Lecture 6
Economic Growth

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Adding Government Spending

- Now $F(k_t) = c_t + i_t + G_t$
- The key equations of the model are:

$$
\begin{aligned}
U'(c_t) &= \beta U'(c_{t+1})[F'(k_{t+1}) + 1 - \delta] \\
k_{t+1} &= (1-\delta)k_t + F(k_t) - c_t - G_t
\end{aligned}
$$

- Consumption dynamics unaffected
- Government spending takes resources, so shifts down capital dynamics Δk_t.
- Difference if government spending increases are permanent or temporary.

Solving the Model in a Special Case

- There is one known case where we can work out an explicit solution.
- Set $\delta = 1$ (full depreciation) use logarithmic utility, Cobb-Douglas:

$$U(c) = \log c, \quad F(k) = zk^{\alpha}$$

- Specialize the key equilibrium equations:

$$\begin{aligned} \frac{1}{c_t} &= \frac{\beta \alpha z k_{t+1}^{\alpha-1}}{c_{t+1}} \\ c_t &= zk_t^{\alpha} - k_{t+1} \end{aligned}$$

- Guess that the solution is a constant savings rate s:

$$c_t = (1-s)y_t$$

- Substitute into conditions:

$$\begin{aligned}\frac{1}{(1-s)zk_t^\alpha} &= \frac{\beta\alpha z k_{t+1}^{\alpha-1}}{(1-s)zk_{t+1}^\alpha} \\ &= \frac{\beta\alpha}{(1-s)k_{t+1}} \\ &= \frac{\beta\alpha}{(1-s)szk_t^\alpha}\end{aligned}$$

- So $s = \beta\alpha$, and $c_t = (1-\beta\alpha)zk_t^\alpha$.

Implications

- In this special case we have the explicit relationship between (c, k), the optimal decision rule or the saddle path.
- We then have the dynamics of k_t:

$$k_{t+1} = szk_t^{\alpha}$$

The steady state is a special case of what we had earlier $(\delta = 1)$:

$$k^* = \left(\frac{\alpha z}{1+\theta}\right)^{\frac{1}{1-\alpha}}$$

- So we can now trace out the dynamics explicitly. For example, if z increases, c_t increases on impact and grows over time to the new steady state.

Economic Growth: Motivation

- Now will turn to analysis of the process of economic growth. A main determinant of living standards.
- Differences across countries:
  - Out of 6.4 billion people, 0.8 do not have access to enough food, 1 to safe drinking water, and 2.4 to sanitation.
  - Life expectancy in rich countries is 77 years, 67 years in middle income countries, and 53 years in poor countries.
- Differences across time:
  - Japanese boy born in 1880 had a life expectancy of 35 years, today 81 years.
  -An American worked 61 hours per week in 1870, today 34.

A Summary

I do not see how one can look at figures like these without seeing them as representing possibilities. Is there some action a government could take that would lead the Indian economy to grow like Indonesia's or Egypt's? If so, what exactly? If not, what is it about the "nature of India" that makes it so? The consequences for human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think about anything else (Lucas 1988, p. 5).

Growth Accounting

- Will develop theories to address growth over time and across countries. First decompose into growth in inputs and growth in productivity.
- As will see later, growth in productivity may be sustainable. Growth in inputs may be important but can't be sustained.
- Decompose output growth into z, K, and N.
  Starting point: Production Function
  $Y(t) = z(t)F(K(t), N(t))$. Now work in continuous time: $t \in (0, \infty)$.

- Assume $F(K, N) = K^\alpha N^{1-\alpha}$. Take logs:

$$\log Y(t) = \log z(t) + \alpha \log K(t) + (1-\alpha) \log N(t)$$

- Let $\dot{x}(t) = \frac{dx}{dt}$. Differentiate respect to t:

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{z}(t)}{z(t)} + \alpha \frac{\dot{K}(t)}{K(t)} + (1-\alpha) \frac{\dot{N}(t)}{N(t)}$$

- We can measure $\dot{Y}/Y$, $\dot{K}/K$, $\dot{N}/N$, and α.
  $\dot{z}/z$ is the Solow residual.

Using Factor Markets to Get α

- Assume the labor and credit markets are competitive:
- Credit Market:

$$r(t) = z(t)F_K(K(t), N(t)) = z(t)\alpha K(t)^{\alpha-1}N(t)^{1-\alpha} = \frac{\alpha Y(t)}{K(t)}$$

$$\Rightarrow \alpha = \frac{r(t)K(t)}{Y(t)}, \quad \text{Capital Share.}$$

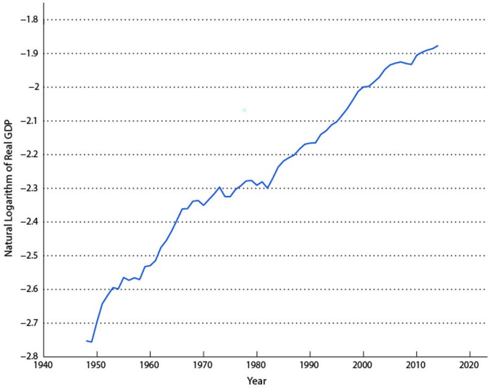
- Labor Market:

$$w(t) = z(t)F_N(K(t), N(t)) = z(t)(1-\alpha)K(t)^{\alpha}N(t)^{-\alpha}$$

$$\Rightarrow 1-\alpha = \frac{w(t)N(t)}{Y(t)}, \quad \text{Labor Share.}$$

- Standard Estimates for U.S.: $\alpha = 0.33 - 0.36$.

Figure 7.23 Natural Log of the Solow Residual



Productivity Growth Rate by Time Period

Using Fernald (SF Fed) measure

Time Period	Growth Rate
1950-1960	2.12
1960-1970	1.81
1970-1980	0.86
1980-1990	0.52
1990-2000	1.12
2000-2007	1.47
2009-2015	0.83

Productivity Slowdowns: 1970s and Present

- There was a big reduction in productivity growth in the 1970s. Following are some reasons:
  - Sharp increases in the price of oil in 70's
  - Structural changes: more services and less and less manufacturing goods produced
  - Slowdown in resources spent on R&D in the late 60's.
  - TFP was abnormally high in the 50's and 60's
  - Information technology (IT) revolution in the 70's
- There has also been slow growth in productivity since recession. Unclear whether this reflects long-run trend (Gordon), or temporary factors/mismeasurement (Varian, Brynjolfsson)

Average Annual Growth Rates

Years	Y	K	N	z
1950-1960	3.6	3.7	1.1	1.7
1960-1970	4.3	3.9	1.8	1.8
1970-1980	3.2	3.0	2.4	0.6
1980-1990	3.3	2.6	1.8	1.3
1990-2000	3.5	2.4	1.4	1.7
2000-2009	1.4	2.1	0.2	0.7
2009-2014	2.1	0.9	0.9	1.1

Growth Accounting for Other Countries

- One key question: was fast growth in East Asian growth miracles mostly due to technological progress or mostly due to capital accumulation?
- East Asia: in late 1980s-early 1990s growth in East Asia declared a "miracle" by many observers.
- Average growth in real GDP 1966-1990: Hong Kong 7%, Singapore 8.5% , South Korea and Taiwan over 8%. But was growth due to growth in inputs or productivity?
- Research by Alwyn Young, summarized by Paul Krugman "The Myth of Asia's Miracle"
- Krugman drew parallel to Soviet economy. Similar also post-WWII Japan.

Country/Region	Per.	$\dot{Y}/Y$	α	$\alpha \frac{\dot{K}/K}{\dot{Y}/Y}$	$(1-\alpha)\frac{\dot{N}/N}{\dot{Y}/Y}$	$\frac{\dot{z}/z}{\dot{Y}/Y}$
Germany	60-90	3.2	0.4	59%	−8%	49%
Italy	60-90	4.1	0.38	49%	3%	48%
UK	60-90	2.5	0.39	52%	−4%	52%
Argentina	40-80	3.6	0.54	43%	26%	31%
Brazil	40-80	6.4	0.45	51%	20%	29%
Chile	40-80	3.8	0.52	34%	26%	40%
Mexico	40-80	6.3	0.63	41%	23%	36%
Japan	60-90	6.8	0.42	57%	14%	29%
Hong Kong	66-90	7.3	0.37	42%	28%	30%
Singapore	66-90	8.5	0.53	73%	31%	−4%
South Korea	66-90	10.3	0.32	46%	42%	12%
Taiwan	66-90	9.1	0.29	40%	40%	20%

Growth "Miracles"?

- Table illustrates finding that most of the growth in East Asian countries was due to growth in inputs.
- Singapore is a prime example. The employed share of the population surged from 27 to 51 percent.
- Increased education: in 1966 more than 1/2 workers had no formal education at all, by 1990 2/3 had completed secondary education.
- Huge investment in physical capital: investment as a share of output rose from 11 % to more than 40%. After accounting for inputs, found *negative* TFP growth.
- Similar arguments for Japan (post WWII) and China (current), not as dire. Was an improvement in Japanese TFP, although slowed.
- China started from low position, has large population, so may grow for a long time before diminishing returns set in. Has slowed lately, somewhat by choice.

Solow Model Overview

- So far have analyzed the sources of growth in an economy, argued that only TFP growth provides sustainable growth.
- Now develop a model showing this and addressing:
    1. What is the relationship between the long-run standard of living and the saving rate, population growth rate, and rate of technical progress?
    2. How does economic growth change over time? (Speed up, slow down, stabilize?)
    3. Are there forces that allow poorer countries to catch up to richer ones?

Basic Assumptions of the Solow Growth Model

- Continuous time. (Book does discrete time.)
- Single good in the economy produced, constant technology.
- No government or international trade.
- All factors of production are fully employed.
- Labor force (population) grows at constant rate

$$n = \frac{\dot{N}}{N}$$

- Initial values for capital, K_0 and labor, N_0 given.

Production

- Cobb-Douglas aggregate production function:

$$Y(t) = z(t)F(K(t), N(t)) = z(t)K(t)^{\alpha}N(t)^{1-\alpha}$$

- For now suppose no TFP growth: $z(t) = 1$.
- Define per worker variables: $y = \frac{Y}{N}, k = \frac{K}{N}$. Then:

$$y = \frac{Y}{N} = \frac{K^{\alpha}N^{1-\alpha}}{N} = \left(\frac{K}{N}\right)^{\alpha}\left(\frac{N}{N}\right)^{1-\alpha} = k^{\alpha} = f(k)$$

- Per worker production function has decreasing returns to scale.
- Again constant returns implies $Y = rK + wN$.

Households

- Suppose households don't value leisure, inelastically supply 1 unit of labor. Aggregate labor supply then N.
- Assume result of household maximization (or social planner's) problem is to save a constant fraction s of income, consume $1 - s$.

$$C = (1 - s)[rK + wN] = (1 - s)Y$$

- Capital evolution:

$$K_{t+1} = (1 - \delta)K_t + sY_t$$

- Rearrange and take limits as $\Delta t \to 0$:

$$\begin{aligned} K_{t+1} - K_t &= -\delta K_t + sY_t \\ \Rightarrow \dot{K} &= sY - \delta K \end{aligned}$$

Capital Accumulation

- Divide by N in the capital accumulation equation:

$$\frac{\dot{K}}{N} = sy - \delta k = sk^{\alpha} - \delta k$$

- Now remember that $k(t) = K(t)/N(t)$, so:

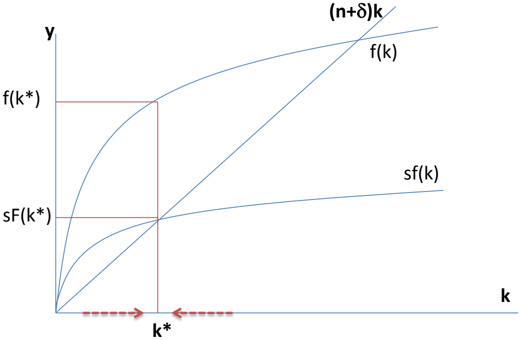
$$\begin{aligned} \dot{k} &= \frac{\dot{K}}{N} - \frac{\dot{N}K}{NN} \\ &= \frac{\dot{K}}{N} - nk \\ &= sk^{\alpha} - (\delta + n)k \end{aligned}$$

- The last line is the fundamental equation of the Solow Model

$$\boxed{\dot{k} = sk^{\alpha} - (\delta + n)\,k}$$

Graphical Analysis

- Change in k, $\dot{k}$ is given by difference of sk^{α} and $(\delta + n)k$
- If $sk^{\alpha} > (\delta + n)k$, then k increases.
- If $sk^{\alpha} < (\delta + n)k$, then k decreases.
- Unique positive steady state. (Trivial steady state at $k = 0$.)
- Positive steady state stable: if move away, will come back to it.



Steady state in the Solow model

Steady State Analysis

- Steady State: $\dot{k} = 0$
- Solve for steady state

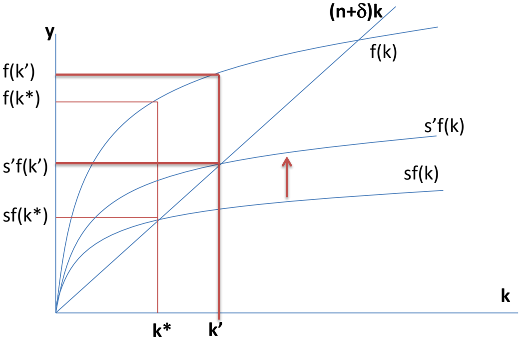
$$0 = s\,(k^*)^\alpha - (n+\delta)k^* \Rightarrow k^* = \left(\frac{s}{n+\delta}\right)^{\frac{1}{1-\alpha}}$$

- Steady state output per worker $y^* = \left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$
- Steady state consumption per worker:

$$c^* = (1-s)(k^*)^\alpha$$

Comparative Statics

- Suppose that of all a sudden saving rate s increases to $s' > s$. Suppose that economy was initially at its old steady state with saving rate s.
- $(n + \delta)k$ curve does not change.
- $sf(k)$ shifts up to $s'f(k)$
- New steady state: higher capital and output per worker.
- Capital stock increases monotonically from old to new steady state.



Increase in savings rate in the Solow model