Lecture 3
General Equilibrium and Applications

Noah Williams

University of Wisconsin - Madison

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Putting Everything Together

- We have a household that decides how much to work, N, and how much to consume, c to maximize utility. It takes as given the wage, w, and the interest rate, r.
- We have a firm that decides how much to produce Y and how much capital, K, and labor, N to hire. It takes as given the wage, w, and the interest rate, r.
- We have a government that raises taxes T, and spends G. We will switch for now to lump sum taxes.
- We are in a static world: we will assume fixed supply $K = \bar{K}$ constant.

Equilibrium

- We will now put these together. Why?
  1. Consistency: we are sure that everyone is doing things that are compatible.
  2. To derive positive predictions from the model.
- We will take as exogenous some objects: $\bar{K}$, G, and z. As well as specifications of u and F.
- Will derive endogenous objects: w, r, N, c, T. These imply Y, π, l.

A Competitive Equilibrium

A Competitive Equilibrium is an allocation $\{Y, N, K, c\}$, a price system $\{w, r\}$ and a government policy $\{T, G\}$ such that:

1. Given the price system and the government policy, households choose N^s and c to maximize their utility.
2. Given the price system and the government policy, firms maximize profits by choice of N^d, K.
3. The government budget is balanced: $G = T$.
4. Markets clear:

$$\begin{aligned} \text{Capital: } K &= \bar{K} \\ \text{Labor: } N^d &= N^s = N \\ \text{Goods: } Y &\equiv zF(K, N) = c + G \end{aligned}$$

Walras Law

- Note that if we impose the other conditions, the goods market clearing condition automatically holds.
- Budget constraint: $c = wN^s + \pi + r\bar{K} - T$
  Profits: $\pi = zF(K, N^d) - wN^d - rK$
  Substitute π into BC, use $K = \bar{K}$, $N^d = N^s = N$:

$$\begin{aligned} c &= wN + (zF(\bar{K}, N) - wN - r\bar{K}) + r\bar{K} - G \\ c + G &= zF(\bar{K}, N) \end{aligned}$$

- This is an implication of Walras law: if all markets but one clear, the other must as well.

Solving for an Equilibrium

- Key here is find w to clear labor market.
- From consumer utility maximization:

$$MRS = \frac{u_l(c,l)}{u_c(c,l)} = w$$

- From firm profit maximization:

$$MPN = zF_N(K, h-l) = w$$

- Equate and impose goods market clearing:

$$\frac{u_l(zF(\bar{K}, h-l) - G, l)}{u_c(zF(\bar{K}, h-l) - G, l)} = zF_N(K, h-l)$$

- Solve for l. Note MRS decreasing in l, MPN increasing in l.

Figure 5.2 The Production Function and the Production Possibilities Frontier

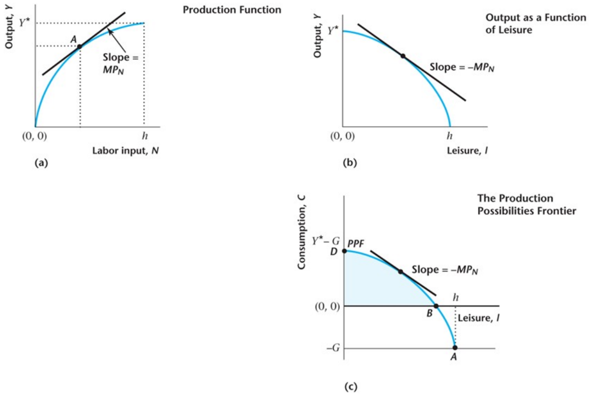

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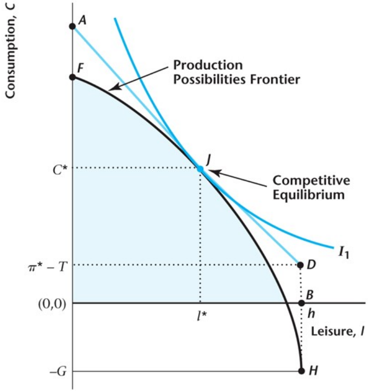
Figure 5.3 Competitive Equilibrium



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Pareto Optimality

- An allocation is Pareto Optimal if there is no way to rearrange production or reallocate goods so that someone is made better off without making someone else worse off. (Limited notion.)
- Let us imagine we have a powerful dictator, the Social Planner, that can decide how much the households consume and work and how much the firms produce.
- The Social Planner does not follow prices. But he understands opportunity cost.
- The Social Planner is benevolent. He searches for the best possible allocation, which will be Pareto optimal.

Social Planner's Problem I

- Maximizes utility of household given G, K.

$$\max_{c,l} u\left(c, l\right)$$

$$\text{subject to: } c + G = zF(K, h - l)$$

- Note: we do not have prices in the budget constraint.
- Again, either Lagrangian or impose constraint. Here impose:

$$\max_{l} u\left(zF(K, h - l) - G, l\right)$$

Social Planner's Problem II

$$\max_{l} u\left(zF(K, h-l) - G, l\right)$$

- First Order Condition with respect to l:

$$-u_c zF_N + u_l = 0$$

$$\frac{u_l}{u_c} = zF_N$$

- Same as the competitive equilibrium.
- A simple example of the first welfare theorem.

The Formal Statement

- First Welfare Theorem: under certain conditions (made clear later), a competitive equilibrium is Pareto optimal.
- We also have the converse.
- Second Welfare Theorem: under certain conditions, a Pareto optimal allocation can be decentralized as a competitive equilibrium.
- To decentralize an optimal allocation, may need a (lump sum) redistribution of wealth.

Some consequences

- First Welfare Theorem states that, under certain conditions, an allocation achieved by a market economy is Pareto optimal.
- Formalization of Adam Smith's "invisible hand" idea.
- Strong theoretical point in favor of decentralized allocation mechanisms: prices direct agents to do what is needed to get a Pareto optimum.
- Second Welfare Theorem states gives the best way to change allocations: redistribute income. Do not change prices.

How robust is the First Welfare theorem?

- Key is the phrase "under certain conditions". Plenty of reasons to deviate from a Pareto optimum:
    1. Distorting (non lump-sum) taxes, as before.
    2. Externalities.
    3. Imperfect Competition.
    4. Asymmetric Information.
    5. Market Incompleteness.
    6. Bounded Rationality of Agents.
- Example: With proportional taxes we saw that household optimality implied.

$$\frac{u_l}{u_c} = (1 - \tau)\, w$$

Opens a wedge between MRS and MPN.

Can we take the planner's problem literally?

- How do we allocate resources in society?
- Could a social planner do as well, or better? Our basic model suggests so.
- Why is this important? a little bit of history
- Could Central Planning work? Mises, Hayek in the 30's: NO.
- Experience is rather clear that it did not, but maybe they just did not implement in properly.

The problem of information

"The problem of rational economic order is determined precisely by the fact that the knowledge of the circumstances of which we must make use never exist in concentrated or integrated form, but solely as the dispersed bits of incomplete knowledge which all the separate individuals possess...

The problem is thus in no way solved if we can show that all the facts, if they were known to a single mind (as we hypothetically assume them to be given to the observing economist) would uniquely determine the solution; instead we must show how a solution is produced by the interactions of people, each of whom possesses only partial knowledge."
–F.A. Hayek, "The Use of Knowledge in Society" (1945)

Using the General Equilibrium Model

- We can now analyze the equilibrium response of the economy to exogenous changes.
- Example: suppose that there is an increase in TFP z.
- Increase in z shifts out MPN, rotates out production possibility frontier. Can produce more with the same amount of labor, giving more scope for consumption.
- Will increase C unambiguously. Effect on N will depend on income and substitution effects.
- Wage likely to increases, due to shift in MPN for any given N. Possible that N falls slightly, but not enough to offset increase in z (which is what leads to the change in N).

Figure 5.9 Competitive Equilibrium Effects of an Increase in Total Factor Productivity

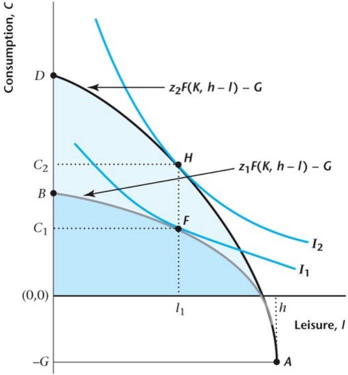

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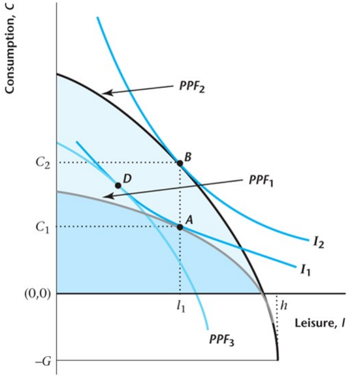
Figure 5.10 Income and Substitution Effects of an Increase in Total Factor Productivity



Application I: WWII and the Increase in G

- During WWII government spending to finance the war effort increased to levels unseen previously in the US.
- What are the predictions of the model for this increase in spending?
- The assumption that government spending is a pure loss of output arguably makes sense here. Pure spending/diversion of resources in short run. Positive effects more long-run and harder to measure.

Figure 1.01 Output of the U.S. economy, 1869-1996

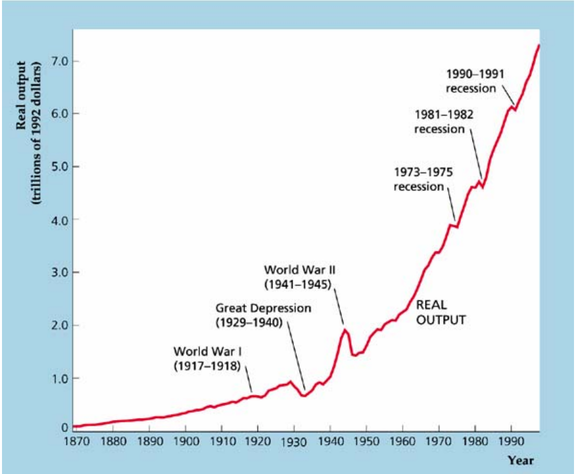
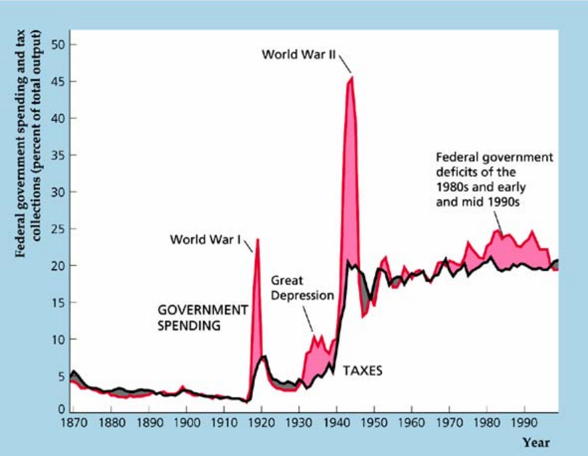


Figure 1.06 U.S. Federal government spending and tax collections, 1869-1999



Analysis of the Change in G

- We'll work out a parametric example with preferences $u(c, l) = \log c + \gamma l$ and Cobb-Douglas technology.
- Production possibilities (goods market):

$$c = Y - G = zK^{\alpha}(h-l)^{1-\alpha} - G$$

To simplify: write $g = G/Y$. So:

$$c = (1-g)zK^{\alpha}(h-l)^{1-\alpha}.$$

- Firm profit maximization:

$$MPN = zF_N(K, h-l) = z(1-\alpha)K^{\alpha}(h-l)^{-\alpha} = w$$

- Household utility maximization:

$$MRS = \frac{u_l(c,l)}{u_c(c,l)} = \frac{\gamma}{1/c} = w$$

- Equate and impose goods market clearing:

$$\begin{aligned}
\frac{u_l(zF(\bar{K},h-l)-G,l)}{u_c(zF(\bar{K},h-l)-G,l)} &= zF_N(\bar{K},h-l) \\
\Rightarrow \gamma(1-g)z\bar{K}^{\alpha}(h-l)^{1-\alpha} &= z(1-\alpha)\bar{K}^{\alpha}(h-l)^{-\alpha} \\
\Rightarrow N = h-l &= \frac{1-\alpha}{\gamma(1-g)}.
\end{aligned}$$

- Government spending has a pure income effect here (since financed by lump sum taxes). Increases labor supply.

- Solve for rest of allocation:

$$\begin{aligned} Y &= z\bar{K}^{\alpha}\left[\frac{1-\alpha}{\gamma(1-g)}\right]^{1-\alpha} \\ c &= (1-g)\,Y = z(1-g)^{\alpha}\,\bar{K}^{\alpha}\left[\frac{1-\alpha}{\gamma}\right]^{1-\alpha} \end{aligned}$$

Output increases with g, consumption decreases.

- Solve for wages and interest rates:

$$\begin{aligned} w &= z(1-\alpha)K^{\alpha}\left[\frac{1-\alpha}{\gamma(1-g)}\right]^{-\alpha} \\ r &= z\alpha K^{\alpha-1}\left[\frac{1-\alpha}{\gamma(1-g)}\right]^{1-\alpha} \end{aligned}$$

So wages decrease with g, interest rates increase.

Summing Up:

- Following increase in $g = G/Y$, the model predicts an increase in (Y, N, r), decrease in (c, w).
- Private consumption spending is "crowded out" by increased government spending.
- Output increases but loss of welfare as both c, l fall.
- These predictions match US experience of WWII.

Figure 5.6 Equilibrium Effects of an Increase in Government Spending

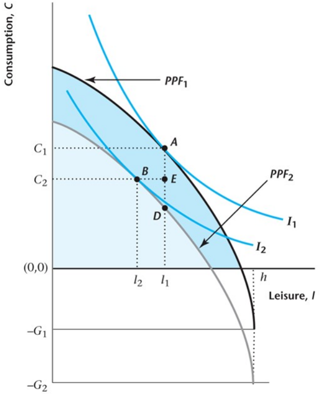
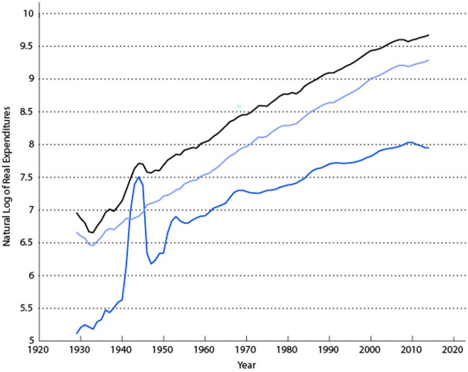


Figure 5.7 GDP, Consumption, and Government Expenditures



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What Does This Analysis Miss?

- **Government debt.** A large fraction of the wartime spending was financed by government debt. Deficit/GDP ratio hit 24% by 1944.
- Debt allows for intertemporal substitution of resources and smoothing burden of taxation. If needed to increase (distortionary) taxes to finance full war spending, production would have been less.
- **Increased productivity.** Wartime mobilization of production increased labor productivity dramatically.
- Led to larger increase in production than our model suggests.

Figure 15.04 Deficits and primary deficits: Federal, state, and local, 1940-1998

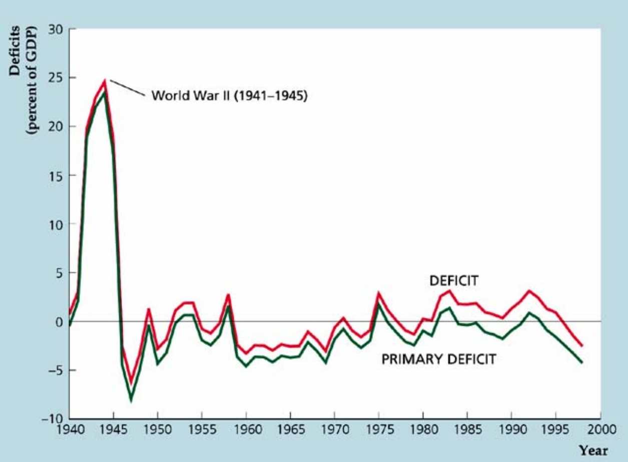


Figure 1.02 Average labor productivity in the United States, 1900-1998

Output Effects of Fiscal Policy

- Can define the multiplier for government spending as the percentage by which output increases for a given increase in government spending:

$$\text{multiplier} = \frac{\Delta Y}{\Delta G} = \frac{Y' - Y}{G' - G}$$

- In our model, using $G = gY$, $G' = g'Y'$:

$$\frac{Y' - Y}{g'Y' - gY} = \frac{z\bar{K}^{\alpha}\left[\frac{1-\alpha}{\gamma}\right]^{1-\alpha}\left((1-g')^{\alpha-1} - (1-g)^{\alpha-1}\right)}{z\bar{K}^{\alpha}\left[\frac{1-\alpha}{\gamma}\right]^{1-\alpha}\left(g'(1-g')^{\alpha-1} - g(1-g)^{\alpha-1}\right)}$$

$$= \frac{(1-g')^{\alpha-1} - (1-g)^{\alpha-1}}{g'(1-g')^{\alpha-1} - g(1-g)^{\alpha-1}}$$

Fiscal Multiplier

- Discussions of fiscal stimulus after 2008 recession, the size of the multiplier a source of some controversy.
  Obama administration suggested ≈ 1.5, Barro suggested ≈ 0.
- In our model with $g = 0.2$, $\alpha = 0.3$, $g' = 0.25$ the multiplier is about 0.75.
- This isn't the best framework for current issues, as there's no unemployment or idle resources. These are the main rationale for the fiscal stimulus.
- In the model here, increase in G always bad for welfare even if output increases.

Evidence on Multipliers

- Many papers have estimated effect government spending on output. Empirical challenge to isolate exogenous change in government spending from endogenous government spending responses
- The multiplier may be higher in recessions where there is “slack” (idle resources) or when monetary policy is constrained by the zero lower bound.
- A recent paper by Ramey and Zubairy (2018) looks at historical US evidence, using measures of unanticipated increases in government spending from two sources: military buildups, and changes in govt spending not accounted for by regression results. Allow multipliers to differ in high and low unemployment periods, zero bound.
- They find multipliers generally less than 1 across periods, with multipliers possibly as high as 1.5 if the zero bound holds.

Figure 3. Military spending news, Blanchard-Perotti shock and unemployment rate

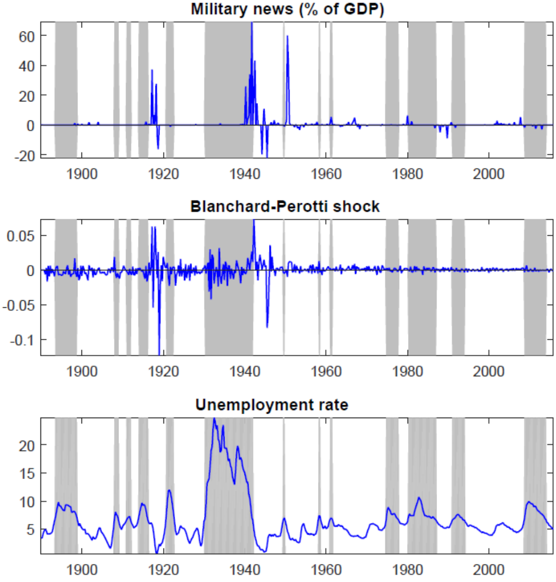


Table 1. Estimates of Multipliers Across States of Slack

	Linear Model	High Unemployment	Low Unemployment	P-value for difference in multipliers across states
Military news shock				
2 year integral	0.66	0.59	0.57	HAC=0.845
	(0.068)	(0.093)	(0.087)	AR=0.843
4 year integral	0.71	0.68	0.64	HAC=0.775
	(0.045)	(0.051)	(0.111)	AR=0.770
Blanchard-Perotti shock				
2 year integral	0.38	0.65	0.31	HAC=0.013
	(0.111)	(0.104)	(0.111)	AR =0.098
4 year integral	0.47	0.75	0.35	HAC=0.001
	(0.111)	(0.074)	(0.108)	AR =0.042
Combined				
2 year integral	0.41	0.60	0.33	HAC=0.110
	(0.101)	(0.094)	(0.110)	AR =0.273
4 year integral	0.56	0.68	0.39	HAC=0.018
	(0.086)	(0.051)	(0.109)	AR =0.214

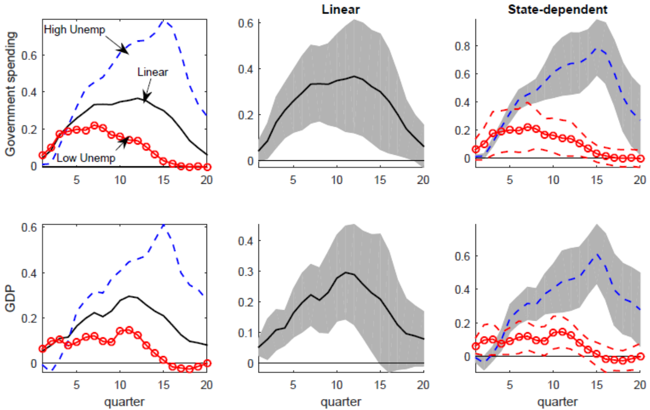
Note: The values in brackets under the multipliers give the standard errors. HAC indicates HAC-robust p-values and AR indicates weak instrument robust Anderson-Rubin p-values.

Table 5. Estimates of Multipliers Across Monetary Policy Regimes: Excluding World War II

	Linear Model	Near Zero Lower Bound	Normal	P-value for difference in multipliers across
Military news shock				
2 year integral	0.78	1.44	0.62	HAC=0.000
	(0.202)	(0.147)	(0.151)	AR =0.258
4 year integral	0.75	1.01	0.76	HAC=0.473
	(0.163)	(0.109)	(0.365)	AR =0.548
Blanchard-Perotti shock				
2 year integral	0.11	0.93	0.08	HAC=0.286
	(0.090)	(0.784)	(0.112)	AR =0.369
4 year integral	0.14	0.69	0.11	HAC=0.467
	(0.099)	(0.785)	(0.124)	AR =0.574
Combined				
2 year integral	0.20	1.56	0.26	HAC=0.012
	(0.093)	(0.496)	(0.109)	AR =0.212
4 year integral	0.25	1.09	0.21	HAC=0.003
	(0.106)	(0.247)	(0.142)	AR =0.364

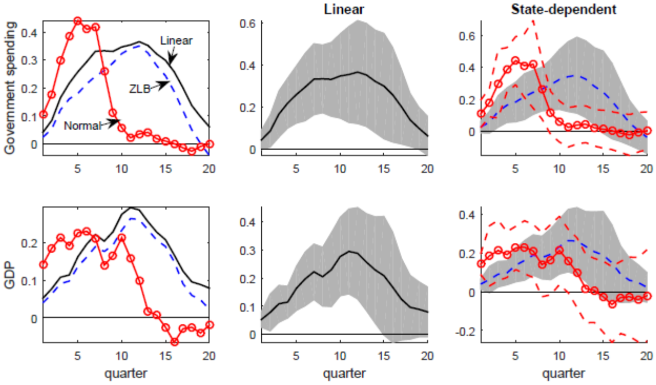
Note: The values in brackets under the multipliers give the standard errors. HAC indicates HAC-robust p-values and AR indicates weak instrument robust Anderson-Rubin p-values.

Figure 5. Government spending and GDP responses to a news shock: Considering slack states



Note: Response of government spending and GDP to a news shock equal to 1% of GDP. The top row shows the response of government spending and the second row shows the response of GDP. The first column shows the responses in the linear and state-dependent model. The second column shows the responses in the linear model. The last column shows the state-dependent responses where the blue dashed lines are responses in the high unemployment state and the lines with red circles are responses in the low unemployment state. 95% confidence intervals are shown in second and third columns.

Figure 11. Government spending and GDP responses to a news shock: Considering zero lower bound



Note: Response of government spending and GDP to a news shock equal to 1% of GDP. The top row shows the response of government spending and the second row shows the response of GDP. The first column shows the responses in the linear and state-dependent model. The second column shows the responses in the linear model. The last column shows the state-dependent responses where the blue dashed lines are responses in the near zero lower bound state and the lines with red circles are responses in the normal state. 95% confidence intervals are shown in second and third columns.