOGH); if the household had neither in 1983, attribute the balance to an IRA; if the household had both in 1983, allocate the 1986 balance by maintaining the 1983 proportions.

We use an average of stock and bond returns, 14 percent, to calculate estimated annual IRA contributions, $Y$. IRA saving is given by

$$S_1 = \begin{cases} 3 \times \max(Y, 0) & \text{if } Y \leq 0 \\ 3 \times \min(Y, L) & \text{if } Y \geq 0 \end{cases}$$

where $L$ is the upper limit on annual contributions (see footnote 1). $Y$ can exceed $L$ because rollovers of other assets into IRAs are allowed under certain circumstances. We describe our efforts to assess the accuracy of the IRA calculation in the text and in Table 1.

**Appendix B**

The consumer maximizes (2), subject to (3), (4), and (5) in each of periods 1, 2, and 3. The third-period problem is trivial and involves consuming all remaining wealth. In period 2, the consumer maximizes

$$V_2(W_2) = U(C_2) + \frac{1}{1 + \rho} V_3(W_3)$$

where the expectations operator is not needed because the value of $Y_2$ (and hence all uncertainty) is revealed before second-period consumption and saving decisions are made.

To maximize (B1) with respect to $C_2$, an expression for $W_3$ as a function of $C_2$ is needed. If there were only one asset, the expression would be $W_3 = (W_2 - C_2)r$, where $r$ is the gross return on the single asset. However, when both IRAs and other assets exist, the expression for $W_3$ is more complicated. Let $X = Y_2 + S_O^1 R_O - C_2$ be liquid resources available for saving in period 2. Since IRAs are perfect substitutes for other saving in period 2, the optimal portfolio allocation rule is to place the first $S_L$ of $X$ into IRAs and the remainder into non-IRA saving. $W_3$ depends on $X$ as shown in Table B1, where $C_{2h}^*$ is optimal consumption in state $h$, and $W_3$ is obtained by substituting $S_1^2$ and $S_2^2$ into the expression for $W_3$ given in (5). Case (a) refers to a consumer who contributes the maximum amount to an IRA. Case (b) refers to someone who contributes a smaller positive amount or zero to an IRA. Case (c) refers to someone who has to withdraw IRA funds in the second period.

Third-period wealth in state $h$ ($h = a, b, c$) can be rewritten as

$$W^*_3 = (W_2^h - C_{2h}^*) R_h$$

where $R_h = [R_O, R_1, (R_f)^2]$ and refers to the marginal interest rate the consumer faces in cases a, b, and c, respectively, and

$$W_2^h = S_O R_O + \frac{S_1^1 (R_1)^2}{R_h} + Y_2 + D_h$$

where

$$D_h = \left[ \frac{L (R_1 - R_O)}{R_O}, 0, 0 \right].$$

55 To keep the notation simple, we ignore the tax-deductibility of IRA contributions. It is easy to show that this does not affect the functional forms that result for IRA and non-IRA saving, although the parameter values would change.
$W^h_2$ is closely related to $W_2$ in (5). The differences are (i) the $R_1^2/R_2^h$ coefficient on $S_1^h$ and (ii) $D_h$. The first difference occurs because the ultimate present discounted value of $S_1^h$ depends on when it is cashed in. The second difference occurs because in case (a) the marginal return on saving does not equal the average return, because the consumer is holding both IRAs and other assets. $D_h$ measures the increased returns due to holding IRAs. Thus, $W^h_2$ may be thought of as effective wealth in period 2.

For each state $h$, the consumer chooses $C_{2h}^*$ to maximize

(B3) $V_2(W^h_2) = U(C^h_2) + \frac{1}{1+\rho}V_3(W^h_3) - (k - C^h_2)^2 - \frac{1}{1+\rho}[k - (W^h_2 - C^h_2)R_h]^2$

using the quadratic functional form assumed in the text. Optimal consumption satisfies

(B4) $C_{2h}^* = q_1^h k + q_2^h W^h_2$

where

$q_1^h = \frac{1+\rho-R_h}{1+\rho+R_h^2}$
$q_2^h = \frac{R_h^2}{1+\rho+R_h^2}$

The consumer then compares utility attained in each state and makes the consumption and savings choices implied by the state with the highest maximum $V_2$.

Substituting (B4) into (B3) yields $V_2$ as a function of $W^h_2$.

(B5) $V_2(W^h_2) = -\left[(1-q_1^h)k - q_2^h W^h_2\right]^2 - \frac{1}{1+\rho} \left[(1+q_1^h R_h)k - R_h (1-q_2^h W^h_2)\right]^2$.

Turning to the first period, the consumer maximizes

(B6) $V_1(W_1) + U(C_1) + \frac{1}{1+\rho}E_1(V_2(W^h_2))$

where the expectation operator is needed because $Y_2$ and hence the state $h$ in period 2 are uncertain when first-period decisions are made. To write (B6) as a function of $S_1^h$ and $S_1^O$, first replace $V_2(W^h_2)$ with (B5) and replace $W^h_2$ with (B2), and then use equation (4), the budget constraint, to replace $C_1$. This yields $V_1$ as a function of $S_1^h$ and $S_1^O$, as given in equation (B7), below. Maximization of (B7) with respect to $S_1^h$ and $S_1^O$ is straightforward, but tedious, and yields

(B7) $V_1(W_1) = -\left[k - W_1 + S_1^O + S_1^h\right]^2 - \frac{1}{1+\rho}E_1\left\{\left[(1-q_1^h)k - q_2^h \left(S_1^O R_h + \frac{S_1^h R_h^2}{R_h} + Y_2 + D_h\right)\right]^2 + \frac{1}{1+\rho} \left[(1-q_1^h R_h)k - R_h (1-q_2^h \left(S_1^O R_h + \frac{S_1^h R_h^2}{R_h} + Y_2 + D_h\right)\right]^2\right\}$
first-order conditions of the following form:

\[ S_0' = \alpha_1 k + \alpha_2 W_1 + \alpha_3 - \alpha_4 S_1' \]
\[ S_1' = \beta_1 k + \beta_2 W_1 + \beta_3 - \beta_4 S_0' \]

where the \( \alpha \)'s and \( \beta \)'s are complicated functions of all the parameters of the model, and \( \alpha_3 \) and \( \beta_3 \) depend on \( E_i[Y_2] \). Equation (B8) is equation (6) in the text.

To derive (11) in the text, note that, dropping the 1 superscript, when \( S_1' = L \), both \( S_1 \) and \( S_0 \) can be written in two ways, by (9) or (10). Using (10) for \( S_0 \), and (9) for \( S_1 \) yields

\[ S_0 = \alpha_1 k + \alpha_2 W_1 + \alpha_3 - \alpha_4 (\delta_1 k + \delta_2 W_1 + \delta_3). \]

By (9), however,

\[ S_0 = \delta_4 k + \delta_5 W_1 + \delta_6 \]

implying that \( \alpha_1 = \alpha_4 \delta_1 + \delta_4 \), \( \alpha_2 = \alpha_4 \delta_2 + \delta_5 \), and \( \alpha_3 = \alpha_4 \delta_3 + \delta_6 \). Substituting these relations into (10) for \( S_0 \) yields

\[ S_0 = \delta_4 k + \delta_5 W_1 + \delta_6 + \alpha_4 (\delta_1 k + \delta_2 W_1 + \delta_3 - L) \]

which is equal to (11) since \( S_1' = \delta_1 k + \delta_2 W_1 + \delta_3 \).

**Appendix C**

The model given by (8), (9), and (11) is

\[ S_1 = \begin{cases} 0 & \text{if } S_1' \leq 0 \\ S_1' = \mathbf{X}\mathbf{\beta} + u & \text{if } 0 < S_1' < L \\ L & \text{if } S_1' \geq L \end{cases} \]

\[ S_0 = \begin{cases} \mathbf{X}\mathbf{\gamma}_1 + \varepsilon_1 & \text{if } S_1' \leq 0 \\ \mathbf{X}\mathbf{\gamma}_2 + \varepsilon_2 & \text{if } 0 < S_1' < 0 \\ \mathbf{X}\mathbf{\gamma}_2 + \eta(S_1' - L) + \varepsilon_2 & \text{if } S_1' \geq L. \end{cases} \]

We jointly estimate \( S_1 \) and \( S_0 \) by maximum likelihood assuming that \( (u, \varepsilon) \) are distributed bivariate normal with standard deviations \( \sigma_u \) and \( \sigma_\varepsilon \) and correlation \( \rho_{u\varepsilon} \), \( j = 1, 2 \).

When \( S_1' \geq L \), the error term in the \( S_0 \) equation is \( \varepsilon_2' = \eta u + \varepsilon_2 \). The standard deviation of this expression is \( \sigma_{\varepsilon_2}^2 = (\eta \sigma_u)^2 + \sigma_\varepsilon^2 + 2 \eta \rho_{u\varepsilon} \sigma_u \sigma_\varepsilon \), and the correlation of \( \varepsilon_2' \) and \( u \) is given by

\[ \rho_{u\varepsilon} = \frac{\eta \sigma_u^2 + \rho_{u\varepsilon} \sigma_u \sigma_\varepsilon}{\sigma_u \sigma_{\varepsilon}^*}. \]

The log-likelihood function consists of the sum over the relevant portions of the following probabilities:

(i) if \( S_1 = 0 \) and the observed value of \( S_0 \) occurs,

\[ f_2(u, \varepsilon_2, \rho_{u\varepsilon}) \]

(ii) if \( 0 < S_1 < L \) and the observed value of \( S_0 \) occurs,

\[ \Phi \left[ \frac{\hat{S}_1 + \rho_{u\varepsilon} \sigma_u \sigma_\varepsilon (S_0 - \hat{S}_0)}{\sigma_u (1 - \rho_{u\varepsilon}^2)^{0.5}} \right] \]

\[ \times \left[ \frac{1}{\sigma_\varepsilon} \phi \left( \frac{S_0 - \hat{S}_0}{\sigma_\varepsilon} \right) \right] \]

(iii) if \( S_1 = L \) and the observed value of \( S_0 \) occurs,

\[ \Phi \left[ \frac{\hat{S}_1 + \rho_{u\varepsilon} \sigma_u \sigma_\varepsilon (S_0 - \hat{S}_0)}{\sigma_u (1 - \rho_{u\varepsilon}^2)^{0.5}} \right] \]

\[ \times \left[ \frac{1}{\sigma_\varepsilon^*} \phi \left( \frac{S_0 - \hat{S}_0}{\sigma_\varepsilon^*} \right) \right] \]
where \( f_2 \) represents the bivariate normal density, \( \Phi \) and \( \phi \) denote the univariate normal distribution and density, \( S_1 = X \hat{\beta} \), and

\[
\hat{S}_o = \begin{cases} 
X \hat{\gamma}_1 & \text{if } S_1 = 0 \\
X \hat{\gamma}_2 & \text{if } 0 < S_1 < 0 \\
X \hat{\gamma}_2 + \hat{\eta}(\hat{S}_1 - L) & \text{if } S_1 = L
\end{cases}
\]

and \( \hat{\eta}, \hat{\gamma}_1, \hat{\gamma}_2, \) and \( \hat{\beta} \) are estimated values of \( \eta, \gamma_1, \gamma_2, \) and \( \beta \).

### Appendix D—Number of Households Excluded by Each Sample-Selection Criterion

<table>
<thead>
<tr>
<th>Sample</th>
<th>Number of households excluded</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial sample</td>
<td>2,822</td>
</tr>
<tr>
<td>Head ≤ 24 years old</td>
<td>31</td>
</tr>
<tr>
<td>Head ≥ 65 years old</td>
<td>466</td>
</tr>
<tr>
<td>Change in marital status</td>
<td>330</td>
</tr>
<tr>
<td>Self-employed</td>
<td>469</td>
</tr>
<tr>
<td>Saving threshold ≤ $100,000</td>
<td>184*</td>
</tr>
<tr>
<td>Remaining sample</td>
<td>1,483</td>
</tr>
</tbody>
</table>

*Note: Some households are excluded by more than one criterion. The number of households excluded after all the previously listed exclusions have been made.

### Appendix E—Unweighted Sample Statistics of Variables Used in Estimation of Table 5

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRA contributions</td>
<td>1.325</td>
<td>2.988</td>
</tr>
<tr>
<td>Non-IRA saving</td>
<td>2.318</td>
<td>22.696</td>
</tr>
<tr>
<td>Age</td>
<td>45.483</td>
<td>12.130</td>
</tr>
<tr>
<td>Age squared</td>
<td>2.215</td>
<td>1.135</td>
</tr>
<tr>
<td>Income</td>
<td>1.035</td>
<td>1.228</td>
</tr>
<tr>
<td>Pension dummy</td>
<td>0.387</td>
<td>0.487</td>
</tr>
<tr>
<td>Years of education</td>
<td>12.857</td>
<td>2.797</td>
</tr>
<tr>
<td>Family size</td>
<td>3.011</td>
<td>1.434</td>
</tr>
<tr>
<td>Non-IRA financial assets</td>
<td>1.699</td>
<td>9.642</td>
</tr>
<tr>
<td>Total debt</td>
<td>2.135</td>
<td>4.174</td>
</tr>
<tr>
<td>Nonliquid assets</td>
<td>0.895</td>
<td>2.115</td>
</tr>
<tr>
<td>1983 IRA dummy</td>
<td>0.213</td>
<td>0.409</td>
</tr>
<tr>
<td>Liquidity-constrained</td>
<td>0.190</td>
<td>0.392</td>
</tr>
<tr>
<td>Home dummy</td>
<td>0.694</td>
<td>0.460</td>
</tr>
</tbody>
</table>

*Note: Units of measurement are listed in Table 5.*

### References


Collins, Julie H. and Wyckoff, James H. “Estimates of Tax-Deferred Retirement Sav-


**Engen, Eric M. and Gale, William G.** “IRAs and Saving in a Stochastic Life-Cycle Model.” Mimeo, University of California at Los Angeles, April 1993.


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