The New Keynesian Model

Noah Williams

University of Wisconsin-Madison

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Research strategy

- policy as systematic \textit{and} predictable

...the central bank’s stabilization goals can be most effectively achieved only to the extent that the central bank not only acts appropriately, but is also understood by the private sector to predictably act in a certain way. The ability to successfully steer private-sector expectations is favored by a decision procedure that is based on a rule, since in this case the systematic character of the central bank’s actions can be most easily made apparent to the public. (Woodford 2003, p. 465)
Flexible price models share a common property – the inverse of the aggregate price level, $1/P_t$, behaves like a speculative asset price. Yet this seems at odds with the evidence. Many researchers accept that some degree of nominal rigidity in prices and/or wages is necessary if a dynamic general equilibrium model is going to have any chance of matching macro time series data and be useful for policy exercises. Some evidence from Bils-Klenow (2004): Half of all goods prices last more that 5.5 months. Varies dramatically over types of goods, amount of competition in industry.
### Table 2

Monthly Frequency of Price Changes for Selected Categories

<table>
<thead>
<tr>
<th>Category</th>
<th>% of Price Quotes with Price Changes</th>
<th>% of Price Quotes with Price Changes, excluding observations with item substitutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>All goods and services</td>
<td>26.1 (1.0)</td>
<td>23.6 (1.0)</td>
</tr>
<tr>
<td>Durable Goods</td>
<td>29.8 (2.5)</td>
<td>23.6 (2.5)</td>
</tr>
<tr>
<td>Nondurable Goods Services</td>
<td>29.9 (1.5)</td>
<td>27.5 (1.5)</td>
</tr>
<tr>
<td>Services</td>
<td>20.7 (1.5)</td>
<td>19.3 (1.6)</td>
</tr>
<tr>
<td>Food</td>
<td>25.3 (1.8)</td>
<td>24.1 (1.9)</td>
</tr>
<tr>
<td>Home Furnishings</td>
<td>26.4 (1.8)</td>
<td>24.2 (1.8)</td>
</tr>
<tr>
<td>Apparel</td>
<td>29.2 (3.0)</td>
<td>22.7 (3.1)</td>
</tr>
<tr>
<td>Transportation</td>
<td>39.4 (1.8)</td>
<td>35.8 (1.9)</td>
</tr>
<tr>
<td>Medical Care</td>
<td>9.4 (3.2)</td>
<td>8.3 (3.3)</td>
</tr>
<tr>
<td>Entertainment</td>
<td>11.3 (3.5)</td>
<td>8.5 (3.6)</td>
</tr>
<tr>
<td>Other</td>
<td>11.0 (3.3)</td>
<td>10.0 (3.3)</td>
</tr>
<tr>
<td>Raw Goods</td>
<td>54.3 (1.9)</td>
<td>53.7 (1.7)</td>
</tr>
<tr>
<td>Processed Goods</td>
<td>20.5 (0.8)</td>
<td>17.6 (0.7)</td>
</tr>
</tbody>
</table>

Notes: Frequencies are weighted means of category components. Standard errors are in parentheses. Durables, Nondurables and Services coincide with U.S. National Income and Product Account classifications. Housing (reduced to home furnishings in our data), apparel, transportation, medical care, entertainment, and other are BLS Major Groups for the CPI. Raw goods include gasoline, motor oil and coolants, fuel oil and other fuels, electricity, natural gas, meats, fish, eggs, fresh fruits, fresh vegetables, and fresh milk and cream.

Figure 3: Price of Triscuit 9.5 oz in Dominick's Finer Foods Supermarket in Chicago

Source: Chevalier, Kashyap and Rossi (2000)

<table>
<thead>
<tr>
<th>Week</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.99</td>
</tr>
<tr>
<td>2</td>
<td>1.14</td>
</tr>
<tr>
<td>3</td>
<td>2.65</td>
</tr>
</tbody>
</table>

Figure 1

![Triscuit 9.5 oz price over time](image-url)
Adding nominal rigidities

Objectives

1. To examine how the introduction of nominal rigidity affects analysis of macro issues.

2. To see how models employed in policy analysis can be derived when some degree of nominal rigidity is introduced into the dynamic general equilibrium models examined so far.
Price Stickiness

- Tendency of prices to adjust slowly in economy.
- Sources: **Monopolistic competition** and **menu costs**.
- Under perfect competition, market forces prices to adjust rapidly. But in many markets, sellers produce differentiated goods with some market power: **monopolistic competition**. Sellers set prices.
- **Menu costs**: costs of changing prices may lead to price stickiness. Even small costs like these may prevent sellers from changing prices often.
- Since competition isn’t perfect, having the wrong price temporarily won’t affect the seller’s profits much. The firm will change prices when demand or costs of production change enough to warrant the price change.
- We’ll actually study the simpler time-dependent pricing rules, rather than menu cost models which lead to state-dependent pricing.
Basic new Keynesian model

Three basic components

1. An expectational “IS” curve (Euler equation)
2. An inflation adjustment equation (Phillips curve/price setting)
3. A specification of policy behavior
An optimizing based model

- The model consists of households who supply labor, purchase goods for consumption, and hold money and bonds, and firms who hire labor and produce and sell differentiated products in monopolistically competitive goods markets.
- The basic model of monopolistic competition is drawn from Dixit and Stiglitz (1977).
- Each firm set the price of the good it produces, but not all firms reset their price each period.
- Households and firms behave optimally: households maximize the expected present value of utility and firms maximize profits.
Households

- The preferences of a representative household defined over a composite consumption good $C_t$, real money balances $M_t/P_t$, and leisure $1 - N_t$, where $N_t$ is the time devoted to market employment.

- Households maximize

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left( \frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right].$$

- The composite consumption good consists of differentiate products produced by monopolistically competitive final goods producers (firms). There are a continuum of such firms of measure 1, and firm $j$ produces good $c_j$. 
Households

The composite consumption good that enters the household’s utility function is defined as

$$C_t = \left[ \int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} \, dj \right]^{\frac{\theta}{\theta-1}} \quad \theta > 1. \quad (2)$$

The parameter $\theta$ governs the price elasticity of demand for the individual goods.
The household’s decision problem can be dealt with in two stages.

1. Regardless of the level of $C_t$, it will always be optimal to purchase the combination of the individual goods that minimize the cost of achieving this level of the composite good.

2. Given the cost of achieving any given level of $C_t$, the household chooses $C_t$, $N_t$, and $M_t$ optimally.
Households

- Dealing first with the problem of minimizing the cost of buying $C_t$, the household’s decision problem is to

$$\min_{c_{jt}} \int_0^1 p_{jt} c_{jt} dj$$

subject to

$$\left[ \int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \geq C_t,$$

where $p_{jt}$ is the price of good $j$. Letting $\psi_t$ be the Lagrangian multiplier on the constraint, the first order condition for good $j$ is

$$p_{jt} - \psi_t \left[ \int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{1}{\theta-1}} c_{jt}^{\frac{1}{\theta}} = 0.$$
Rearranging, \( c_{jt} = (p_{jt}/\psi_t)^{-\theta} C_t \). From the definition of the composite level of consumption (2), this implies

\[
C_t = \left[ \int_0^1 \left( \frac{p_{jt}}{\psi_t} \right)^{-\theta} C_t \right]^{\frac{\theta}{\theta-1}}^{\frac{1}{\theta-1}} = \left( \frac{1}{\psi_t} \right)^{-\theta} \left[ \int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{\theta}{\theta-1}} C_t.
\]

Solving for \( \psi_t \),

\[
\psi_t = \left[ \int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \equiv P_t.
\]
Households

- The Lagrange multiplier is the appropriately aggregated price index for consumption.
- The demand for good $j$ can then be written as

$$c_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\theta} C_t. \quad (5)$$

The price elasticity of demand for good $j$ is equal to $\theta$. As $\theta \to \infty$, the individual goods become closer and closer substitutes, and, as a consequence, individual firms will have less market power.
Households

- Given the definition of the aggregate price index in (4), the budget constraint of the household is, in real terms,

\[
C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \left( \frac{W_t}{P_t} \right) N_t + \frac{M_{t-1}}{P_t} + R_{t-1} \left( \frac{B_{t-1}}{P_t} \right) + \Pi_t, \tag{6}
\]

where \( M_t \) (\( B_t \)) is the household’s nominal holdings of money (one period bonds). Bonds pay a gross nominal rate of interest \( R_t \). Real profits received from firms are equal to \( \Pi_t \).

- In the second stage of the household’s decision problem, consumption, labor supply, money, and bond holdings are chosen to maximize (1) subject to (6).
Households

The following conditions must also hold in equilibrium

1. the Euler condition for the optimal intertemporal allocation of consumption

\[ C_t^{-\sigma} = \beta R_t E_t \left( \frac{P_t}{P_{t+1}} \right) C_{t+1} \]  

(7)

2. the condition for optimal money holdings:

\[ \gamma \left( \frac{M_t}{P_t} \right)^{-b} \frac{1}{C_t^{-\sigma}} = \frac{R_t - 1}{R_t} \]  

(8)

3. the condition for optimal labor supply:

\[ \frac{\chi N_t^\eta}{C_t^{-\sigma}} = \frac{W_t}{P_t} \]  

(9)
Firms

- Firms maximize profits, subject to three constraints:
  1. The first is the production function summarizing the technology available for production. For simplicity, we have ignored capital, so output is a function solely of labor input $N_{jt}$ and an aggregate productivity disturbance $Z_t$:

$$c_{jt} = Z_t N_{jt}, \quad \mathbb{E}(Z_t) = 1.$$

  2. The second constraint on the firm is the demand curve each faces. This is given by equation (5).

  3. The third constraint is that each period some firms are not able to adjust their price. The specific model of price stickiness we will use is due to Calvo (1983).
Price adjustment

- Each period, the firms that adjust their price are randomly selected: a fraction $1 - \omega$ of all firms adjust while the remaining $\omega$ fraction do not adjust.
  
  - The parameter $\omega$ is a measure of the degree of nominal rigidity; a larger $\omega$ implies fewer firms adjust each period and the expected time between price changes is longer.

- For those firms who do adjust their price at time $t$, they do so to maximize the expected discounted value of current and future profits.
  
  - Profits at some future date $t + s$ are affected by the choice of price at time $t$ only if the firm has not received another opportunity to adjust between $t$ and $t + s$. The probability of this is $\omega^s$. 
First consider the firm’s cost minimization problem, which involves minimizing $W_t N_{jt}$ subject to producing $c_{jt} = Z_t N_{jt}$. This problem can be written as

$$\min_{N_t} W_t N_t + \varphi^n_t (c_{jt} - Z_t N_{jt}).$$

where $\varphi^n_t$ is equal to the firm’s nominal marginal cost. The first order condition implies

$$W_t = \varphi^n_t Z_t,$$

or $\varphi^n_t = W_t / Z_t$. Dividing by $P_t$ yields real marginal cost as $\varphi_t = W_t / (P_t Z_t)$. 

Price adjustment
The firm’s decision problem

The firm’s pricing decision problem then involves picking $p_{jt}$ to maximize

$$E_t \sum_{i=0}^\infty \omega^i \Delta_{i,t+i} \Pi \left( \frac{p_{jt}}{P_{t+i}}, \varphi_{t+i}, c_{t+i} \right) =$$

$$E_t \sum_{i=0}^\infty \omega^i \Delta_{i,t+i} \left[ (\frac{p_{jt}}{P_{t+i}})^{1-\theta} - \varphi_{t+i} \left( \frac{p_{jt}}{P_{t+i}} \right)^{-\theta} \right] C_{t+i},$$

where the discount factor $\Delta_{i,t+i}$ is given by $\beta^i (C_{t+i}/C_t)^{-\sigma}$ and profits are

$$\Pi(p_{jt}) = \left[ \left( \frac{p_{jt}}{P_{t+i}} \right)^{c_{jt+i}} - \varphi_{t+i} c_{jt+i} \right]$$
Price adjustment

- All firms adjusting in period $t$ face the same problem, so all adjusting firms will set the same price.

- Let $p_t^*$ be the optimal price chosen by all firms adjusting at time $t$. The first order condition for the optimal choice of $p_t^*$ is

  $$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ (1 - \theta) \left( \frac{1}{p_{jt}} \right) \left( \frac{p_t^*}{P_{t+i}} \right)^{1-\theta} + \theta \varphi_{t+i} \left( \frac{1}{p_t^*} \right) \left( \frac{p_t^*}{P_{t+i}} \right)^{-\theta} \right]$$

- Using the definition of $\Delta_{i,t+i}$,

  $$\left( \frac{p_t^*}{P_t} \right) = \left( \frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\theta}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left( \frac{p_{t+i}}{P_t} \right)^{\theta-1}}.$$

  (10)
The case of flexible prices

- If all firms are able to adjust their prices every period ($\omega = 0$):

$$\left( \frac{p_t^*}{P_t} \right) = \left( \frac{\theta}{\theta - 1} \right) \varphi_t = \mu \varphi_t. \quad (11)$$

- Each firm sets its price $p_t^*$ equal to a markup $\mu > 1$ over nominal marginal cost $P_t \varphi_t$.

- When prices are flexible, all firms charge the same price, and $\varphi_t = \mu^{-1}$. 

The case of flexible prices

- Using the definition of real marginal cost, this means

$$\frac{W_t}{P_t} = \frac{Z_t}{\mu}.$$  

- However, the real wage must also equal the marginal rate of substitution between leisure and consumption to be consistent with household optimization:

$$\frac{\chi N_t^\eta}{C_t^{-\sigma}} = \frac{Z_t}{\mu}. \quad (12)$$
The case of flexible prices

Flexible-price output

- Let a $\hat{x}_t$ denote the percent deviation of a variable $X_t$ around its steady-state. Then, the steady-state yields

$$\eta \hat{n}_t + \sigma \hat{c}_t = \hat{z}_t.$$ 

- Now using the fact that $\hat{y}_t = \hat{n}_t + \hat{z}_t$ and $\hat{y}_t = \hat{c}_t$, flexible-price equilibrium output $\hat{y}_t^f$ can be expressed as

$$\hat{y}_t^f = \left[ \frac{1 + \eta}{\eta + \sigma} \right] \hat{z}_t.$$  \hspace{1cm} (13)
The case of sticky prices

- When prices are sticky ($\omega > 0$), the firm must take into account expected future marginal cost as well as current marginal cost when setting $p_t^*$.  
- The aggregate price index is an average of the price charged by the fraction $1 - \omega$ of firms setting their price in period $t$ and the average of the remaining fraction $\omega$ of all firms who set prices in earlier periods.  
- Because the adjusting firms were selected randomly from among all firms, the average price of the non-adjusters is just the average price of all firms that was prevailing in period $t - 1$.  
- Thus, the average price in period $t$ satisfies  

$$P_t^{1-\theta} = (1 - \omega)(p_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta}.$$  

(14)
Inflation adjustment

- Using the first order condition for $p_t^*$ and approximating around a zero average inflation, flexible-price equilibrium,

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa} \hat{\phi}_t$$

where

$$\tilde{\kappa} = \frac{(1 - \omega) [1 - \beta \omega]}{\omega}$$

- Equation (15) is often referred to as the New Keynesian Phillips curve.
The New Keynesian Phillips curve is forward-looking; when a firm sets its price, it must be concerned with inflation in the future because it may be unable to adjust its price for several periods.

Solving forward,

\[ \pi_t = \hat{\kappa} \sum_{i=0}^{\infty} \beta^i E_t \hat{\phi}_{t+i}, \]

Inflation is a function of the present discounted value of current and future real marginal cost.

Inflation depends on real marginal cost and not directly on a measure of the gap between actual output and some measure of potential output or on a measure of unemployment relative to the natural rate, as is typical in traditional Phillips curves.
Real marginal cost and the output gap

- The firm's real marginal cost is equal to the real wage it faces divided by the marginal product of labor: $\varphi_t = W_t / P_t Z_t$.
- Because nominal wages have been assumed to be completely flexible, the real wage must equal the marginal rate of substitution between leisure and consumption.
- In a flexible price equilibrium, all firms set the same price, so (11) implies that $\varphi = \mu^{-1}$. From equation (9), $\hat{w}_t - \hat{p}_t = \eta \hat{n}_t + \sigma \hat{y}_t$. Recalling that $\hat{c}_t = \hat{y}_t$, $\hat{y}_t = \hat{n}_t + \hat{z}_t$, the percentage deviation of real marginal cost around the flexible price equilibrium is

$$\hat{\varphi}_t = [\eta \hat{n}_t + \sigma \hat{y}_t] - \hat{z}_t = (\eta + \sigma) \left[ \hat{y}_t - \left( \frac{1 + \eta}{\eta + \sigma} \right) \hat{z}_t \right].$$
Real marginal cost and the output gap

- But from (13), this can be written as

\[ \hat{\phi}_t = (\eta + \sigma) \left( \hat{y}_t - \hat{y}_t^f \right). \]  

(16)

- Using these results, the inflation adjustment equation (15) becomes

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \]  

(17)

where \( \kappa = (\eta + \sigma) \tilde{\kappa} = (\eta + \sigma) (1 - \omega) \left[ 1 - \beta \omega \right] / \omega \) and \( x_t \equiv \hat{y}_t - \hat{y}_t^f \) is the gap between actual output and the flexible-price equilibrium output.

- This inflation adjustment or forward-looking Phillips curve relates output, in the form of the deviation around the level of output that would occur in the absence of nominal price rigidity, to inflation.
The demand side of the model

- Start with Euler condition for optimal consumption choice

\[ C_t^{-\sigma} = \beta R_t E_t \left( \frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\sigma} \]

- Linearize around steady-state:

\[ -\sigma \hat{c}_t = (\hat{i}_t - E_t p_{t+1} + p_t) - \sigma E_t \hat{c}_{t+1} \]

or

\[ \hat{c}_t = E_t \hat{c}_{t+1} - \left( \frac{1}{\sigma} \right) (\hat{i}_t - E_t p_{t+1} + p_t). \]

- Goods market equilibrium (no capital)

\[ Y_t = C_t. \]
The demand side of the model

Linearization

- Euler condition becomes

\[ \hat{y}_t = E_t \hat{y}_{t+1} - \left( \frac{1}{\sigma} \right) (\hat{i}_t - E_t \rho_{t+1} + \rho_t). \]

- This is often called an “expectational IS curve”, to make the comparisons with old-style Keynesian models clear.
Demand and the output gap

- Express in terms of the output gap $x_t = \hat{y}_t - \hat{y}_f$:

$$\hat{y}_t - \hat{y}_f = E_t (\hat{y}_{t+1} - \hat{y}_{t+1}^f) - \left( \frac{1}{\sigma} \right) (\hat{i}_t - E_t p_{t+1} + p_t) + (E_t \hat{y}_{t+1}^f - \hat{y}_f^t)$$

or

$$x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) (r_t - r_t^n),$$

where $r_t = \hat{i}_t - E_t p_{t+1} + p_t$ and

$$r_t^n \equiv \sigma \left( E_t \hat{y}_{t+1}^f - \hat{y}_f^t \right).$$

- Notice that the nominal interest rate affects output through the interest rate gap $r_t - r_t^n$. 

Noah Williams (UW Madison)

New Keynesian model

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The general equilibrium model

- Two equation system

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa x_t
\]

\[
x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right) (\hat{i}_t - E_t \pi_{t+1} - r^n_t)
\]
The general equilibrium model

- Consistent with
  - optimizing behavior by households and firms
  - budget constraints
  - market equilibrium

- Two equations but three unknowns: $x_t$, $\pi_t$, and $i_t$ – need to specify monetary policy
Adding lagged inflation

- To capture the inflation persistence found in the data, it is common to augment the basic forward-looking inflation adjustment equation with the addition of lagged inflation:

\[ \pi_t = (1 - \phi) \beta E_t \pi_{t+1} + \kappa x_t + \phi \pi_{t-1} + \varepsilon_t. \]  \hspace{1cm} (18)

- In this formulation, the parameter \( \phi \) is often described as a measure of the degree of backward-looking price setting behavior.
  - Fuhrer (1997) finds little role for future inflation once lagged inflation is added to the inflation adjustment equation.
  - Rudebusch (2000) estimates (18) using U. S. data and argues that \( \phi \) is on the order of 0.7, suggesting that inflation is predominantly backward-looking.
Christiano, Eichenbaum, and Evans (2001) make a distinction between firms that reoptimize it setting their price and those that do not:

- each period a fraction $1 - \omega$ of all firms optimally set their price;
- the remaining firms either simply adjust their price based on the average rate of inflation, so that $p_{jt} = \bar{\pi} p_{jt-1}$ where $\bar{\pi}$ is the average inflation rate, or they adjust based on the most recently observed rate of inflation, so that $p_{jt} = \pi_{t-1} p_{jt-1}$.

Costly to optimize
Indexation and decision lag

- This specification results in an inflation adjustment equation of the form

\[ \pi_t = \left( \frac{\beta}{1 + \beta} \right) E_t \pi_{t+1} + \left( \frac{1}{1 + \beta} \right) \pi_{t-1} + \tilde{\kappa} \hat{\phi}_t. \]

The presence of lagged inflation in this equation introduces inertia into the inflation process.

- CEE also assume prices set before time \( t \) information is available:

\[ \pi_t = \left( \frac{\beta}{1 + \beta} \right) E_{t-1} \pi_{t+1} + \left( \frac{1}{1 + \beta} \right) \pi_{t-1} + \tilde{\kappa} E_{t-1} \hat{\phi}_t. \]
Estimates of new Keynesian Phillips curve yield values of $\omega$ that too high.

Estimates range from 0.758 to 0.911 (Dennis 2006)

Value of 0.8 implies prices adjusted on average every $(1 - 0.8)^{-1} = 5$ quarters.

Micro evidence for U.S. suggests duration between price changes closer to 2 quarters, implying $\omega = 0.5$. 
The sensitivity of marginal cost to output

- Empirically, inflation does not seem to respond strongly to the output gap: $\kappa$ is small.
- In basic theory,

$$
\kappa = (\eta + \sigma) \frac{(1 - \omega) [1 - \beta \omega]}{\omega}
$$

where $1 - \omega$ is the fraction of adjusting firms, $\sigma$ is the coefficient of relative risk aversion, and $\eta$ is the (inverse) of the wage elasticity of labor supply.
The sensitivity of marginal cost to output

So $\kappa$ small if

- $\omega$ large – high degree of price rigidity (estimates often imply unrealistic values around 0.8)
- $\sigma$ small – very little risk aversion
- $\eta$ is small – high degree of labor supply elasticity.
The sensitivity of marginal cost to output

- Researchers have extended basic model to make marginal cost less sensitive to output.
- Christiano, Eichenbaum, and Evans (2001) – variable capital utilization
- Basic idea:
  - In standard model, increase in demand can only increase production if real wage rises to induce an increase in labor supply. If wage elasticity of labor supply is small, the real wage has to rise a lot. This boosts real marginal cost and inflation.
  - If output can increase by utilizing capital more intensely, wages and marginal cost will rise less.
The sensitivity of marginal cost to output

Firm-specific capital

- Generates decreasing returns to labor;
- Marginal cost varies across firms;
- Marginal cost is increasing in firm’s output;
- Elasticity of marginal cost to output depends on short-run returns to scale in variable factor.
Firm-specific capital

- **Intuition**
  - if aggregate real wages rise, firms that adjust price raise their price;
  - but rise in price lowers output at firm;
  - with fixed capital, marginal cost falls as output declines;
  - this dampens amount firm will raise price.

- If capital at firm is costly to adjust, firm only slowly adjusts its capital.

- Eichenbaum and Fisher find they get estimates of $\omega$ around one half – more plausible.
Demand persistence

The trouble with Euler conditions

- Euler condition is purely forward looking – same problems arise as with inflation equation.

- Output is discounted value of future interest rate gaps:

$$x_t = - \left( \frac{1}{\sigma} \right) E_t \sum_{i=0}^{\infty} (r_{t+i} - r^m_{t+i}).$$
Habit persistence

- To match the hump shaped response of output seen in the data, habit persistence has become a standard component of new Keynesian models (Fuhrer 2000, Christiano, Eichenbaum, and Evans 2001).

- External Habit persistence: Marginal utility of current consumption depends on past aggregate consumption.

- Internal Habit Persistence: Marginal utility of current consumption depends on household’s past consumption.
General equilibrium, estimated models

- Christiano, Eichenbaum, and Evans (2001)
- Smets and Wouters (2003)
- Levin, Onatski, Williams, and Williams (2005)

  Components:
  - Habit persistence
  - Variable capital utilization
  - Investment with 2nd-order adjustment costs
  - Price adjustment at start of period (based on expectations – information delay)
  - Wage and price stickiness
Conclusions

- Basic model fairs poorly when faced with data – too forward-looking;
- Habit persistence, variable capital utilization, firm specific capital, sticky wages all help.
- Models fit data, but decomposition into flexible-price and gap may miss major historical episodes.
Policy analysis

Key issues

- What are the objectives of optimal policy?
- Is the policy environment one of commitment or discretion?
- What instrument rule implements the optimal policy?
- What are the properties of the resulting equilibrium?
Given the specification of the economic environment, what are the appropriate objectives of the central bank?

Standard to assume central bank is concerned with minimizing a quadratic loss function that depended on output and inflation – plausible, but ultimately *ad hoc*. Common in the Barro-Gordon tradition

Woodford (2003) has provided the most detailed analysis of the link between a welfare criteria derived as a log-linear approximation to the utility of the representative agent and the type of quadratic loss functions so common in the literature.
Woodford demonstrates that deviations of the expected discounted utility of the representative agent around the level of steady-state utility can be approximated by

$$E_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda (x_{t+i} - x^*)^2 \right]. \quad (19)$$

- $x_t$ is the gap between output and the output level that would arise under flexible prices, and $x^*$ is the gap between the steady-state efficient level of output (in the absence of the monopolistic distortions) and the steady-state level of output.
Comparison to a standard loss function

This looks a lot like the standard quadratic loss function. There are, however, two critical differences.

1. The output gap is measured relative to the rate of output under flexible prices.
2. Inflation variability enters because, with price rigidity, higher inflation results in an inefficient dispersion of output among the individual producers.

★ Because prices are sticky, higher inflation results in an increase in overall price dispersion.
Policy weights

- Theory says something about the weights in the loss function:

\[ E_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda (x_{t+i} - x^*)^2 \right], \]

where

\[ \Omega = \frac{1}{2} \bar{Y} U_c \left[ \frac{\omega}{(1-\omega)(1-\omega\beta)} \right] \left( \theta^{-1} + \eta \right) \theta^2 \]

and

\[ \lambda = \left[ \frac{(1-\omega)(1-\omega\beta)}{\omega} \right] \frac{(\sigma + \eta)}{(1 + \eta \theta) \theta}. \]

- Greater nominal rigidity (larger \( \omega \)) reduces \( \lambda \).
- Loss function endogenous.
- Calvo specification implies \( \lambda \) is small – Taylor specification leads to larger weight on output gap.
A common approach to “optimal” policy is in terms of simple rules. The most famous of such instrument rules is the Taylor Rule (Taylor 1993):

\[ i_t = \pi_t + 0.5x_t + 0.5 \left( \pi_t - \pi^T \right) + r^*, \]

where \( \pi^T \) was the target level of average inflation (Taylor assumed it to be 2%) and \( r^* \) was the equilibrium real rate of interest (Taylor assumed this too was equal to 2%).

The Taylor Rule for general coefficients is

\[ i_t = r^* + \pi^T + \delta_x x_t + \delta_\pi \left( \pi_t - \pi^T \right). \quad (20) \]
A larger literature has now developed that has estimated the Taylor Rule, or similar simple rules, for a variety of countries and time periods.

- For example, Clarida, Galí, and Gertler (2000) do so for the Federal Reserve, the Bundesbank, and the Bank of Japan.
- Estimates for the United States under different Federal Reserve Chairman are reported by Judd and Rudebusch (2000).
- In general, the basic Taylor Rule, when supplemented by the addition of the lagged nominal interest rate, does quite well in matching the actual behavior of the policy interest rate.

The argument for simple rules relies not on their optimality but on their simplicity; they may serve as a useful benchmark for policy or aid in promoting policy transparency.
Policy Implication of forward-looking models

- The basic new Keynesian inflation adjustment equation took the form
  \[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t. \]

- That is, there is no additional disturbance term.
  \[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \Rightarrow \pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i} \]

- The absence of a stochastic disturbance implies there is no conflict between a policy designed to maintain inflation at zero and a policy designed to keep the output gap equal to zero.

- Just set \( x_{t+i} = 0 \) for all \( i \); keeps inflation equal to zero.
Thus, the key implication of the basic new Keynesian model is that price stability is the appropriate objective of monetary policy.

No policy conflicts.

When prices are sticky but wages are flexible, the nominal wage can adjust to ensure labor market equilibrium is maintained in the face of productivity shocks. Optimal policy should then aim to keep the price level stable.
Models that combine optimizing agents and sticky prices have very strong policy implications.

When the price level fluctuates, and not all firms are able to adjust, price dispersion results. This causes the relative prices of the different goods to vary. If the price level rises, for example, two things happen.

1. The relative price of firms who have not set their prices for a while falls. They experience an increase in demand and raise output, while firms who have just reset their prices reduce output. This production dispersion is inefficient.

2. Consumers increase their consumption of the goods whose relative price has fallen and reduce consumption of those goods whose relative price has risen. This dispersion in consumption reduces welfare.
Optimal policy

- The solution is to prevent price dispersion by stabilizing the price level.
- What is critical for this result is that nominal wages are assumed to be completely flexible.
- But the same argument would apply if wages are sticky and prices flexible. With sticky wages and flexible prices, monetary policy should stabilizes the nominal wage.
Woodford versus Friedman

- The basic new Keynesian model suggests price stability (i.e., zero inflation) is optimal.
  - Zero inflation eliminates inefficient price dispersion.

- Friedman rule: zero nominal rate of interest is optimal.
  - Zero nominal rate eliminates inefficiency in money holdings.
  - Optimal inflation is negative (deflation) at rate equal to real rate of interest.

- Khan, King, and Wolman (2000) analysis model with both distortions.

- The conclude optimal inflation is closer to zero than to the Friedman rule.
Cost shocks

- Assume

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t \]

where \( e \) represents an inflation or cost shock.

- Then

\[ \pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i} + \sum_{i=0}^{\infty} \beta^i E_t e_{t+i} \]

- Cannot keep both \( x \) and \( \pi \) equal to zero.

- Trade-offs must be made.
Implications of cost shocks

- Stochastic wedge between marginal rate of substitution and real wage.
- Wedge between flexible-price output and efficient level of output.
- Sticky nominal wages.
- Cost channel.
- Exchange rate movements, imperfect pass through.