Financial Frictions in DSGE Models

Noah Williams

University of Wisconsin-Madison
Overview

Conventional Model with Perfect Capital Markets:

1. Arbitrage between return to capital and riskless rate

\[ E_t \beta \Lambda_{t,t+1} R_{kt+1} = E_t \beta \Lambda_{t,t+1} R_{t+1} \]

where \( \beta \Lambda_{t,t+1} \) is the household’s stochastic discount factor.

2. Financial structure irrelevant.
Overview (con’t)

With capital market frictions:

1. External finance premium ⇒

\[ E_t^{\beta} \Lambda_{t,t+1} R_{kt+1} > E_t^{\beta} \Lambda_{t,t+1} R_{t+1} \]

2. Premium depends inversely on borrower balance sheets ⇒

3. If borrower balance sheets move procyclically, external finance premium move countercyclically:
   ⇒ feedback between financial and real sectors ("financial accelerator,"
   ⇒ disturbances originating in the financial sector can have real effects.
Objective

Illustrate the following key concepts:

1. Asymmetric information and/or costly contract enforcement as foundations of financial market imperfections

2. Premium for external finance

3. Rationing vs. non-rationing equilibria

4. Balance sheets and the external finance premium

5. Idiosyncratic risk and the external finance premium.
Objective (con’t)

Illustrate with two simple models:

1. Costly State Verification Model (CSV) (Townsend, 1979)

2. Costly Enforcement Model
Basic Environment

- Two Periods: 0 and 1.

- Risk Neutral Entrepreneur:
  Has project that requires funding in 0 and pays off in 1.

- Competitive Risk Neutral Lender:
  Has opportunity cost of funds $R$. 
Basic Environment (con’t)

Project Finance:

\[ QK = N + B \]

\( K \equiv \) Capital Input
\( N \equiv \) Entrepreneurs’s Net Worth (Equity Finance)
\( B \equiv \) Debt Finance
Basic Environment (con’t)

Period 1 Payoff

\[ \tilde{\omega} R_k \cdot QK \]

\( R_k \equiv \text{Average Gross Return on Capital} \)

\( \tilde{\omega} \equiv \text{Idiosyncratic Shock} \)

Entrepreneur takes \( \tilde{\omega} R_k \) as given, but \( K \) is a choice variable.
Basic Environment (con’t)

Idiosyncratic Shock Distribution:

\[ E\{\tilde{\omega}\} = 1 \]

\[ \tilde{\omega} \in [\omega, \bar{\omega}] \]

\[ H(\omega) = \text{prob}(\tilde{\omega} \leq \omega) \]

\[ h(\omega) = \frac{dH}{d\omega} \]
Perfect Information and Perfect Contract Enforcement

- Given $E \omega R_k = R_k$, entrepreneurs operates if

  $$ R_k \geq R $$

  where $R$ is the opportunity cost.

- If $R_k > R$, entrepreneur's demand for funds is infinite
  Competitive market forces $\Rightarrow R_k = R$ in equilibrium.

- Miller-Modigliani theorem applies:
  Real Investment Decision is independent of financial structure
  Financial Structure is indeterminate
Private Information and Limited Liability

- Private Information:
  
  Only entrepreneurs can costlessly observe returns.

  Lenders must pay a cost equal to a fixed fraction $\mu$ of the realized return $\omega R_k K$.

  Interpretable as a bankruptcy cost.

- Limited Liability:

  Entrepreneurs minimum payoff bounded at zero.
Private Information and Limited Liability (con’t)

Implications:

- Entrepreneur has incentive to misreport returns.

- Financial structure matters to real investment decisions, due to expected bankruptcy costs.

- Financial structure determinate: Designed to reduce expected bankruptcy costs.
Entrepreneur’s Optimization Problem:

1. Investment Decision (choice of K)

2. Financial contract: payment schedule based on $\omega$ and decision to monitor

3. Constraint: Lender must receive opportunity cost in expectation.
Optimal Contract

1. Induce Truth-Telling (revelation principle)

2. Minimize Expected Monitoring Costs

⇒

- Optimal Contract is Standard Debt: i.e, Debt with bankruptcy
Optimal Contract (con’t)

Let $D \equiv$ face value of debt and $\omega^* \equiv$ the cutoff value of $\omega$

$$D = \omega^* R_k Q K$$

The contract then works as follows:

- If $\omega \geq \omega^*$:
  Lender’s payoff is $D = \omega^* R_k Q K$; Borrower’s payoff is $(\omega - \omega^*) R_k Q K$

- If $\omega < \omega^*$,
  The borrower announces default and then the lender monitors.
  Lender’s payoff is $(1 - \mu) \omega R_k Q K$; Borrower’s payoff is 0.

- Observe that the deadweight bankruptcy cost is $\mu \omega R_k Q K$. 
Optimal Contract (con’t)

Intuition for Optimal Contract

1. There is no incentive for the entrepreneur to lie:
   - In non-default states the payment to lenders is fixed
   - In default states there is monitoring.

2. Expected bankruptcy costs are minimized.

By giving the lender everything in the default state, the non-default payment $D$ is minimized.

Given $D = \omega^* R_k K$, the bankruptcy probability $H(\omega^*)$ is

$$H(\omega^*) = H\left(\frac{D}{R_k Q K}\right)$$

which is increasing in $D$. 
Optimal Contract (con’t)

Given the form of the optimal contract ⇒

Lender’s expected payment:

\[ \int_{\omega^*}^{\bar{\omega}} \omega^* R_k Q K dH + \int_{\omega}^{\omega^*} (1 - \mu)\omega R_k Q K dH \equiv [\Gamma(\omega^*) - \mu G(\omega^*)] R_k Q K \]

with

\[ \Gamma(\omega^*) = \int_{\omega^*}^{\bar{\omega}} \omega^* dH + \int_{\omega}^{\omega^*} \omega dH \]

\[ = \omega^*[1 - H(\omega^*)] + \int_{\omega}^{\omega^*} \omega dH \]

\[ G(\omega^*) = \int_{\omega}^{\omega^*} \omega dH \]
Optimal Contract (con’t)

- $\Gamma(\omega^*)$ is increasing and concave
  \[
  \Gamma'(\omega^*) = 1 - H(\omega^*) > 0 \\
  \Gamma''(\omega^*) = -h(\omega^*) < 0
  \]

- $G(\omega^*)$ is increasing and convex, assuming $\omega^* h(\omega^*)$ is increasing
  \[
  G'(\omega^*) = \omega^* h(\omega^*) > 0 \\
  G''(\omega^*) > 0
  \]

- $\Gamma(\omega^*) - \mu G(\omega^*)$ is increasing so long as the default prob $H(\omega^*)$ is not too large
  \[
  \Gamma'(\omega^*) - \mu G'(\omega^*) = 1 - H(\omega^*) - \mu \omega^* h(\omega^*)
  \]
  which is positive under reasonable values for $H(\omega^*), \mu$ and $\omega^* h(\omega^*)$
Entrepreneur’s Decision Problem

- Objective:

\[
\max_{\omega^*,K} \left\{ \max \left\{ \left[ 1 - \Gamma(\omega^*) \right] R_k Q K - RN, 0 \right\} \right\}
\]

- subject to

\[
[\Gamma(\omega^*) - \mu G(\omega^*)] R_k Q K = R(Q K - N)
\]

\[\lambda \equiv \text{constraint multiplier} = \text{shadow value of } N\]
Entrepreneur’s Decision Problem

Combining equations:

$$\max_{\omega^*, K} \{ \max \{ [R_k - R - \mu G(\omega^*) R_k] Q K, 0 \} \}$$

subject to

$$[\Gamma(\omega^*) - \mu G(\omega^*)] R_k Q K = R(Q K - N)$$

- where $\mu G(\omega^*) R_k$ \equiv expected default costs
  (related to premium for external finance)
Entrepreneur’s Decision Problem (con’t)

F.O.N.C:

- $\omega^*$

- $K$

\[
\lambda = 1 + \frac{\mu G'(\omega^*)}{\Gamma'(\omega^*) - \mu G''(\omega^*)}
\]

\[
R_k - \frac{\lambda}{\{[1 - \Gamma(\omega^*)] + \lambda[\Gamma(\omega^*) - \mu G(\omega^*)]\}} \cdot R = 0
\]

- $\lambda$

\[
[\Gamma(\omega^*) - \mu G(\omega^*)]R_k = R(1 - \frac{N}{K})
\]
Entrepreneur’s Decision Problem (con’t)

Given $\Gamma'(\omega^*) - \mu' G(\omega^*) > 0 \Rightarrow$ three observations:

1. $\lambda$ is increasing in $\omega^*$ (from FONC for $\omega^*$)

2. $\omega^*$ increasing in $R_k/R$ (from FONC for $K$)

3. $\frac{\lambda}{\{1-\Gamma(\omega^*)+\lambda[\Gamma(\omega^*)-\mu G(\omega^*)]\}} > 1$ is the premium for external finance.

Note that the premium is increasing in $\omega^*$. 
Optimal Choices of $\omega^*$ and $K$

The following two equations determine $\omega^*$ and $QK$:

- Lender's voluntary participation constraint:

$$[\Gamma(\omega^*) - \mu G(\omega^*)] R_k = R \left(1 - \frac{N}{QK}\right)$$

- Optimal Choice of Capital

$$R_k - \chi(\omega^*) R = 0$$

with

$$\chi(\omega^*) = \frac{\lambda(\omega^*)}{\{[1 - \Gamma(\omega^*)] + \lambda(\omega^*)[\Gamma(\omega^*) - \mu G(\omega^*)]\}} > 1; \quad \chi'(\omega^*) > 0$$
The Demand for Capital and Net Worth

- Inverting the lender’s voluntary participation constraint:

\[ \frac{QK}{N} = \frac{1}{1 - [\Gamma(\omega^*) - \mu G(\omega^*)] R_k/R} \]

- \( \omega^* \) is increasing in \( R_k/R \) from FONCs for \( \omega^* \) and \( K \).

\[ \frac{QK}{N} = \phi\left( \frac{R_k}{R} \right) \]

with

\[ \phi'\left( \frac{R_k}{R} \right) > 0 \]
Aggregate Demand for Capital and Financial Crises

- Capital demand

\[ QK = \phi\left(\frac{R_k}{R}\right)N \]

where \( \phi\left(\frac{R_k}{R}\right) \) is the optimal leverage ratio.

- \( \phi\left(\frac{R_k}{R}\right) \) does not depend on firm specific factors \( \Rightarrow \)

  Can aggregate capital demand across entrepreneurs:

\[ Q\overline{K} = \phi\left(\frac{R_k}{R}\right)\overline{N} \]

where \( \overline{N} \) is aggregate net worth and \( \overline{K} \) is aggregate capital demand.

- Financial Crisis: Sharp drop in N or in \( \phi\left(\frac{R_k}{R}\right) \) that reduces \( Q\overline{K} \).
Balance Sheet Strength and the Spread

- Inverting yields

\[ \frac{R_k}{R} = \chi \left( \frac{QK}{N} \right) \]

with

\[ \chi' \left( \frac{QK}{N} \right) > 0 \]

where \( \chi \) is the gross spread.

- Thus, in the market equilibrium, the spread is inversely related to aggregate balance sheet strength

  \[ \Rightarrow \] during a crisis the balance sheet weakens and the spread increases.
Bernanke/Gertler/Gilchrist Financial Accelerator Model

Dynamic General Equilibirum Framework with

1. Money

2. Imperfect Competition

3. Nominal Price Rigidities (Calvo staggered price setting.)

4. Financial Accelerator as in Bernanke/Gertler(1989), featuring asset price mecha-
nism in Kiyotaki and Moore (1997)
Sectors

1. Households

2. Business Sector
   (a) entrepreneur/firms
   (b) capital producers
   (c) retailers

3. Central Bank
Households

- **Objective**

\[
\max E_t \sum_{i=0}^{\infty} \beta^i [\log(C_{t+i}) + a_m \log\left(\frac{M_{t+i}}{P_{t+i}}\right) - a_n \frac{1}{1 + \gamma_n} L_{t+i}^{1+\gamma_n}]
\]  

subject to

\[
C_t = \frac{W_t}{P_t} L_t + \Pi_t - T_t - \frac{M_t - M_{t-1}}{P_t} - \frac{1}{1+i_t} D_t - D_{t-1}
\]

where \(D_t \equiv \) intermediary deposits.

- **As in Woodford (2003), we restrict attention to the cashless limit of the economy** (the limit as \(a_m \to 0\)).
Decision Rules

- labor supply

\[
\frac{W_t}{P_t} = a_n L_{t+i}^{\alpha_n} / \left( \frac{1}{C_t} \right)
\]  

(3)

- consumption/saving;

\[
\frac{1}{C_t} = E_t \left\{ (1 + i_t) \frac{P_t}{P_{t+1}} \beta \frac{1}{C_{t+1}} \right\}
\]

(4)
Entrepreneurs/Firms

- Produce wholesale output

- Competitive, risk neutral, face capital market frictions.

- A measure unity in the market at any time.

- i.i.d survival probability $\theta$: The expected horizon is accordingly $\frac{1}{1-\theta}$. $1 - \theta$ enter to replace exiting entrepreneurs. Ensures borrowers do not save their way out of the financial constraint. (A way of modeling dividend payouts).
  
  - Optimal to retain earnings until exit. Consume wealth upon exit.

- Exiting entrepreneurs make a small transfer to new entrepreneurs and then consume the rest.
Production Technology

Gross firm output $GY_t$ (≡ firm output $Y_t$ plus leftover firm capital):

$$GY_t = \omega_t [A_t (K_t)^\alpha (L_t)^{(1-\alpha)} + (1 - \delta)K_t].$$  \hspace{1cm} (5)

with

$$Y_t = \omega_t A_t (K_t)^\alpha (L_t)^{(1-\alpha)}$$

where $\omega_t$ is i.i.d across firms and across time, with

$$E\{\omega_t\} = 1$$
Labor Demand

\[ \frac{W_t}{P_{wt}} = (1 - \alpha) \frac{Y_t}{L_t} \]
Capital Demand

- Gross Return to Capital

\[ E_t \{ R_{kt+1} \} = E_t \left\{ \frac{P_{w+1} \alpha Y_{t+1}}{P_{t+1} K_{t+1}} + (1 - \delta) Q_{t+1} \right\} \]

- Opportunity Cost

\[ E_t \left\{ (1 + i_t) \frac{P_t}{P_{t+1}} \right\} \]
Capital Demand (con’t)

Under perfect markets, capital demand given by

\[ E_t \{ R_{kt+1} \} = E_t \left\{ (1 + i_t) \frac{P_t}{P_{t+1}} \right\} \]

With imperfect markets:

\[ E_t \{ R_{kt+1} \} > E_t \left\{ (1 + i_t) \frac{P_t}{P_{t+1}} \right\} \]
Capital Demand (con’t)

The finance of capital is divided between net worth and debt:

\[ Q_t K_{t+1} = N_t + \frac{B_t}{P_t}. \]

\( N_t \) is accumulated via retained earnings.
Costly State Verification

• Assume:
  – costly state verification and limited liability
  – one period loan contracts between bank and firm
  – entrepreneurs absorb aggregate risk:
    * banks diversify idiosyncratic risk →
    * households receive sure nominal return from banks
      · no need for households to monitor banks

⇒

1. (Aggregate state-contingent) debt with costly default is optimal

2. Agency costs of external finance (expected default costs)

3. Net worth reduces expected default costs
Costly State Verification with Aggregate Risk

- \( R_{kt+1} \equiv \tilde{u}_{t+1} \bar{R}_{kt+1} \)
  - \( \tilde{u}_{t+1} \equiv \text{aggregate risk} \)
  - \( \bar{R}_{kt+1} \equiv E_t\{R_{kt+1}\} \)

- Ex post return on capital
  \[
  R_{kt+1} = \omega_{t+1} \tilde{u}_{t+1} \bar{R}_{kt+1} Q_t K_t
  \]

- Expected real return on deposits \( R_{t+1} \)
  \[
  R_{t+1} = (1 + i_t) E_t\left(\frac{P_t}{P_{t+1}}\right)
  \]
  - Deposits offer sure nominal return
Costly State Verification with Aggregate Risk (con’t)

- Given $\tilde{u}$, bank’s expected debt payoff $= \text{opportunity cost}$

$$[\Gamma(\omega^*) - \mu G(\omega^*)] \tilde{u} Q K = R(Q K - N)$$

$$\Gamma(\omega^*) = [1 - H(\omega^*)] \omega^* + \int_{\omega}^{\omega^*} \omega dH$$

$$G(\omega^*) = \int_{\omega}^{\omega^*} \omega dH$$

- Entrepreneur’s objective:

$$\max_K E \left\{ \max_{\omega^*} \left\{ [1 - \Gamma(\omega^*)] \tilde{u} R_k K Q K - R N, 0 \right\} \right\}$$

where $\omega^*$ is chosen ex post and $K$ ex ante.
Entrepreneur’s Decision Problem

- Objective:

\[
\max_K E\{\max_{\omega^*} \{[\tilde{u}\overline{R}_k - R - \mu G(\omega^*)\tilde{u}\overline{R}_k]QK, 0\}\}
\]

- subject to

\[
[\Gamma(\omega^*) - \mu G(\omega^*)]\tilde{u}\overline{R}_k QK = R(QK - N)
\]

\[\lambda(\tilde{u}) \equiv (\text{state-contingent}) \text{ constraint multiplier} = (\text{state-contingent shadow value of net worth})\]
Optimal Choices of $\omega^*$ and $K$

- Optimality conditions:

$$ [\Gamma(\omega^*_u) - \mu G(\omega^*_u)]\bar{u}\bar{R}_k = R(1 - \frac{N}{QK}) $$

$$ \bar{R}_k - \bar{\chi}R = 0 $$

$$ \bar{\chi} = E\{\chi(\omega^*_u)\} $$

$$ \chi(\omega^*_u) = \frac{\lambda(\omega^*_u)}{\{[1 - \Gamma(\omega^*_u)] + \lambda(\omega^*_u)[\Gamma(\omega^*_u) - \mu G(\omega^*_u)]\}} > 1; \quad \chi'(\omega^*) > 0 $$

$$ \lambda(\bar{u}) = 1 + \frac{\mu G'(\omega^*_u)}{\Gamma'(\omega^*_u) - \mu G'(\omega^*_u)} $$
Optimal Choice of Capital

- Take the expectation of the lenders vpc and combine equations (see lecture 6)

\[ Q_t K_{t+1} = v \left( \frac{R_{kt+1}}{R_{t+1}} \right) N_t; \quad v'(\cdot) > 0 \]

- Aggregate Demand for Capital (Inverting the previous equation)

\[ \bar{R}_{kt+1} = \bar{\chi}_t R_{t+1} \]

with

\[ \bar{\chi}_t = \chi \left( \frac{Q_t K_{t+1}}{N_t} \right) \]

\[ \chi'(\cdot) > 0; \; \chi(0) = 1; \; \chi(\infty) = \infty \]
Evolution of Net Worth

\[ N_t = \theta V_t + (1 - \theta)X \]

where

\[ V_t = (1 - m_t)R_{kt}Q_{t-1}K_t - \left[ (1 + i_{t-1}) \frac{P_{t-1}}{P_t} \right] \frac{B_t}{P_{t-1}} \]

and \( X = \) total transfers to new entrepreneurs, with

\[ R_{kt} = \frac{P_{wt}}{P_t} \alpha \frac{Y_{tt}}{K_{tt}} + (1 - \delta)Q_t \]

\[ m_t = \mu G(\omega_{t-1}^*) \]

where \( G(\omega_{t-1}^*) \) is total defaults, so that \( m_t R_{kt} Q_{t-1} K_t \) is total default costs.
Evolution of Net Worth (con’t)

- Main Sources of Net Worth Fluctuations

  Unexpected movements in $Q_t$ and $P_t$

- Irving Fisher’s debt-deflation hypothesis: unanticipated declines in price level raises real debt burdens.
The Role of Leverage

Given \( Q_{t-1}K_t = N_{t-1} + \frac{B_{t-1}}{P_{t-1}} \)

\[ V_t = \{[(1 - m_t)R_{kt} - R_t]\phi_{t-1} + R_t\}N_{t-1} \]

with

\[ \phi_{t-1} = \frac{Q_{t-1}K_t}{N_{t-1}} \]

\[ R_t = (1 + i_{t-1}) \frac{P_{t-1}}{P_t} \]

- The sensitivity of net worth to unanticipated returns is increasing in the leverage ratio \( \phi_{t-1} \).
Capital Producers

- Capital Producers are competitive. They produce new capital and sell at the price $Q_t$.

- Evolution of capital

\[ K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right)K_t + (1 - \delta)K_t \]

\[ \Phi' > 0, \Phi'' < 0, \Phi\left(\frac{I}{K}\right) = \frac{I}{K} \]
Optimal Choice of Investment

- no lags

\[ Q_t - \Phi'(\frac{I_t}{K_t}) = 0 \]

\( Q \) is increasing \( \frac{I_t}{K_t} \) as in Tobin’s Q theory

- planning lags

\[ E_{t-1}\{Q_t - [\Phi'(\frac{I_t}{K_t})]^{-1}\} = 0 \]

\( I_t \) picked at \( t-1 \) based on expected \( Q_t \). (to get investment delays to shocks)

Note: Marginal product of capital used in producing new capital goods is zero within a local region of the steady state. See BGG.)
Retailers

- Buy wholesale output and sell as differentiated product

- Set prices on a staggered basis as in Calvo (1983)

\[
\frac{P_t}{P_{t-1}} \approx (\mu \frac{P^w_t}{P_t})^\lambda E_t(\frac{P_{t+1}}{P_t})^\beta
\]

in loglinear form

\[
\pi_t = \lambda (p_{wt} - p_t) + \beta E_t \pi_{t+1}
\]

- Note: \( p_t - p_{wt} \) is the log price markup.
Resource Constraint and Asset Markets

- $C_t^e \equiv$ entrepreneurial consumption; $M_t \equiv$ total monitoring costs:

$$Y_t = C_t + C_t^e + I_t + G_t + M_t$$

with

$$C_t^e = (1 - \phi)(V_t - X)$$

$$M_t = m_t R_t Q_{t-1} K_t$$

- Bank balance sheet

$$B_t = D_t$$
Monetary and Fiscal Policy

Monetary Rule:

\[ i_t = \rho i_{t-1} + (1 - \rho)[\gamma_\pi \pi_t + \gamma_y(y_t - y^n_t)] + \varepsilon^{rn}_t \]

\[ i_t = r_{t+1} - E_t \pi_{t+1} \]

Fiscal Policy:

Gov't spending exogenous and finance by lump sum taxes.
Investment, Finance and Monetary Policy in BGG

\[ I_t/K_t = \phi(Q_t) \]  \hspace{1cm} (6)

\[ E_t R_{t+1}^k = \chi \left( \frac{Q_t K_{t+1}}{N_{t+1}} \right) \left\{ (1 + i_t) \frac{P_t}{P_{t+1}} \right\} \]  \hspace{1cm} (7)

where

\[ E_t R_{t+1}^k = E_t \left\{ \frac{P_{w+1}}{P_{t+1}} \alpha \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta) Q_{t+1} \right\} \]  \hspace{1cm} \frac{Q_t}{Q_t} (8)
Investment, Finance and Monetary Policy in BGG (con’t)

Note:

\[ N_t = \theta \{(1 - m_t)R_{kt}Q_{t-1}K_t - (1 + i_{t-1}) \frac{P_{t-1}}{P_t} \frac{B_t}{P_{t-1}} \} + (1 - \theta)D \]

Thus:

i. Positive feedback between asset prices and investment (financial accelerator)

ii. Strength depends positively on leverage ratio ratio \( Q_t K_{t+1}/N_t \).

iii. Monetary Policy has additional impact via balance sheets
Log-linearized BGG model

Aggregate demand

\[
y_t = \frac{C}{Y}c_t + \frac{I}{Y}\text{inv}_t + \frac{G}{Y}g_t + \frac{C^e}{Y}c^e_t + \ldots
\]

\[
c_t = -\sigma r_{t+1} + E_tC_{t+1}
\]

\[
c^e_t = \frac{1 - \phi}{\phi}n_{t+1}
\]
\[(inv_t - k_t) = \varphi q_t\]

\[E_t r_{kt+1} = (1 - \varphi)E_t(p_{wt+1} - p_{t+1} + y_{t+1} - k_{t+1}) + \varphi E_tq_{t+1} - qt\]

\[E_t r_{kt+1} - r_{t+1} = -\nu(n_t - q_t - k_{t+1})\]
LOG-LINEARIZED BGG MODEL (con’t)

Aggregate supply

\[ y_t = a_t + \alpha k_t + (1 - \alpha) l_t \]

\[ y_t - l_t = \mu_t + \gamma l_t + c_t \]

\[ \pi_t = \kappa (p_{wt} - p_t) + \beta E_t \pi_{t+1} \]
LOG-LINEARIZED BGG MODEL (con’t)

Evolution of state variables

\[ k_{t+1} = \delta \text{inv}_t + (1 - \delta)k_t \]

\[ n_t = \frac{\theta RK}{N} [r_t^k - r_t] + \theta R(r_t + n_{t-1}) \]

with

\[ r_r = i_{t-1} - \pi_{t-1} \]
LOG-LINEARIZED BGG MODEL (con’t)

Monetary Policy Rule

\[ i_t = \rho i_{t-1} + (1 - \rho)[\gamma_\pi \pi_t + \gamma_y (y_t - y^n_t)] + \varepsilon_t^{rn} \]

\[ i_t = r_{t+1} - E_t \pi_{t+1} \]
Calibrating Financial Sector Parameters

Choose (i) survival probability $\theta$, (ii) monitoring costs $\mu$, and (iii) the moments of the idiosyncratic shock to match evidence on:

1. Steady state external finance premium: $R_k/R_\ast$.

2. Steady state leverage ration $QK/N$

3. Annual business failure rate.
Figure 3: Monetary Shock - No Investment Delay

Output

Investment

Nominal Interest Rate

Premium

All Panels: Time Horizon in Quarters
Figure 4: Output Response - Alternative Shocks

Technology Shock

Demand Shock

Wealth Shock

All Panels: Time Horizon in Quarters
Figure 5: Monetary Shock - One Period Investment Delay

Output

Investment

Nominal Interest Rate

Premium

All Panels: Time Horizon in Quarters
Figure 6: Monetary Shock - Multisector Model with Investment Delays

Aggregate Output

Sectoral Output

Premium and Nominal Interest Rate

Sectoral Investment

All Panels: Time Horizon in Quarters; Panels 2-4: Model with Financial Accelerator.
Figure 1: DSGE forecasts of the Great Recession

Notes: The figure is taken from Del Negro and Schorfheide (2013). The panels show for each model/vintage the available real GDP growth (upper panel) and inflation (GDP deflator, lower panel) data (black line), the DSGE model’s multi-step (fixed origin) mean forecasts (red line) and bands of its forecast distribution (shaded blue areas; these are the 50, 60, 70, 80, and 90 percent bands, in decreasing shade), the Blue Chip forecasts (blue diamonds), and finally the actual realizations according to the May 2011 vintage (black dashed line). All the data are in percent, Q-o-Q shows the filtered mean of $\lambda_t$ (solid black line) and the 50%, 68% and 90% bands in shades of blue.

the Geweke and Amisano approach can be seen as choosing the weights so to optimize the portfolio’s historical performance. Geweke and Amisano show that because of the benefits from diversification, these pools fare much better in a pseudo-out-of-sample forecasting exercise than “putting all your eggs in one basket” – that is, using only one model to forecast – as well as forecasts combinations based on Bayesian Model Averaging (BMA).