Non-Participation

- In previous we assumed an interior solution, \( l < h \) or \( N > 0 \). But if \( \pi > 0 \) there may be corner solutions of individuals who choose not to work.

- Impose additional constraint \( l < h \) via Kuhn-Tucker multiplier \( \mu \). \( \mu = 0 \) if \( l < h \), \( \mu > 0 \) if \( l = h \).

\[
\max_{c,l} u(c, l) + \lambda[(h - l)w + \pi - c] + \mu[h - l]
\]

- First order conditions:

\[
\begin{align*}
uc &= \lambda \\
ul &= \lambda w + \mu \\
\Rightarrow \frac{ul}{uc} &= w + \frac{\mu}{\lambda} \geq w
\end{align*}
\]

- For non-participant, marginal rate of substitution > wage.
Figure 4.6  The Representative Consumer Chooses Not to Work
A Parametric Example

- $u(c, l) = \log c + \gamma \log l$

\[ MRS = \frac{u_l}{u_c} = \frac{\gamma \frac{1}{l}}{\frac{1}{c}} = \gamma \frac{c}{l} \]

- FOC+Budget constraint:

\[ \gamma \frac{c^*}{h - N^*} = w \]
\[ c^* = N^* w + \pi \]

- Then:

\[ N^* = \frac{wh - \gamma \pi}{(1 + \gamma)w} \]
We use the Hicksian decomposition to see effects of a change in $w$.

**Income Effect:** changes in $w$ induce changes in total income even if $l^*$ stays constant. Reduces work incentive: use more income to “buy” leisure.

**Pure income effect:** Increase in $\pi$.

**Substitution Effect:** changes in $w$ make leisure change its relative price with total utility constant. Increases work incentive.

**(Almost) pure substitution effect:** One-time change in wage, say in peak sales period.
Figure 4.7 An Increase $\pi - T$ for the Consumer
Figure 4.8  Increase in the Real Wage Rate—Income and Substitution Effects
Income effect:

\[ \frac{\partial N^*}{\partial \pi} = -\frac{\gamma}{(1 + \gamma)w} < 0 \]

Suppose \( \pi = 0 \), then

\[ N^* = \frac{h}{1 + \gamma} \]

Labor supply does not respond to the wage at all! So income and substitution effects completely offset.

With \( \pi > 0 \) income effect only partly offsets substitution effect.
Labor supply curve $N(w)$ plots response of labor supplied by households to a change in wage, holding fixed unearned income (and preferences).

For individual workers, slope of labor supply unclear. Depends on income and substitution effects. For high enough wage, may be backward bending. That is $N'(w) > 0$ for low $w$ but $N'(w) < 0$ for $w$ high enough.

May also be discontinuous if there are fixed costs of work (commuting costs). Won’t work low number of hours.
In the aggregate, labor supply supply curve embodies both intensive and extensive margins, and is is upward sloping.

Intensive margin: for those already working, increase in wage has income and substitution effects.

Extensive margin: increases in wages may induce some who were not in labor force to enter and supply labor. Always increasing in $w$.

Aggregate labor supply curve also smooths out kinks in individual supply, for example due to fixed costs of work.
Figure 4.10  Effect of an Increase in Dividend Income or a Decrease in Taxes
What is a firm?

- A firm uses capital $K$ and labor $N$ to produce output $Y$ via a production function $F$:

$Y = zF(K, N)$

$z$ is the level of technology or total factor productivity (TFP).

- The main example we’ll use is Cobb-Douglas production function, with $\alpha \in (0, 1)$:

$Y = zK^\alpha N^{1-\alpha}$

- We will start with a static model: $K$ is constant.
Properties of the Technology

We’ll make several assumptions on the technology $F$, all of which are satisfied by Cobb-Douglas.

1. Inputs are essential.

$$F(0, N) = F(K, 0) = 0$$

2. Constant returns to scale:

$$F(\lambda K, \lambda N) = \lambda F(K, N)$$

Doubling inputs doubles output. Compared to decreasing (increasing) returns to scale where doubling inputs leads to less (more) than double output.

$$F(K, N) = K^\alpha N^{1-\alpha}$$

$$F(\lambda K, \lambda N) = (\lambda K)^\alpha (\lambda N)^{1-\alpha}$$

$$= \lambda K^\alpha N^{1-\alpha}$$
3. Marginal productivities of capital and labor are positive and decreasing.

\[ MPK = F_K > 0, \quad F_{KK} < 0 \]
\[ MPN = F_N > 0, \quad F_{NN} < 0 \]

Increasing each factor gives more output, but at a decreasing rate.

\[ F(K, N) = K^\alpha N^{1-\alpha} \]
\[ F_K = \alpha K^{\alpha-1} N^{1-\alpha} > 0 \]
\[ F_{KK} = \alpha(\alpha - 1) K^{\alpha-2} N^{1-\alpha} < 0 \]
\[ F_N = (1 - \alpha) K^\alpha N^{-\alpha} > 0 \]
\[ F_{NN} = -\alpha(1 - \alpha) K^\alpha N^{-\alpha-1} < 0 \]
Figure 4.14 Production Function, Fixing the Quantity of Capital and Varying the Quantity of Labor

Output, $y$

Slope $= MP_N$

$F(K^*, N^d)$

$N^*$

Labor Input, $N^d$
4. Marginal productivity of each factor increases in the other.

\[
\frac{\partial MPK}{\partial N} = \frac{\partial}{\partial N} F_K = F_{KN} > 0 \\
\frac{\partial MPN}{\partial K} = \frac{\partial}{\partial K} F_N = F_{KN} > 0
\]

Note one implies other since \( F_{KN} = F_{NK} \).

Additional capital makes workers more productive: spread workers among more machines (and vice versa).

\[
\begin{align*}
F_K &= \alpha K^{\alpha-1} N^{1-\alpha} \\
F_{KN} &= \alpha(1 - \alpha) K^{\alpha-1} N^{-\alpha} > 0 \\
F_N &= (1 - \alpha) K^\alpha N^{-\alpha} \\
F_{NK} &= \alpha(1 - \alpha) K^{\alpha-1} N^{-\alpha} > 0
\end{align*}
\]
Figure 4.18  Total Factor Productivity Increases
Problem of the Firm I

- Competitive firm rents capital at rate $r$, hires labor at wage $w$.
- Profits: output minus costs

$$\pi = zF(K, N) - rK - wN$$

- Note everything in real terms – same as setting price of output to 1.
- Firm maximizes profits by hiring labor and capital. We take first order conditions for choice of $K$ and $N$:

$$zF_K(K, N) = r$$
$$zF_N(K, N) = w$$

(1)

- Factors are paid their marginal products.
- (1) can be solved to give the labor demand: $N^d(w)$
Figure 4.20  The Marginal Product of Labor Curve Is the Labor Demand Curve of the Profit-Maximizing Firm
With Cobb-Douglas production, we have:

$$\pi = zK^\alpha N^{1-\alpha} - rK - wN$$

We take first order conditions for $K$ and $N$:

$$zF_K = z\alpha K^{\alpha-1} N^{1-\alpha} = r \quad (2)$$

$$zF_N = z(1-\alpha) K^\alpha N^{-\alpha} = w \quad (3)$$

(3) can be solved to give the labor demand:

$$N^d(w; K) = MPN^{-1}(w; K) = K \left( \frac{z(1-\alpha)}{w} \right)^{\frac{1}{\alpha}}$$

$N^d$ decreasing in $w$. Increases in $z$, $K$ shift labor demand.
To fully solve the example, we want to solve for $K$ and $N$.

We begin dividing (2) by (3):

$$\frac{\alpha zK^{\alpha-1} N^{1-\alpha}}{(1 - \alpha) zK^\alpha N^{-\alpha}} = \frac{r}{w}$$

or

$$\frac{\alpha}{1 - \alpha} \frac{N}{K} = \frac{r}{w}$$

or

$$K = \frac{w}{r} \frac{\alpha}{1 - \alpha} N$$ (4)

This gives us the optimal capital/labor ratio of the firm. Depends on relative prices $w/r$, relative productivities $\alpha, 1 - \alpha$. 

But if we substitute (4) in (1), trying to solve for $N$, we get:

\[
(1 - \alpha) z K^\alpha N^{-\alpha} = w
\]

\[
(1 - \alpha) z \left( \frac{w}{r} \frac{\alpha}{1 - \alpha} N \right)^\alpha N^{-\alpha} = w
\]

\[
(1 - \alpha) z \left( \frac{w}{r} \frac{\alpha}{1 - \alpha} \right)^\alpha = w
\]

$N$ disappears!

You can check that the same happens with $K$ if we substitute (4) in (2).

What is wrong?
We have constant returns to scale.

The size of the firm is indeterminate: we can have just one!

Can show constant returns equivalent to

\[ F(K, N) = F_K K + F_N N. \]

This implies profits are always zero if firm maximizes, since

\[ r = MPK, \ w = MPN. \]

\[
\pi = zK^\alpha N^{1-\alpha} - rK - wN \\
= zK^\alpha N^{1-\alpha} - MPK \cdot K - MPN \cdot N \\
= zK^\alpha N^{1-\alpha} - \alpha zK^{\alpha-1} N^{1-\alpha} K - (1 - \alpha) zK^\alpha N^{-\alpha} N = 0
\]

Example of Euler's theorem for homogeneous functions

So the firms really only pick the labor-capital ratio given relative prices:

\[
\frac{N}{K} = \frac{1 - \alpha}{\alpha} \frac{r}{w}
\]
In equilibrium, markets clear so:

\[ N^d (w, K) = N^s (w) \]

\[ K^d (r, N) = \bar{K} \quad \text{i.e., fixed} \]

\[ r = \alpha z K^{\alpha - 1} N^{1-\alpha} \]

\[ w = (1 - \alpha) z K^\alpha N^{-\alpha} \]

- We have a system of four equations in four unknowns \((K, N, r, w)\).
- In other words, firms choose \(K/N\) ratio, but by assumption \(K\) is fixed at \(\bar{K}\) in short run, which gives labor demand \(N^d\).
- We will return to discussing general equilibrium shortly.
A Simple Example

A representative worker has preferences:

\[ u(C, l) = 2\sqrt{C} - a \frac{(1 - l)^{1.5}}{1.5} \]

The budget constraint is:

\[ C = w(1 - l), \]

where 1 is the hours in the day, so 1 − l is labor supply. Capital is fixed at 1, and the representative firm technology is:

\[ Y = zN^{0.5} \]

1. Find the household labor supply function.
2. Find expressions for the equilibrium values of the labor input and the wage.
3. Suppose that a increases but z is unchanged. What is the effect of this change on labor and the wage?
What are we missing?

- Wages are often different from marginal productivity. Labor market not completely a spot market.
- Some reasons:

We will return to some of these frictions later in the class. For now continue with competitive, spot labor market.
Operational definition: takes in taxes $T$ and spends $G$.

We’ll assume a balanced budget. No debt in this static model.

Also assume household does not value government spending. Not crucial here.

We’ll also assume proportional labor income taxes on households. Later will consider lump sum taxes.

\[
G = T \\
T = \tau wN
\]
Set unearned income equal to capital income $rK$. Incorporate labor income taxes.

Problem for household is now:

$$\max_{c,N} u(c, l)$$

subject to

$$c = (1 - \tau) wN + rK$$

First order conditions:

$$\frac{u_l}{u_c} = (1 - \tau) w$$

Household now equates $MRS$ to after-tax wage.

Taxes affect labor supply!
Reconsider the log example from earlier:

\[ u(c, l) = \log c + \gamma \log l \]

Then we get:

\[ \gamma \frac{c^*}{h - N^*} = (1 - \tau)w \]

\[ c^* = N^*(1 - \tau)w + \pi \]

Therefore we see that taxes affect labor supply:

\[ N^* = \frac{(1 - \tau)wh - rK}{(1 + \gamma)(1 - \tau)w} \]
Laffer Curve

Tax revenue has a Laffer curve:

\[ T = \tau wN = w\tau \frac{(1 - \tau) \, wh - rK}{(1 + \gamma) (1 - \tau) \, w} \]
If you tax a product less results in a subsidy to more...

We've been taxing work, output, and income and subsidizing non-work, leisure, and unemployment. The consequences are obvious!

\[
\frac{\Delta IR}{\Delta t} < 0
\]

\[
\text{Prohibitive Range}
\]

\[
\text{Normal Range}
\]

To Den Rumsfeld, at our Two Contests, Longview 9/13/74

Casey B. Leff
There were arguments in 1981 and 2001 that US economy was on the bad side of the Laffer curve, and revenue could increase with tax cuts.

Although these tax cuts may have increased economic growth, there is no evidence that revenue increased.

Estimates of peak tax revenue in US are at 60% or greater federal income tax rate.

A 2010 ECB study found Sweden was on the bad side, with a top labor income tax of 57% and a payroll tax of 31%. Historically it had been even higher, up to 90%.

Laffer curve arguments are more likely apply to narrower categories of goods which have higher elasticities of substitution, like luxury goods or possibly capital gains.