Lectures 8-9: Fiscal Policy in the Growth Model

and Optimal Taxation

Economics 714, Spring 2017

1 Fiscal Policy in the Growth Model

1.1 Lump-Sum Taxes

Level of taxes and spending (even if lump sum) matter because of wealth effects.

Assume $U(C_t, N_t) = U(C_t)$, so $N_t \equiv 1$, define $F(K, 1) = f(K)$.

Equilibrium conditions:

$$u'(C_t) = \beta u'(C_{t+1})[r_{t+1} + 1 - \delta]$$

$$= \beta u'(C_{t+1})[f'(K_{t+1}) + 1 - \delta]$$

$$K_{t+1} = f(K_t) + 1 - \delta K_t - C_t - G_t$$

Dynamics:

$$\Delta c = 0 \implies f'(K) = \rho + \delta$$

$$\Delta k = 0 \implies f(K) = C + \delta K + G$$

1.2 Distorting Taxes

Now consider linear tax $\tau^N_t$ on labor income, $\tau^K_t$ on capital income:

$$T_t = \tau^N_t w_t N_t + \tau^K_t (r_t - \delta) K_t$$
Government budget constraint as before with this definition of revenue.

Define after-tax gross return $R^K_t = 1 + (1 - \tau^K_t)(r_t - \delta)$.

Household budget constraint, using capital law of motion:

$$\sum_{t=0}^{\infty} q_t[C_t + K_{t+1}] = \sum_{t=0}^{\infty} q_t[(1 - \tau^N_t)w_t N_t + R^K_t K_t]$$

Firm problem unaffected: $w_t = F_N, r_t = F_K$.

Households: now allow elastic labor supply, so solve

$$\max_{\{C_t, K_{t+1}, N_t\}} \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - N_t) \quad \text{s.t. BC}$$

Equilibrium conditions:

$$u_C(C_t, 1 - N_t) = \beta u_C(C_{t+1}, 1 - N_{t+1})[1 + (1 - \tau^K_{t+1})(F_K(K_{t+1}, N_{t+1}) - \delta)]$$

$$\frac{u_L(C_t, 1 - N_t)}{u_C(C_t, 1 - N_t)} = (1 - \tau^N_t)F_N(K_t, N_t)$$

$$K_{t+1} = F(K_t, N_t) + 1 - \delta K_t - C_t - G_t$$

2 Ramsey Optimal Taxation

2.1 Setup

Look for linear taxes that fund given $\{G_t\}$ and maximize household welfare

First, find implementability constraint summarizing equilibria. Rewrite HH BC:

$$\sum_{t=0}^{\infty} q_t[C_t - (1 - \tau^N_t)w_t N_t] = \sum_{t=0}^{\infty} q_t[R^K_t K_t - K_{t+1}] = q_0 R^K_0 K_0$$

Then use HH first order conditions to substitute out for prices

$$\sum_{t=0}^{\infty} \beta^t[u_C(C_t, 1-N_t)C_t - u_L(C_t, 1-N_t)N_t] = U_C(C_0, 1-N_0)K_0[1+(1-\tau^K_N)(F_K(K_0, N_0) - \delta)]$$
Primal approach: solve for allocation first, back out supporting taxes from equilibrium conditions:

\[
\left( \frac{u_C(C_t, 1 - N_t)}{\beta u_C(C_{t+1}, 1 - N_{t+1})} - 1 \right) \frac{1}{F_K(K_{t+1}, N_{t+1}) - \delta} = 1 - \tau_{t+1}^K \\
\frac{u_L(C_t, 1 - N_t)}{u_C(C_t, 1 - N_t) F_N(K_t, N_t)} = 1 - \tau_t^N
\]

\( \tau_0^K \) only affects period zero: initial capital is inelastic, tax it as much as possible. To make problem interesting, restrict \( \tau_0^K \leq \tau^K \).

Define \( U(C, N) = U(C, 1 - N) \) so \( U_N = -U_L \). Also define:

\[
W(C_t, N_t; \lambda) = U(C_t, N_t) + \lambda [U_C(C, N)C + U_N(C, 1 - N)N]
\]

Then Ramsey problem can be written:

\[
\max_{\{C_t, K_{t+1}, N_t\}} \sum_{t=0}^{\infty} \beta^t W(C_t, N_t; \lambda) - \lambda U_C(C_0, N_0)K_0[1 + (1 - \tau^K_0)(F_K(K_0, N_0) - \delta)]
\]

subject to (multiplier \( \mu_t \)):

\[
C_t + G_t + K_{t+1} = F(K_t, N_t) + 1 - \delta K_t
\]

2.2 Characterization

First order conditions for \( t \geq 1 \):

\[
W_C(t) = \mu_t \\
W_N(t) = -\mu_t F_N(t)
\]

\[
\mu_t = \beta \mu_{t+1}(F_K(t + 1) + 1 - \delta)
\]
These imply modified Euler equation and intra-temporal optimality condition:

\[ W_C(C_t, N_t) = \beta W_C(C_{t+1}, N_{t+1})[1 + F_K(K_{t+1}, N_{t+1}) - \delta] \]

\[ -\frac{W_N(C_t, N_t)}{W_C(C_t, N_t)} = F_N(K_t, N_t) \]

Implications:

- zero long-run capital tax: if \( G_t \to \bar{G} \), allocation converges to steady state, then \( \tau^K_t \to 0 \).

- smoothing of tax rates: allocation smooths tax response to changes in \( G_t \).

Example: \( U(C, N) = C^{1-\gamma} \frac{1-\gamma}{1-\gamma} - N^{1+\theta} \frac{1+\theta}{1+\theta} \)

Then \( W_C = U_C(1 + \lambda - \lambda \gamma) \), \( W_N = U_N(1 + \lambda - \lambda \phi) \).

Implies \( \tau^K_t = 0 \) for \( t \geq 2 \), and \( \tau^N_t \) constant for \( t \geq 1 \).