

Selective Incentives in an Apex Game

AN EXPERIMENT IN COALITION FORMATION

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The effect of a selective incentive system on the likelihood of collective action is tested using an experiment with an Apex Game, a power-imbalanced game in which the weak players choose between competing against each other to form an alliance with the strong player or cooperating with each other in a unanimous alliance of weak players (excluding the strong player). A theoretical introduction analyzes the nature and importance of selective incentives for collective action and demonstrates the relevance of Apex Game experiments for studies of collective action. Results confirm the predictions: Formation of the coalition of weak players rises from 20% in the control condition to 62% when a negative selective incentive system is added.

Selective incentives are private goods given to individuals to induce them to participate in collective action to provide some public good. Although their importance for collective action has been acknowledged since the publication of Mancur Olson's (1965) *The Logic of Collective Action*, little work has been done to analyze the impact of selective incentives or to assess the nature of the processes which govern their use.

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This report describes the results from an experiment designed to demonstrate the impact of selective incentives on collective action in a power-imbalanced game. The game chosen for research is the **Apex Game**, which has only occasionally been used in coalition game experiments, but which is especially well-suited for the study of collective action.

Because this research synthesizes several strands of past work, preliminary theoretical discussions provide its context by analyzing the importance of selective incentives for collective action and by demonstrating the relevance of the Apex Game for studies in collective action. The actual experiment has a simple design which demonstrates the overwhelming impact of selective incentives in the situation. These results indicate the usefulness of pursuing this line of research in future experiments with more elaborate designs.

THE LOGIC OF COLLECTIVE ACTION

In his now-famous introduction, Mancur Olson (1965) persuasively attacked the common belief that rational individuals should act to provide themselves with collective benefits:

But it is *not* in fact true that the idea that groups will act in their self-interest follows logically from the premise of rational and self-interested behavior. It does *not* follow, because all of the individuals in a group would gain if they achieved their group objective, that they would act to achieve that objective, even if they were all rational and self-interested. Indeed, unless the number of individuals in a group is quite small, or unless there is coercion or some other special device to make individuals act in their common interest, *rational, self-interested individuals will not act to achieve their common or group interests.* [1965: 1-2, emphasis in original].

The logic of Olson's argument is founded in economic theories of public goods, where a good is a public good if its provision to some member of a group means that it cannot feasibly be withheld from others in that group (Olson, 1965: 14). Economists have shown that consumers will not act to "buy" public goods, a problem often referred to as the "theory of market failure." (See, for example, Head, 1974; Rowley and Peacock, 1975.) Olson's contribution was to see: "*The achievement of any common goal or the satisfaction of any common interest means that a public or collective good has been provided for*

that group. The very fact that a goal or purpose is *common* to a group means that no one in the group is excluded from the benefit or satisfaction brought about by its achievement" (1965: 15, emphasis in original).

Olson goes on to argue that apparent instances of collective action have actually been due to the presence of *selective incentives*:

Only a *separate and "selective" incentive* will stimulate a rational individual in a latent group to act in a group-oriented way. In such circumstances group action can be obtained only through an incentive that operates, not indiscriminately, like the collective good, upon the group as whole, but rather *selectively* toward the individuals in the group. The incentive must be "selective" so that those who do not join the organization working for the group's interest, or in other ways contribute to the attainment of the group's interest, can be treated differently from those who do. These "selective incentives" can be either negative or positive, in that they can either coerce by punishing those who fail to bear an allocated share of the costs of the group action, or they can be positive inducements offered to those who act in the group interest [1965: 51].

Olson's work has been criticized, principally by showing that his argument depends upon implicit assumptions which restrict the scope of his conclusions more than his prose would imply. Frohlich and Oppenheimer (1970) argue that predictions about a person's willingness to cooperate with collective action *must* include a probability estimate of the likely behavior of others. Frohlich et al. (1975) demonstrate that Olson's conclusions hold only if the value of the collective good to the group and the amount of collective good produced are linear functions of the number of persons who cooperate; when either or both of these functions are not linear, Olson's conclusions may not hold. Frohlich and Oppenheimer (1978: 48-65) demonstrate that provision of "lumpy" goods may be rational when provision of comparable, continuously divisible goods would not be.

Several authors have addressed Olson's assertion (1965: 44) that the probability of collective action declines with group size. It should be noted that this assertion is *not* formally derived from the preceding mathematics, which appear about eight pages earlier. Chamberlin (1974) argues that Olson fails to consider effects of income elasticity; when these are considered, the probability of collective action may increase, decrease, or be unaffected by group size, depending on the elasticity. Schofield (1975) approaches the issue as a game-theoretic coalition formation problem; he argues that if a large enough subset of actors from a cooperative coalition exists, this coalition will expand to include all actors, although they will not distribute the costs

of cooperation equally. Bonacich et al. (1976) define a typology in which the gain from cooperating and the temptation of not cooperating each may increase, decrease, or remain constant with group size. They provide examples for five of the nine possible types. Again, the likelihood of collective action decreases with group size for some types, while for others it increases or remains constant.

Selective Incentives

Olson argues that collective action is generally impossible for rational individuals unless the collective good is augmented with selective incentives, private goods which can be presented contingent upon cooperation with collective action. In his book, he reviews and analyzes the history of labor unions, class groups, and pressure groups to support his claim that successful collective action requires selective incentives.

It is fortunate that others have shown that Olson's derivations are not as general as he claims, because, as Frohlich and Oppenheimer (1970: 120) argue, selective incentives cannot be a solution to collective action if collective action is always impossible, for the provision of a good as a selective incentive to motivate others' collective action is itself a form of collective action. It is only because collective action sometimes *is* rational that selective incentives may be used and be important.

Frohlich and Oppenheimer miss a key insight when they use this argument to dismiss the importance of selective incentives. The insight is this: Selective incentives can turn a collective action situation in which cooperation is *irrational* into one in which collective action is *rational*. The utility functions and cost curves for the decision to use the incentive may be different from those for the original collective action decision. It may be rational for individuals to use their incentives even when it is not rational for individuals to act collectively. If the incentives are used, they change the original situation to one in which collective action *is* rational.

There are five properties a good must have to be a selective incentive:

- (1) The good must be a private good. This is Olson's central point and requires no elaboration.

1. These points are elaborated and their implications discussed in greater detail in Oliver (1978).

- (2) It must be possible to defer distribution of the incentive until *after* actors have made their cooperative or noncooperative responses, or have been irrevocably committed to them. Norms of reciprocity might be thought to allow incentives to precede action, but they really imply the presence of a larger system of incentives which punishes those who break agreements.
- (3) The good's value to the potential participants must be large enough to make the expected value of cooperation greater than the expected value of noncooperation. That is, if M is the magnitude of the incentive, and c is the cooperative response, while d is the uncooperative response, then $E(d) - E(c) < M$.
- (4) The good must be controlled by actors who would benefit from its use as an incentive. That is, the good must be controlled by actors whose payoff is higher if the collective action occurs than if it does not occur. The actors who control the incentive may or may not be potential participants in the collective action.
- (5) Actors who control the good must determine that the expected value of allocating the good contingent upon cooperation is greater than the expected value of not doing so. The difference in the expected value of using a good as an incentive and not using it as an incentive can be shown to be a multiplicative function of three factors: the difference the use of the incentive makes in the probability of the other's cooperation; the value of the collective good to the incentive-controller; and the difference the other's cooperation makes in the probability of a successful collective action.²

Positive and Negative Incentives

Olson's discussions of selective incentives make no distinctions between positive and negative incentives. And, at the individual level, there is no fundamental difference between them. Regardless of whether it is positive or negative, the magnitude of the incentive must be greater than $E(d) - E(c)$ if it is to make collective action rational. There may be differences in people's emotional responses to rewards and punishments, but in terms of rational decision-making, there is no particular difference between them.

This equivalence at the individual level does *not* hold structurally. Positive incentives are given to people who cooperate; when collective action is successful, positive incentives will be expended to reward cooperators. Negative incentives, on the other hand, are given to those who do not cooperate; this means that when collective action is successful, the incentive may not have to be used at all, if everybody cooperates.

Positive incentives are more "efficient" when a relatively small portion of the total population must cooperate for the collective good to

2. The derivation of this result is presented in the appendix.

be provided. Negative incentives, on the other hand, are more efficient when unanimity, or near-unanimity, is required. If only 5% of the population needs to contribute to the arts fund to make it successful, they can be rewarded by having their names printed in a program; it would be silly and wasteful to try to punish the 95% who did not contribute. Conversely, a strike requires near-unanimity to succeed, and it is most efficient to threaten to punish "scabs."³

THE APEX GAME

The Apex Game is a power-imbalanced coalition formation game first described by von Neumann and Morgenstern (1947: 473-503) and extensively analyzed by Horowitz and Rapoport (1974). An n -person Apex Game has one player called an Apex and $n-1$ players called Bases; its essential feature is that the only legal winning coalitions (those with positive payoffs) either must include the Apex or must include all other players except the Apex. Apex Games may be defined either with characteristic functions or as weighted-majority games.

The Apex Game creates choice dilemmas for the Base players which are particularly intriguing for a researcher interested in the problems of power imbalance and collective action. The Apex Game is heavily power-imbalanced; the more Bases there are, the more imbalanced the game is, since the Apex essentially has the weight of $n-2$ Bases. Horowitz and Rapoport (1974) describe this situation as follows:

The Apex's position may be compared to that of a monopolist, with the only limitation that he must find at least one ally. Only the coalition of all other players against him may defeat him. The Base's position poses an intriguing dilemma: he must either cooperate with all other Base players regardless of their number, or he must join the Apex and possibly some other Base players. If the first course

3. Consider incentives of some magnitude M , one positive and one negative. Let n be the total number of people in the group of potential cooperators, and let m be the number who actually cooperate. If collective action occurs, a positive incentive must be given to m actors, meaning that the total amount of incentive given to all actors must be Mm . On the other hand, the amount of negative incentive necessary if collective action occurs is $M(n-m)$. $M(n-m) = Mm$ only in the special case where $m = n/2$, the special situation where exactly half of the group cooperates. The total amount of incentive necessary is greater for positive incentives than negative when $n/2 > m$; conversely, a greater amount of negative incentive is necessary when $n/2 < m$. Thus positive incentives are more "efficient" when a small proportion of the total group cooperates, and negative incentives are more efficient when a large proportion of the total group cooperates. The implications of this are elaborated in Oliver (1978: 27ff).

of action is chosen, the Base player risks being frozen out of a winning coalition, if one or more Base players yield to the temptation of extra gain by forming a coalition with the Apex. On the other hand, if he chooses to negotiate with the Apex, the Base must consider the highly competitive environment produced by the Apex's multitude of choices in stating his demand for his share [1974: 162].

The standard triad game which has been the subject of so many experiments in coalition formation is formally an Apex Game, but it is a degenerate case which makes it inappropriate for generalizing about behavior in power-imbalanced situations. If a triad game can be represented in characteristic function form (this includes weighted-majority games), if it has a constant sum, and if no one player can win alone, the game resolves to the abstract game in which any two players can defeat a third (Rapoport, 1979: 85-86). Thus, there is no true power imbalance in the three-person Apex Game. Experiments with triads have produced contradictory and mixed findings. (Reviews of research with triads may be found in Tedeschi, et al., 1973; Chertkoff, 1979; and Burhans, 1973.) All theoretical analyses and empirical results described in this article refer to Apex Games with four or more players.

Horowitz and Rapoport (1974) have derived predictions from Apex Games from several standard game-theoretic solutions for n -person games. These are summarized in Table 1. To facilitate interpretation of the results, Horowitz and Rapoport's expressions have been translated so that the basic term, m , represents the number of Bases in the game; k represents the number of Bases who coalesce with the Apex when $k > 1$. All these approaches predict two or more possible coalitions as "rational" outcomes, except the Shapley value which assumes formation of the grand coalition of all players. These approaches provide no predictions about the relative probabilities of the predicted coalitions. For example, Horowitz and Rapoport's competitive bargaining set, which is identical to von Neumann and Morgenstern's (1947) main simple solution, predicts that *either* the coalition of the Apex and one Base, with the indicated payoff division, *or* the weak union of all the Bases, with equal division of payoff, is the expected outcome. This is consistent with the focus of game theory on how coalition members will apportion their joint payoff, rather than on which coalition will form (Rapoport, 1970: 286).

Every theory predicts that the Base coalition will divide the payoff equally when it forms. The theories differ in the division they predict within an Apex-Base coalition, and to some extent in *which* coalitions they predict will form. All theories except the Shapley value predict

TABLE 1
 Predictions for Which Coalitions Will Form and How They Will Divide the Payoff in the Apex Game,
 as Given by Several Game Theorists (Summarized from Horowitz and Rapoport, 1974; Notation Modified)

Solution Name	Original Theorists	Payoff Division Predicted for Each Coalition (Blanks indicate coalition is predicted not to occur)						
		Grand Coalition		Apex and k Bases		Apex and 1 Base		Base Coalition
		Apex	Each Base	Apex	Each Base	Apex	Base	Each Base
Main simple solution	von Neumann and Morgenstern					$\frac{m-1}{m}$	$\frac{1}{m}$	$\frac{1}{m}$
Shapley Value	Shapley	$\frac{m-1}{m+1}$	$\frac{2}{m(m+1)}$					
Bargaining Set (range of predicted outcomes)	Aumann and Maschler	$\frac{m-1}{2m-1}$	$\frac{1}{2m-1}$	$\frac{1}{2}$	$\frac{1}{2k}$	$\frac{1}{2}$	$\frac{1}{2}$	
		to	to		to	to	to	$\frac{1}{m}$
		$\frac{(m-1)^2}{m(m-1)+1}$	$\frac{1}{m(m-1)+1}$	$\frac{k(m-2)+1}{k(m-1)+1}$	$\frac{1}{k(m-1)+1}$	$\frac{m-1}{m}$	$\frac{1}{m}$	
Kernel	Davis and Maschler			$\frac{1}{2}$	$\frac{1}{2k}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{m}$
Competitive bargaining set	Horowitz and Rapoport					$\frac{m-1}{m}$	$\frac{1}{m}$	$\frac{1}{m}$

m = number of Base players in the game; k = number of Base players in Apex-Base coalition.

that the Base coalition is as likely to form as one or more of the Apex-Base coalitions.

Empirical studies with Apex Games do not support these implicit predictions on which coalition will form. Only five previous experiments with Apex Games having four or more players have been located: one small experiment conducted by Selten and Schuster (1968); a cumulative series of three experiments by Chertkoff, Komorita, and Meek (Chertkoff, 1971; Komorita and Chertkoff, 1973; Komorita and Meek, 1972); and a computer-assisted experiment by Horowitz and Rapoport (1974). These experimenters were interested in payoff divisions within coalitions, not in which coalition formed. However, this research focuses on the relative frequency of the all-Base coalition versus coalitions involving the Apex, so Table 2 summarizes the frequency of formation of coalitions of various types found in past experiments. The data from the control condition of my experiment are added for comparability.

The most striking result from all research with the Apex Game is that the Base coalition rarely forms. Past game-theoretic analyses do not account for this result; all except the Shapley value predict the Base coalition as one rational outcome. The Shapley value's prediction of the grand coalition is clearly not supported. The prediction of the competitive bargaining set and the main simple solution that coalitions involving the Apex will include only one Base tends to be supported, although the data are meager since other coalitions were ruled out in most experiments. Thus, none of the previous theory that has been applied in these experiments has predicted or explained the most striking consistency in the results.⁴

The Apex Game as a Prisoner's Dilemma

Previous game theory analyses have failed to explain the results of Apex Games precisely because they have failed to recognize the

4. A reviewer (Joe Oppenheimer) has argued that the result is not surprising if each possible coalition is viewed as equally probable, since there are four possible coalitions in the four-person Apex Game (the all-Base coalition and three Apex-Base coalitions) and five possible coalitions in the five-person Apex Game, making the expected probability of the all-Base coalition .25 and .20 respectively in these games. The observed proportions are in all instances smaller than these expected values, although only slightly smaller in the two experiments allowing face-to-face bargaining and in the control condition of this experiment. The proportions in experiments which rotated positions and did not allow face-to-face communication are far lower than these expected values.

TABLE 2
Summary of Results of Apex Game Experiments

<i>Researchers</i>	<i>Numbers of Trials</i>	<i>Payment</i>	<i>Mode of Communication</i>	<i>Grand Coalition</i>	<i>Percentage of Trials in which Each Coalition Formed</i>		
					<i>Apex and k Bases</i>	<i>Apex and l Base</i>	<i>Base Coalition</i>
Selten and Schuster	12 5-man groups (one-shot)	about \$10 per group, to divide	face-to-face	8%	8%	67%	17%
Chertkoff	20 4-person groups* (one-shot)	none	face-to-face	--**	0%	70%	20%
Komorita and Meek	10 4-person groups played 10 rounds each, rotating roles (N = 100 rounds)	some; amount not reported	system of messages	--**	5%	89%	6%
Komorita and Chertkoff	10 4-person groups played 28 rounds each, rotating roles (N = 280 rounds)	some; amount not reported	system of messages	--**	2%	9%	4%

Table 2 (Continued)

Researchers	Numbers of Trials	Payment	Mode of Communication	Percentage of Trials in which Each Coalition Formed		
				Grand Coalition	Apex and k Bases	Apex and l Base Base Coalition
Horowitz and Rapoport		\$3.60 or \$5.40 per round to divide, plus 75¢ per hour	computer terminals			
4-man	24 rounds***			--**	--**	92%
5-man	24 rounds***			--**	--**	96%
Oliver	9 4-person groups played 9 rounds each****	\$2.70 per round to divide	written messages read aloud by experimenter	--**	--**	90%

*Two groups, 10%, failed to reach any agreement.

**This coalition impossible in this experiment.

***Twelve 5-man groups played two 4-person and two 5-person games each, rotating positions.

****Players retained positions across all nine rounds; Apex was a confederate of the experimenter who always attempted a 50-50 division of payoff with one Base, rotating among Bases across rounds.

qualitative difference between the Apex and the rest of the players. Only by examining the Apex and the Bases as different types of actors can the clear regularities in the way subjects play the game be interpreted. It should not be surprising that the impact of power imbalance must be taken into account in order to explain behavior in a power-imbalanced situation.

Dissection of the possibilities of the game (always assuming that each player seeks to maximize his own earnings) reveals that the Bases' situation is equivalent to a prisoner's dilemma.

At one extreme, if the Base coalition is effectively impossible, possibly because of missing communication networks or some history of intragroup hostilities, the expected payoff to each Base is zero, because the Bases can be expected successively to underbid one another, each one preferring something to nothing. The Apex's expected payoff in this situation is the total payoff.⁵

The other extreme occurs when the Base coalition *is* viable, but the Apex attempts to play for maximum gain as if it were not, refusing to be "loyal" to any Base, always being willing to accept a better offer, attempting to drive his share as close as possible to the total payoff. Such pseudorational behavior on the part of the Apex would force formation of the Base coalition, leaving the Apex with nothing. To see this, assume (in line with the overwhelming consistency of theoretical and empirical results) that each Base's share of the payoff in a Base coalition is P/m , where P is the total payoff, and m is the number of Bases. The Bases have a viable option of receiving P/m , so the Apex cannot force his share to be more than $P - (P/m) = P(m-1)/m$. This is the basic idea behind Horowitz and Rapoport's (1974) competitive bargaining set.

If all legal coalitions are viable and if all players behave rationally (rather than pseudorationally), Horowitz and Rapoport argue that either the Base coalition, giving each Base P/m , or the Apex-Base coalition, giving the Base P/m and the Apex the rest, is a rational solution. The Base would be indifferent as to either choice. But rational players should not only consider their payoffs, they should consider the probability of obtaining that payoff. Look at this from the Apex's perspective: If he offers a Base P/m , the expected value to him of that proposal is $\frac{1}{2}P(m-1)/m$, since if the Base is indifferent as to two

5. This analysis is based on continuous (infinitely divisible) payoffs. Practically, the Base's share would be the smallest unit different from zero (or psychologically different from zero), and the Apex's share would be the total payoff minus this smallest unit.

options, he will choose each with probability .5. But if the Apex offers the Base just the next psychologically meaningful unit more than P/m (i.e., $P/m + 1$), then the Base's probability of acceptance should approach unity, since he would prefer $P/m + 1$ to P/m . If his probability of acceptance is 1.0, then the expected value to the Apex of making this proposal would be $P(m-1)/m - 1$. The expected value of proposing P/m equals the expected value of proposing $P/m + 1$ only when $P = 2m/(m-1)$, a condition ruled out by our assumption that $m > 2$ and that P is expressed in the smallest psychologically meaningful units, making it at least several times larger than 1.⁶

Thus, a rational Apex should offer one Base a little more than P/m and should act in ways that make the Base's probability of acceptance as close as possible to unity.

But what about the Bases? If the Apex behaves as prescribed, the Bases' situation is equivalent to a multiperson prisoner's dilemma. Every Base receives P/m if the Base coalition forms, but a defector can obtain more than this by forming a stable alliance with the Apex. Regardless of what the other Bases are doing, each Base's rational choice is to negotiate with the Apex: If the other Bases are trying to promote the Base coalition, the defector can be assured of a better payoff by bargaining with the Apex, and if any other Base is negotiating with the Apex, a Base's only choice is to enter the competition, since the Base coalition requires the unanimous cooperation of the Bases.

There are extensive debates in the prisoner's dilemma literature about whether it is more "rational"—in an evaluative sense—to defect or to cooperate (since the individually irrational cooperators achieve higher payoffs than rational defectors), but the empirical facts are clear: Prisoner's dilemmas usually but not always result in players locking into the noncooperative options, and the amount of cooperation declines rapidly with the number of players involved (Bixenstine

6. Alternatively, if the Apex makes the same offer to all Bases and coalesces with the first one who accepts, the probability that at least one of m Bases who are indifferent as to their two options will accept the Apex's offer is $1 - .5^m$. The expected value of offering $P/m + 1$ is still $P(m-1)/m - 1$, since virtual certainty of acceptance is still assumed. The expected value of offering P/m is now $P(m-1)/m \cdot (1 - .5^m)$. The two expected values are equal when $P = m/(m-1) \cdot (1/.5^m) = 2^m m/(m-1)$. As m increases, P must increase as 2^m ; this is the number of times greater P must be than the smallest "psychologically meaningful" unit. For small m (as in experiments), this is relatively small. (For example, if $m = 5$, $P = 40$.) For large m , P gets quite large. (For example, if $m = 10$, $P = 1138$.) This means that it is not necessary for Apex to take one unit less than P/m to assure virtual acceptance by some Base if m , the number of Bases, is fairly large.

et al., 1966; Hamburger et al., 1975; Kahan, 1973; Kalisch et al., 1962; Goehring and Kahan, 1976; Rapoport, 1975). Thus, once it is seen that the Bases in an Apex Game face a decision equivalent to a multiperson prisoner's dilemma, the low frequency of the Base coalition is easily explained.

The Apex Game as Collective Action

If rational play by the Apex places the Bases in a situation equivalent to a prisoner's dilemma, then cooperation with the Base coalition is an instance of collective action in Olson's sense of providing some common good. This equivalence was first demonstrated by Hardin (1971) and elaborated upon by Dawes (1975). Bases who seek only to form the Base coalition raise other Bases' expected winnings, regardless of what the others wish to do. To look at it another way, each Base prefers that other Bases bargain solely with the Base coalition so that he may obtain his share of the Base coalition's payoff *or* obtain a favorable distribution of the payoff in bargaining with the Apex without competition.

Thus it is relevant to follow up on Olson's claim that selective incentives make collective action rational when it otherwise would not be. As the theoretical review of the collective action literature revealed, selective incentives are not the only possible solution to the dilemma of collective action. Howard (1971) shows that the dilemma may be resolved with metagames, in which players essentially make choices of strategies of play ("I'll cooperate if you will and not if you won't"). This probably accounts for findings of increased cooperation in multiperson prisoner's dilemmas when communication between subjects is allowed (Dawes et al., 1977) or when each subject's choice is "public" (Fox and Guyer, 1978); however, Caldwell (1976) found that just letting the subjects talk did not significantly increase their cooperativeness. Significantly for this research, however, Caldwell's third condition, in which subjects were not only allowed to talk but to punish each other if two or more players voted to punish the same person, produced substantially higher levels of cooperation than the other two conditions.

The central proposition of this experiment is that collective action in a mixed-motive or prisoner's dilemma situation (in this case, formation of the Base coalition in an Apex Game) is more likely to occur in the presence of a potential selective incentive than in its absence. In

this experiment, the Base players are given access to a system whereby they *may* reward or punish one another if they so choose, but they are *not* required to do so.

If subjects were merely rewarded or punished by the experimenter according to their behavior in the Apex Game, their behavior would probably change, but the experiment would hardly be interesting for students of collective action. The problematic, and therefore interesting, feature of this research is that subjects are free to use the incentive system, to ignore it, or to use it in ways other than intended by the experimenter. Thus, the prediction is not simply that people will do what they are reinforced for doing. Rather, persons in a power-imbalanced situation will be aware of the qualitative differences between themselves and the strong person, will be motivated to form a union of the weak players if they have the means to do so, and will choose to use an incentive system available to them to achieve the goal of cooperation among the weak players. The power imbalance in the game makes the Apex Game different from prisoner's dilemma games, because the weak players may choose how to perceive their situation in a primitive experimental analogue of class consciousness.

METHODS AND PROCEDURES

The Apex Game

Eighteen groups of three subjects each played the Bases in a four-person Apex Game with characteristic function $AB = AC = AD = BCD = \$2.70$, with all other coalitions defined as impossible. A lengthy instruction tape (with transcripts for subjects to follow along) explained the game in terms of coalitions and bargaining in neutral terms which made no mention of inequality, collective action, or other such concepts. Subjects were seated in the small room and separated by partitions which prevented them from seeing one another; they were asked not to talk. Subjects seem to have cooperated with the experimenter's request that they remain anonymous to one another; there is no evidence of distortion of data due to a few subjects' glimpses of one another. Subjects communicated by passing bargaining slips to the experimenter, who read them aloud. Communication was restricted to making, accepting, or rejecting proposed divisions of payoffs to the

coalitions of which one could be a member. Play was in alphabetical order, rotating who went first on each round. Subjects were allowed to bargain as long as they wished, until a subset of players stayed with a particular proposed division of the payoff for three complete turns of bargaining, after which they could ratify the agreement and end the round. Subjects were told their cumulative earnings at the end of each round, but they were not actually paid until the end of the experiment. Each group played nine rounds.⁷

The Apex

Position A (the Apex) was a student confederate paid by the hour. He played according to a set of instructions that exactly determined his actions but was sufficiently complex to appear fairly "natural." His instructions were designed to place the Bases in a multiperson prisoner's dilemma: He sought an even split of the payoff with one Base on each round, and rotated from round to round which Base he would bargain with. He always sought to win; if the player whose "turn" it was would not cooperate, he would make a proposal to someone else. If he could not strike a bargain for an even split, he would take less in a proposal, except that he would not take less than five points.

The Selective Incentive

The Bases in an Apex Game must cooperate unanimously if they are to form the Base coalition, so an efficient incentive must be negative.⁸ The incentive system was called the "Ding Game" and was constructed to meet the theoretically specified characteristics of a selective incentive. After each round of bargaining, each Base player was allowed to give out up to 90¢ worth of negative points ("dings"). Punishment was public but anonymous: Everyone knew how much each player had lost, but not who had given what to whom. Negative scores did not accumulate: The worst a player could do after punishment was to end up with zero. It was emphasized that players could give anywhere between none and 90¢ worth of punishment, that there was no penalty

7. See Oliver (1977) for more detailed information on methods and procedures.

8. A positive incentive would have to be worth $m - 1$ times the total payoff of the game, while a negative incentive needs to be worth only $(m - 1)/m$ times as much.

for not using the punishment, and that recipients would really lose the money and not get it back.⁹

Design and Sample

The design simply contrasted a control group which played the unadorned Apex Game with a treatment group which played the Apex Game augmented by the selective incentives game. Assignment of three-person groups to treatment condition was entirely random; this was done after subjects had been seated in their cubicles, and the instructions tape for the Apex Game was running. There were no violations of randomization.¹⁰

Subjects were summer school undergraduates at the University of North Carolina, who were recruited on the basis of availability for a

9. These rules were designed to make using the incentive always rational. Criterion 5 for a selective incentive identifies three components to consider in determining the expected value of using an incentive:

- (i) The value to the actor of the collective good. Each Base always benefits from others' not attempting to bargain with the Apex.
- (ii) The difference the others' behavior makes in the chances of obtaining the collective good. Again, any Base's withdrawal from competition for the Apex's favor increases the others' chances of winning a higher payoff in the round.
- (iii) The difference using the incentive makes in the other's behavior. This should be positive. If the player controlling the incentive does not understand simple principles of reinforcement or otherwise decides that making something unprofitable to others will reduce their chances of doing it, or if the player receiving the punishment for some reason persists in doing what he is punished for, this term might be zero in a player's evaluation.

By attempting to make the cost of using the punishments as close to zero as possible, and assuming that the benefit of using the incentive is always positive, using the incentive should always be rational.

The rules described were developed during pretesting when it became clear that the threat of retaliation for punishment was a serious "cost" of using the incentive; these rules are designed to minimize the possibility and impact of retaliation. Because there were only three Bases, it was sometimes still possible for a player to identify the source of his punishment. (Other times he *believed* he knew the source, but was wrong.)

10. Although there were no violations of randomization, two situations contributed to minor deviations from the ideal of no influence on the subjects that could be confounded with treatment. First, the experimenter knew that the last two groups had to be control groups, since the last slip for the incentive condition was drawn on the eighteenth trial; this occurred after the subjects had been scheduled for the trials, so the only possible "contamination" could have occurred in the period of face-to-face interaction when the experimenter met each subject and conducted him to the experiment room. Second, two groups (one in each condition) had to be eliminated due to excessive suspicion on the part of one or more subjects, which effectively disrupted the BCD coalition. This leaves the results unaffected, since one group favored the hypothesis and the other did not.

three-hour time block and a desire to participate in an experiment in which earnings would depend on how well they played a game with "some competitive and some cooperative elements." A truthful estimate was given of the range of possible earnings in the experiment. Students were assigned to particular time blocks solely on the basis of availability. The difficulties involved in scheduling three persons for the same long time period precluded blocking by sex, race, or any other characteristic.

Blind luck and a small sample put a disproportionate number of females in the control condition. Preliminary analyses revealed that females tended more often than males to form a coalition with the Apex, as did persons in the control condition. To be sure that the unfortunate sex distribution did not account for the relation between the independent and dependent variables, all analyses were also performed controlling for sex. Sex of subjects in each condition accounts for a significant, but small, proportion of the total variance in the dependent variable, as shown below.

RESULTS

The hypothesis of this experiment was overwhelmingly supported: Subjects were much more likely to form the weak union, the Base coalition, when they had the ability to punish one another than when they did not. Table 3 summarizes this result.

To verify that the unfortunate sex distribution does not account for the results, the relationship is controlled for sex. Since sex is an individual characteristic, the dependent variable in this instance is the percentage of trials in which the individual subject coalesced with the Apex: The total of this variable for all three subjects in a group is the complement of the percentage of rounds in which the group formed the Base coalition. Table 4 gives the appropriate means and measures of association. Although sex has an effect on behavior, the effect is much weaker than that of the presence of the incentive system.

These sharp differences in behavior in the two conditions were mirrored by differences in subjects' responses on postexperimental questionnaires. When asked what they thought would have been the best way to earn the most money in the Apex Game, only 15% in the control condition responded with an answer indicating that repeated formation of the Base coalition (BCD) was best, while 81% of the in-

TABLE 3
Effect of Treatment Condition (presence or absence of ability to punish) on Mean Percentage of Rounds in Which the Base Coalition Formed (N=18 groups of three subjects each)

<i>MEANS</i>					
<i>Condition</i>	<i>Percentage Base Coalitions</i>		<i>Standard Deviation</i>		<i>n</i>
Control	20		17		9
Incentive	62		23		9

<i>ANALYSIS OF VARIANCE</i>					
<i>Source</i>	<i>Sum of Squares</i>	<i>d.f.</i>	<i>Mean Square</i>	<i>F</i>	<i>p</i>
Between conditions	64.22	1	64.22	17.784	.001
Within conditions	57.78	16	3.61		
Total	122.00	17			

ETA² = .53

TABLE 4
Relationship Between Experimental Condition and Percentage of Trials in Which the Individual Subject Coalesced with the Apex, Controlling for Sex

	<i>MEANS</i>				<i>ETA²</i>
	<i>Control</i>	<i>(n)</i>	<i>Incentive</i>	<i>(n)</i>	
Total Sample	27%	(27)	13%	(27)	.28
Females	28%	(21)	16%	(14)	.24
Males	24%	(6)	9%	(13)	.24

centive-condition subjects gave this response ($\phi^2 = .89$). Not only did their perceptions of the Apex Game differ, but their perception of the Apex (player A) differed dramatically between conditions. Subjects were asked to state how they "felt about" each of the other players. Responses concerning player A were grouped into the categories shown in Table 5. About a quarter of the subjects in each condition had no particular feelings about player A, but those who did react to A reacted very differently in the two conditions. Particularly striking are the nearly half of the incentive-condition subjects who felt sorry for A, when none of the control subjects did, and the much higher hostility toward A in the control condition.

TABLE 5
 Percentage Distribution of Subjects' Feelings about Player A,
 by Treatment Condition (N=54 individual subjects)

<i>Feeling About Player A</i>	<i>Condition</i>	
	<i>Control</i>	<i>Incentive</i>
Hostility or other negative feelings	56%	15%
Positive response, i.e., respect, admiration, except sympathy or pity	19	4
Sympathy, pity, feel sorry	0	44
Mixed negative and positive or sympathetic	0	11
Indifference, or no feelings expressed	26	26
Total %	101% ^a	100%
N	(27)	(27)

a. Rounding error.

The presence of the incentive clearly made a large difference. The prediction was that subjects would punish each other contingent upon cooperation with the Apex, and not otherwise, thus making it rational for all of them to cooperate in the Base coalition. Inspection of actual punishing behavior reveals that subjects generally behaved as expected, but they tended to punish less than expected contingent upon coalescing with the Apex, and occasionally punished each other at other times. The mean number of points of punishment of each type is given in Table 6.

Contrary to expectations, it was not the case that groups who had the highest levels of contingent punishment (or the lowest levels of other punishment) had the highest levels of formation of the Base coalition: The correlations, while weak, are in the opposite direction from the prediction. Based on a qualitative assessment of the records of bargaining, the explanation appears to be that many subjects refrained from punishing or punished only a small amount at first; only if a subject persisted in his refusal to cooperate with the Base coalition could he count on consistent maximum punishment from the other subjects. Thus the correlation is negative, since groups who hit early upon cooperation never needed to punish one another heavily.

There is no doubt that the presence of the selective incentives had a dramatic effect on subjects' behavior, but isolating the nature and reasons for this effect is more difficult. Table 7 summarizes subjects'

TABLE 6
 Summary of Incentive Use
 by Subjects in the Incentive Condition

	<i>Points</i>
Maximum possible to give on a round	90
Mean given to a player who had coalesced with the Apex in that round	50
Mean given to a player who had cooperated with the Base Coalition in that round	9
Mean given to a player who had not formed a coalition in that round (harmless punishment)	6

TABLE 7
 Subjects' Reports of How They Played the Ding Game, and
 Conclusions on How the Game Should be Played

<i>Ding Game Playing Policy</i>	<i>Actually Played</i>		<i>Should Play</i>	
	<i>f</i>	<i>%</i>	<i>f</i>	<i>%</i>
Contingent: punish player for not cooperating with BCD (and not otherwise)	9	33%	16	59%
Retaliation for other's punishment	3	11	0	0
Equalize payoffs	3	11	0	0
Never punish	1	4	5	18
Other articulated policy (no two alike)	3	11	4	15
No consistent policy	3	11	1	4
Mixture of any of the above	5	18	0	0
No answer, or uninterpretable answer	0	0	1	4
Total	27	99% ^a	27	100%

a. Rounding error.

responses to two open-ended questions, one asking them how they actually used their punishments, the other asking them "now that the game is over" how the punishments should be used. Actual behavior was spread out widely, but subjects' final views zero in sharply on contingent punishment: Nearly three-fifths (59%) gave this response. All but one of the five subjects who felt that punishment should never

be used also felt that the BCD coalition was best, apparently feeling that BCD was an obvious choice which should not have to be enforced by punishment. Just over a fifth of the subjects (6, or 22%) gave various complicated answers which were neither similar to each other nor particularly clear to the researcher.

It appears that at some point during the nine trials, three-fifths of the subjects came to understand the rational structure of the compound Apex-Ding game. Their actions, coupled with the possibly nonaggressive actions of those who decided it was rational not to punish at all, were sufficient to influence the behavior of other players so that nearly everyone came to see BCD as the coalition of choice, even the substantial minority who never understood the rational structure of the game. Rational actors who are equipped with some means of influencing others' behavior may be able to lead the whole group to a collectively rational solution, even when the others respond only to the incentives which impinge directly upon them and lack a correct understanding of their situation.

Attempts to identify antecedents of subjects' understanding of the game indicated that females were much more likely than males to state one of the confusing "other" strategies for using the punishments; other antecedents of these answers (which tend to be correlated with sex) were seeming not to understand the game, reporting a goal other than maximizing earnings for the game, and perceiving anything other than formation of BCD as the best strategy for the Apex Game. (Table 8 summarizes these results.) None of these variables distinguished those who felt it was best not to punish at all. Table 9 shows essentially the same results for actual punishment policy, except that those who wanted to maximize their earnings were slightly *less* likely to punish contingently.

In sum, relatively few players of the compound Apex-Ding game understood the game and played it "rationally" from the start, but the majority of subjects came to appreciate its structure and to use this understanding to induce others to form the BCD coalition. There were some holdouts who refused to cooperate with BCD even in the face of massive punishment, and some groups who never generated much punishment for cooperating with A, but the overall pattern of results was strongly in line with the predictions of the model for rational players.

TABLE 8
 Cross-Tabulation of Subject's Conclusion on Best Punishment
 Policy and Individual Attributes (N = 27)

<i>Individual Attributes</i>	<i>Perceived Best Punishment Policy</i>			<i>U.C.^a</i>
	<i>Contingent</i>	<i>Not Punish</i>	<i>Other</i>	
Sex of Subject				
Female	7 (50%) ^b	2 (14%)	5 (36%)	.06
Male	9 (69%)	3 (23%)	1 (8%)	
Understanding				
Understands	15 (68%)	4 (18%)	3 (14%)	.10
Does not understand	1 (20%)	1 (20%)	3 (60%)	
Goal in Game				
Maximize earnings	10 (71%)	2 (14%)	2 (14%)	.04
Any other	6 (46%)	3 (23%)	4 (31%)	
Perceived Best Strategy				
Always form BCD	15 (68%)	4 (18%)	3 (14%)	.10
Any other	1 (20%)	1 (20%)	3 (60%)	

a. U.C. = Uncertainty coefficient, a nominal measure of association.
 b. Row percentages are given in this table.

TABLE 9
 Cross-Tabulation of Subject's Reported Punishment and
 Individual Attributes (N = 27)

<i>Individual Attributes</i>	<i>Punishment Policy</i>		<i>Statistics^a</i>
	<i>Contingent</i>	<i>Other</i>	
Sex of Subject			
Female	3 (21%) ^b	11 (79%)	G = .52
Male	6 (46%)	7 (54%)	T _b = .26
Understanding			
Understands	9 (41%)	13 (59%)	G = 1.00
Does not understand	0 (0%)	5 (100%)	T _b = .34
Goal in Game			
Maximize earnings	4 (29%)	10 (71%)	G = -.22
Any other	5 (38%)	8 (62%)	T _b = -.10
Perceived Best Strategy			
Always form BCD	8 (36%)	14 (64%)	G = .39
Any other	1 (20%)	4 (80%)	T _b = .13

a. G = gamma; T_b = Kendall's Tau-B; both are ordinal measures of association.
 b. Row percentages are given in this table.

DISCUSSION

The results of the experiment provide overwhelming confirmation for the hypothesis. Subjects who played an Apex Game in which they could (but did not have to) punish one another were far more likely to act collectively and to form the Base coalition than those who could not punish one another. The punishments were nearly always used in the predicted fashion—as selective punishments for cooperating with the Apex—and not otherwise.

There is always a legitimate question as to the generalizability of laboratory experiments. What, it may be asked, does the behavior of college students playing an abstract and rather esoteric four-person game tell us about labor unions, or rent strikes, or antipollution campaigns? The answer is that there is no sense in which direct inferences from experiments to “natural” phenomena may be made. However, this experiment and these natural phenomena may all be viewed as concrete instances of the larger theory of selective incentives and collective action. Since the experimental data strongly confirm the theory, this strongly suggests that similar predictions may be made about the effect of selective incentives on collective action in other, more natural, settings. Whether these predictions are true or not must of course be subjected to empirical test.

To the extent that it is acknowledged that experimental games may provide interesting information about collective action and the responses of persons to power-imbalanced situations, this particular line of research seems especially appropriate. As others have noted, the special dilemmas of choice facing the Bases in an Apex Game capture in microcosm the choices facing weak actors in a power-imbalanced situation: Should they view each other as antagonists in the effort to align with those who have power, or should they view each other as allies in an attempt to overcome the power differential? The fact that Base players in an Apex Game are free to interpret the situation in any fashion they desire allows the game to capture a simple analogue of class consciousness.

This research takes the results of game-theoretic analyses as givens, insofar as they predict likely divisions of the payoff in various coalitions under varying game structures. But instead of pursuing further the question of how payoffs should be divided, this research builds upon these past results and seeks to use them in answering the questions: “*Which* coalitions will form?” and “What factors affect which coalitions

will form?" It is a particular contribution of this article to argue that asking the question of which coalition will form leads to the adoption of principles of rational decision-making under risk, especially the calculation of expected values for various courses of action which take into account the likely actions of others.

Thus this article brings together several lines of research in what will hopefully prove to be a productive synthesis in the study of collective action. Continuing research by the author involves using varying levels of positive and negative incentives in Apex Games.

Appendix

DERIVATION OF EXPRESSION FOR USE OF INCENTIVE

An expression may be derived for the value to a person who controls a private good of using that good as a selective incentive to motivate others' cooperation in collective action. The general approach in this development is to write the expected value of various courses of action. The only possibly unusual procedure is to express one actor's payoffs as a function of *others'* actions. Some notational conventions are necessary. Let

- Q be a set of actors whose combined cooperative action would produce a collective good;
- I be an actor who controls a good which meets the criteria for a potential selective incentive with respect to the members of Q and who would benefit from collective action by Q;
- y_j be the value to I of the j^{th} possible outcome of the collective action situation;
- c_i' be I's act of using the private good as a selective incentive, that is, of presenting the good to Q contingent upon Q's cooperation;
- $E(c')_i$ be the expected value to I of using the incentive to induce collective action;
- d_i' be I's act of *not* using the good as an incentive, of using it in some alternate fashion;
- $E(d')_i$ be the expected value of I of *not* using the good as a selective incentive;

- $E_I(c_Q)$ be the value to I of the members of Q choosing c;
- $E_I(d_Q)$ be the value to I of the members of Q choosing d.

Using these notational conventions, we can express I's decision to use the incentive or not. First we consider the difference Q's choice (c or d) makes in I's payoff:

$$E_I(c_Q) - E_I(d_Q) = \sum_j p(y_j | c_Q) \cdot y_j - \sum_j p(y_j | d_Q) \cdot y_j. \tag{1}$$

Notice that this is an expression for the difference in an actor's expected payoff depending upon some behavioral choice, expect that I's payoffs depend upon Q's behavior. Rearranging terms allows us to rewrite 1 as 2:

$$E_I(c_Q) - E_I(d_Q) = \sum [y_j] \cdot [p(y_j | c_Q) - p(y_j | d_Q)] \tag{2}$$

or the sum of the products of each possible outcome value y_j and the *difference* in the probability of that outcome depending on whether Q cooperates or does not cooperate.

Now we consider the difference I's choice (c' or d') makes in his own payoff. First we express the expected value of each choice separately, in 3 and 4.

$$E(c')_I = [p(d_Q | c')] \cdot [E_I(d_Q)] + [p(c_Q | c')] \cdot [E_I(c_Q)] \tag{3}$$

probability value to probability value to
 Q chooses d I if Q Q chooses c I if Q
 if I does c' chooses d if I does c' chooses c

$$E(d')_I = [p(d_Q | d')] \cdot [E_I(d_Q)] + [p(c_Q | d')] \cdot [E_I(c_Q)] \tag{4}$$

Since c_Q and d_Q are dichotomous choices,

$$p(d_Q | c') = 1 - p(c_Q | c') \tag{5}$$

and

$$p(d_Q | d') = 1 - p(c_Q | d'). \tag{6}$$

Substituting 5 and 6 into 3 and 4, subtracting 4 from 3, and simplifying algebraically,

$$E(c') - E(d') = [p(c_Q | c') - p(c_Q | d')] \cdot [E_I(c_Q) - E_I(d_Q)]. \tag{7}$$

Substituting 2 into 7, we obtain our result:

$$E(c'_i) - E(d_i) = \text{difference using incentive makes to I} = \left[p(c_Q | c'_i) - p(c_Q | d'_i) \right] \cdot \left[\sum_j [y_j] [p(y_j | c_Q) - p(y_j | d_Q)] \right]. \quad [8]$$

difference using incentive makes in probability of Q's cooperation	value to I of each outcome	difference Q's cooperation makes in probability of each outcome value
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Costs of using an incentive may be treated in one of two ways. Either they are considered to be taken account of in the net value (y_j) of each outcome, in which case equation 8 stands as it is, or they may be considered as a separate term, in which case we impose the constraint $E(c'_i) - E(d'_i) > C$, where C is the cost of using the incentive. Depending upon the context and purpose of a particular analysis, either is a legitimate treatment of the cost term.

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