Lecture 14
Real Business Cycles

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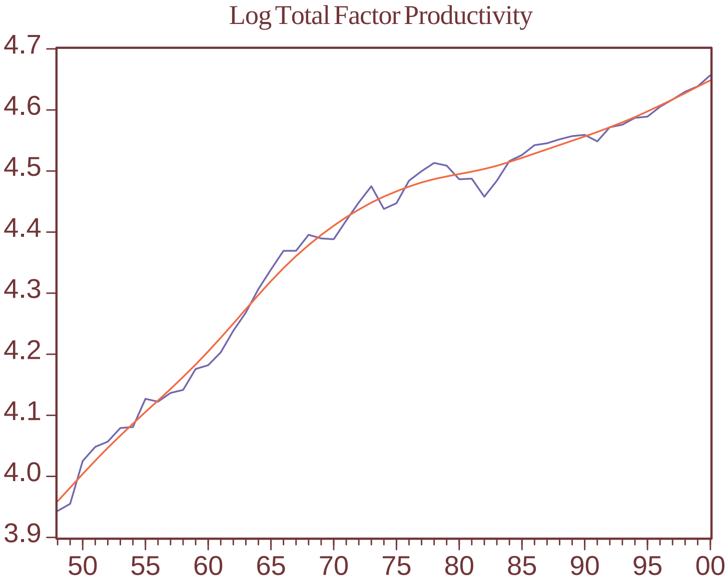
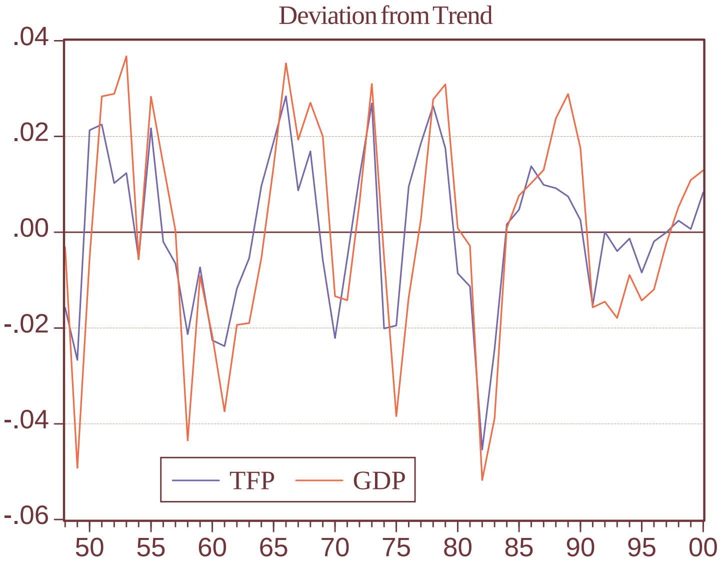
Economics 702

- We learned how to map preferences (for the household), technology (for the firm) and a government policy into a Competitive Equilibrium.
- If we let preferences, technology or the government preferences change over time, the equilibrium sequence will also fluctuate.
- We will use a model of this type to analyze business cycle fluctuations.
- All these (preferences, technology, policy) are real factors (as opposed to monetary).
- This is why we call this approach Real Business Cycles.
- Basic model: Brock and Mirman (1972)
  Big innovation: Kydland and Prescott (1982), 2004 Nobel Prize winners.

- We will have a shock: change in technology or policy.
- Then we will have a propagation mechanism: intertemporal labor substitution and capital accumulation.
- We will have fluctuations as an equilibrium outcome.
- Main driving force is changes in productivity. Solow model emphasized TFP as source of growth. Now emphasize (random) variations in TFP as source of business cycles.
- Basic idea: intertemporal substitution. When productivity is high, want to work more, produce more. When it is low, the reverse. Changes in productivity drive output.
- Observation: TFP (Solow residual) and GDP are highly correlated.


Log Total Factor Productivity
4.7
4.6
4.5
4.4
4.3
4.2
4.1
4.0
3.9
50
55
60
65
70
75
80
85
90
95
00


Deviation from Trend
.04
.02
.00
-.02
-.04
-.06
50
55
60
65
70
75
80
85
90
95
00
TFP
GDP

- Before studying business cycle models, let's first discuss the features in the data that characterize a business cycle.
- How different are long-run growth and the business cycle?

Changes in Output/Worker	*Secular Growth*	*Business Cycle*
Due to capital	1/3	0
Due to labor	0	2/3
Due to productivity	2/3	1/3

- We will use the same models with a slightly different focus.
- View growth models as determining the trend in the data, now analyze business cycles as deviations from trend.

Business Cycle Definitions and Regularities

- Recession: Period of declining GDP (relative to trend).
- Boom: Period of increasing GDP (relative to trend).
- Peak: End of boom, beginning of recession.
- Trough: End of recession, beginning of boom.
- No regular amplitude or frequency of fluctuations.
- But there are some regularities or "business cycle facts":
    - Correlations (comovements)
    - Relative variabilities
    - Phase: leading/coincident/lagging

Table 1
Business Cycle Statistics for the U.S. Economy

	Standard Deviation	Relative Standard Deviation	First Order Auto-correlation	Contemporaneous Correlation with Output
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30	2.93	0.87	0.80
N	1.79	0.99	0.88	0.88
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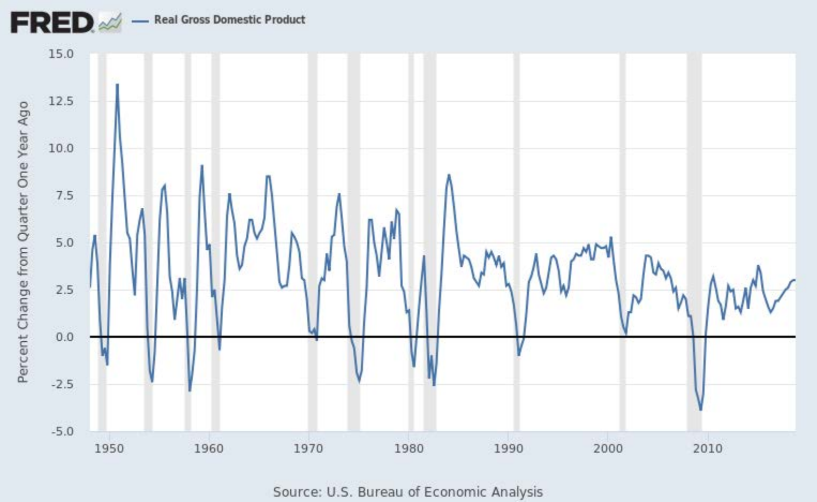
w	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	-0.35
A	0.98	0.54	0.74	0.78

- US experience: more volatile business cycles pre-war, moderation in fluctuations post-war and especially post 1984.
- Of obvious recent relevance. On December 1, 2008 NBER determined US entered a recession in December 2007. On September 20, 2010 declared recession ended in June 2009.
- Issue of stabilization policy for business cycles: Should Fed cut rates? Should govt. cut taxes? Should government increase spending?

Percentage Change in Real GDP



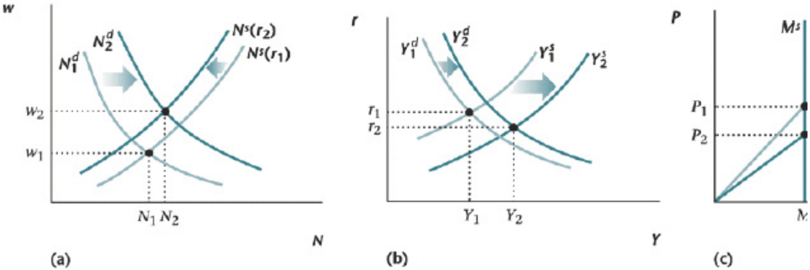
NBER Business Cycle Dates

BUSINESS CYCLE REFERENCE DATES		DURATION IN MONTHS			
Peak	**Trough**	**Contraction**	**Expansion**	**Cycle**	
Quarterly dates are in parentheses		*Peak to Trough*	*Previous trough to this peak*	*Trough from Previous Trough*	*Peak from Previous Peak*
	December 1854 (IV)	--	--	--	--
June 1857(II)	December 1858 (IV)	18	30	48	--
October 1860(III)	June 1861 (III)	8	22	30	40
April 1865(I)	December 1867 (I)	32	46	78	54
June 1869(II)	December 1870 (IV)	18	18	36	50
October 1873(III)	March 1879 (I)	65	34	99	52
March 1882(I)	May 1885 (II)	38	36	74	101
March 1887(II)	April 1888 (I)	13	22	35	60
July 1890(III)	May 1891 (II)	10	27	37	40
January 1893(I)	June 1894 (II)	17	20	37	30
December 1895(IV)	June 1897 (II)	18	18	36	35
June 1899(III)	December 1900 (IV)	18	24	42	42
September 1902(IV)	August 1904 (III)	23	21	44	39
May 1907(II)	June 1908 (II)	13	33	46	56
January 1910(I)	January 1912 (IV)	24	19	43	32
January 1913(I)	December 1914 (IV)	23	12	35	36
August 1918(III)	March 1919 (I)	7	44	51	67
January 1920(I)	July 1921 (III)	18	10	28	17
May 1923(II)	July 1924 (III)	14	22	36	40
October 1926(III)	November 1927 (IV)	13	27	40	41
August 1929(III)	March 1933 (I)	43	21	64	34
May 1937(II)	June 1938 (II)	13	50	63	93
February 1945(I)	October 1945 (IV)	8	80	88	93
November 1948(IV)	October 1949 (IV)	11	37	48	45
July 1953(II)	May 1954 (II)	10	45	55	56
August 1957(III)	April 1958 (II)	8	39	47	49
April 1960(II)	February 1961 (I)	10	24	34	32
December 1969(IV)	November 1970 (IV)	11	106	117	116
November 1973(IV)	March 1975 (I)	16	36	52	47
January 1980(I)	July 1980 (III)	6	58	64	74
July 1981(III)	November 1982 (IV)	16	12	28	18
July 1990(III)	March 1991(I)	8	92	100	108
March 2001(I)	November 2001 (IV)	8	120	128	128
December 2007 (IV)	June 2009 (II)	18	73	91	81
Average, all cycles:					
1854-2009 (33 cycles)		17.5	38.7	56.2	56.4*
1854-1919 (16 cycles)		21.6	26.6	48.2	48.9**
1919-1945 (6 cycles)		18.2	35.0	53.2	53.0
1945-2009 (11 cycles)		11.1	58.4	69.5	68.5

Persistent Productivity Increase

- Now consider a persistent increase: productivity stays high for “a while” but comes back down.
- Substitution effect outweighs income effect in labor supply.
  - Labor demand increases more than labor supply falls.
  - Wages and employment increase.
  - Output supply increases
- Consumption and investment demand increase slightly.
- Output supply increases more than output demand.
- Interest rates fall and output increases.

Figure 11.3 Effects of a Persistent Increase in Total Factor Productivity the Real Business Cycle Model



A Quantitative Business Cycle Model

- Let's now suppose that we have an economy that is hit over time by productivity shocks with the same characteristics that the ones that hit the US economy.
- How does this economy behave? In particular, how do the variances and covariances of the main variables in our economy compare with those observed in the US economy?
- Basic real business cycle model due to Kydland and Prescott (1982). One of the two main contributions for which they won the Nobel prize in 2004.

Contribution of Kydland and Prescott

- In addition to its importance as a business cycle model, the Kydland-Prescott paper had a number of other methodological contributions.
- Part of a then-new literature on rational expectations. Agents within the model understand fully the equilibrium laws of motion.
- Used computer to solve and simulate the model to derive predictions to a broader extent then before.
- Focused on calibration of the model rather than formal estimation.

Logic of the Model

- As described earlier, persistent shocks to TFP are the driving source of fluctuations in the model. No other randomness.
- Static effects of change in TFP: Implies higher labor productivity, increasing wages. Substitution effect leads to higher labor supply, thus increasing output.
- Dynamic effects of change in TFP: Part of increased output is consumed, but part is saved. The more persistent the effect, the more saved. Also greater returns to capital so more investment, yielding higher capital stock.
- So for extended period get greater output due to increases in labor and capital inputs as well as direct TFP effect.
- Effects of a single shock eventually die out, but they may be long-lived. However new shocks continually arrive.

Household Problem

- Add uncertainty, due to productivity shocks, to our infinite horizon model. This will imply fluctuations in wages and interest rates.
- Households must forecast future wages and interest rates when deciding how much to work and save. So the household maximizes expected utility.

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, 1 - N_t\right)$$

$$C_t + K_{t+1} = w_t N_t + (1 + r_t) K_t, \; \forall \; t > 0$$

Household Optimization

- First order conditions for consumption choice at any dates t and $t+1$ gives an Euler equation, now modified for uncertainty.

$$u_C(C_t, 1 - N_t) = \beta E_t \left[u_C(C_{t+1}, 1 - N_{t+1}) \left(1 + r_{t+1}\right) \right]$$

  Since r_{t+1} is unknown at t, so are C_{t+1} and N_{t+1}. So we must take expectations.

- First order condition for leisure choice is static with no effect of uncertainty. Implies same marginal condition:

$$MRS = \frac{u_l(C_t, 1 - N_t)}{u_C(C_t, 1 - N_t)} = w_t$$

Problem of the Firm

- Firm's problem is the same as before, however technology changes over time. Now z_t is random.
- Firm invests at date t, when future productivity z_{t+1} and interest rate r_{t+1} uncertain
- The firm optimality conditions are then:

$$\begin{aligned} z_t F_N(K_t, N_t) &= w_t \\ E_t[z_{t+1} F_K(K_{t+1}, N_{t+1})] - \delta &= E_t r_{t+1} \end{aligned}$$

- These are essentially the same as before, with the second modified for uncertainty.
- Firms invest until the expected future marginal product of capital is equal to the expected interest rate plus depreciation.

Optimality Conditions

- Euler equation under uncertainty:

$$u_C(C_t, 1-N_t) = \beta E_t \left[u_C(C_{t+1}, 1-N_{t+1})\left(1 + z_{t+1}F_K(K_{t+1}, N_{t+1}) - \delta\right)\right]$$

- Labor market optimality:

$$\frac{u_l(C_t, 1-N_t)}{u_C(C_t, 1-N_t)} = z_t F_N(K_t, N_t)$$

- Goods market clearing:

$$K_{t+1} = z_t F(K_t, N_t) + (1-\delta)K_t - C_t$$

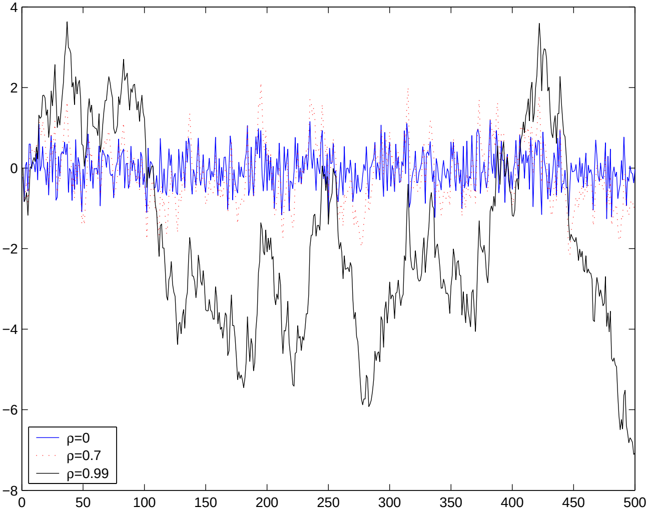
Evolution of the Technology

- z_t changes randomly over time. Ignore growth and just think of fluctuations around a trend.
- We assume it follows the process:

$$\begin{aligned} \log z_t &= \rho \log z_{t-1} + \varepsilon_t \\ \varepsilon_t &\sim N(0, \sigma^2) \end{aligned}$$

- This process is called $AR(1)$: an autoregression of order 1.
- The parameter ρ governs how persistent are the changes in TFP. If $\rho = 1$ they are permanent. If $0 < \rho < 1$ they are persistent but eventually die out.

Examples of TFP Processes



A Competitive Equilibrium

- This economy has a unique competitive equilibrium.
- This economy satisfies the conditions that assure that both welfare theorems hold.
- Why is this important?
- Practical: We can solve instead the Social Planner's Problem associated with it.
- Normative: Business cycles in the model are efficient.
- Fluctuations are the optimal response to a changing environment. They are not sufficient for inefficiencies or for government intervention. In this model the government can only worsen the allocation.

Solving the Model

- The previous problem does not have a known "paper and pencil" analytic solution.
- Analysis of the model requires some approximations (such as linearization) or numerical analysis.
- Computational methods are a crucial part of modern macroeconomics. We can solve and simulate the model numerically, then estimate or calibrate parameters to match aspects of the data.
- In addition, we can then use the structural models for policy analysis and so study counterfactuals.
- For example, the program suite DYNARE for use in Matlab is convenient toolbox (integrated with Matlab) to solve, estimate, and simulate dynamic stochastic general equilibrium models.

Solving the Model in a Special Case

- There is one known case where we can work out an explicit solution.
- Set $\delta = 1$ (full depreciation) use our Cobb-Douglas production, and log utility:

$$u(C, 1-N) = (1-a)\log C + a\log(1-N).$$

- Specialize the key equilibrium conditions:

$$\begin{aligned}
\frac{aC_t}{(1-a)1-N_t} &= (1-\alpha)\, z_t K_t^{\alpha} N_t^{-\alpha} \\
\frac{1}{C_t} &= \beta E_t \left[\frac{\alpha z_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha}}{C_{t+1}} \right] \\
K_{t+1} &= z_t K_t^{\alpha} N_t^{1-\alpha} - C_t
\end{aligned}$$

- Make the following guesses:

$$C_t = (1-s)Y_t, \;\; N_t = \bar{N}$$

Constant saving rate s, constant labor supply $\bar{N}$.

- Substitute into conditions:

$$\begin{aligned}
\frac{a(1-s)z_t K_t^{\alpha}\bar{N}^{1-\alpha}}{(1-a)(1-\bar{N})} &= (1-\alpha)\, z_t K_t^{\alpha}\bar{N}^{-\alpha}. \\
\frac{1}{(1-s)z_t K_t^{\alpha}\bar{N}^{1-\alpha}} &= \beta E_t\left[\frac{\alpha z_{t+1}K_{t+1}^{\alpha-1}\bar{N}^{1-\alpha}}{(1-s)z_{t+1}K_{t+1}^{\alpha}\bar{N}^{1-\alpha}}\right] \\
&= \beta E_t\left[\frac{\alpha}{(1-s)K_{t+1}}\right] \\
&= \beta E_t\left[\frac{\alpha}{(1-s)s z_t K_t^{\alpha}\bar{N}^{1-\alpha}}\right] \\
\Rightarrow s &= \beta\alpha
\end{aligned}$$

Implications

- This special case is then similar to the Solow model: constant savings rate. Constant labor supply (no growth). Difference is random shocks.
- Now $K_{t+1} = sY_t$, so

$$\begin{aligned} Y_{t+1} &= z_{t+1} K_{t+1}^{\alpha} \bar{N}^{1-\alpha} \\ &= z_{t+1} \left(sY_t \right)^{\alpha} \bar{N}^{1-\alpha}. \end{aligned}$$

- Taking logs:

$$\begin{aligned} \log Y_{t+1} &= \mu + \log z_{t+1} + \alpha \log Y_t \\ &= \mu + \rho \log z_t + \alpha \log Y_t + \varepsilon_{t+1}. \end{aligned}$$

where $\mu = \alpha \log s + (1-\alpha) \log \bar{N}$

Implications: Output Persistence

$$\log Y_{t+1} = \mu + \rho \log z_t + \alpha \log Y_t + \varepsilon_{t+1}.$$

- Output and technology together follow a (vector) $AR(1)$.
- Can simplify further, using:

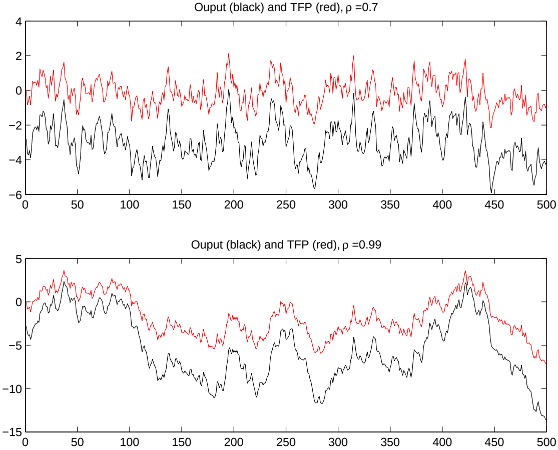
$$\log z_t = \log Y_t - \mu - \alpha \log Y_{t-1}$$

So then:

$$\log Y_{t+1} = (1-\rho)\mu + (\rho + \alpha) \log Y_t - \alpha\rho \log Y_{t-1} + \varepsilon_{t+1}.$$

- Output follows an AR(2) process.
- Output is persistent because of the TFP shocks and because of capital accumulation.

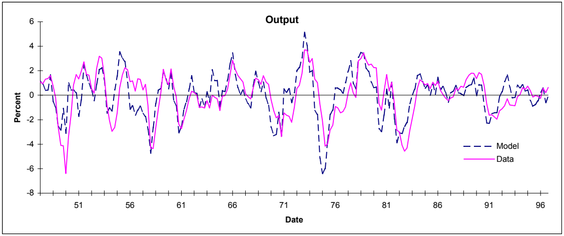
Output and TFP Co-movements

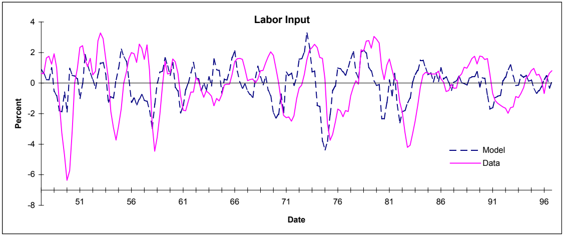


Simulations from a Quantitative Version

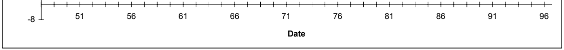
- We have seen the qualitative behavior of the model, showing that the real business cycle model is consistent with the data.
- Apart from the special case we studied, to fully solve the model we need to use numerical methods.
- Calibrate the model: choose parameters to match some key economic data.
  Example: set β so that steady state real interest rate matches US data.
- Program up on computer and simulate: use random number generator to draw technology shocks, feed them through the model.
- Compute correlations and volatilities and compare to US data.

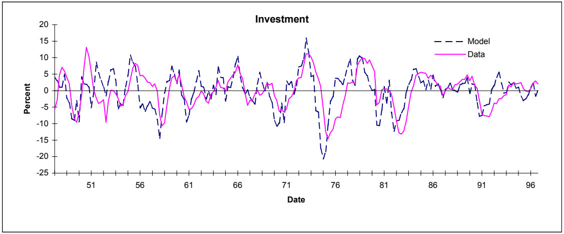
Figure 7

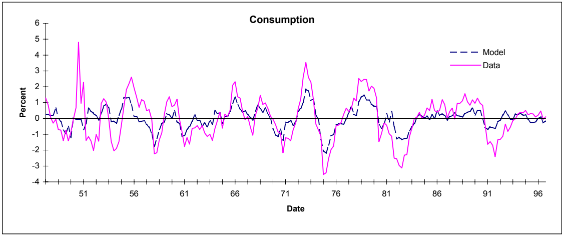

Output
Percent
6
4
2
0
-2
-4
-6
-8
51
56
61
66
71
76
81
86
91
96
Date
Model
Data


Labor Input
Percent
4
2
0
-2
-4
-6
-8
51
56
61
66
71
76
81
86
91
96
Date
Model
Data


Investment
20
15
Model

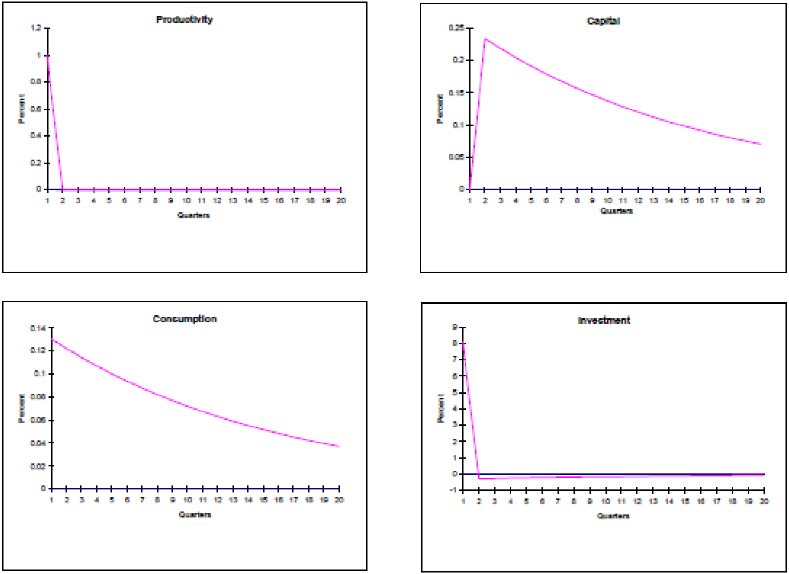


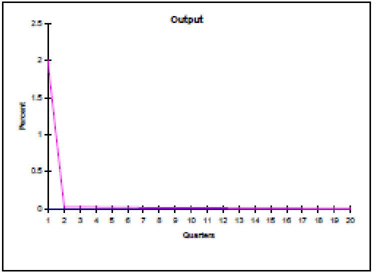


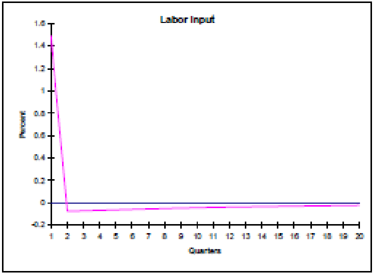


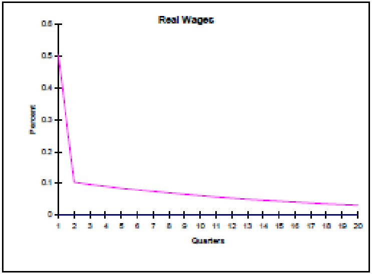
Note: Sample period is 1947:2 - 1996:4. All variables are detrended using the Hodrick-Prescott filter.

Figure 9:
Impulse Responses to a Purely Transitory Shock




Output
Percent
2.5
2
1.5
1
0.5
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Quarters


Labor Input
Percent
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Quarters


Real Wages
Percent
0.6
0.5
0.4
0.3
0.2
0.1
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Quarters

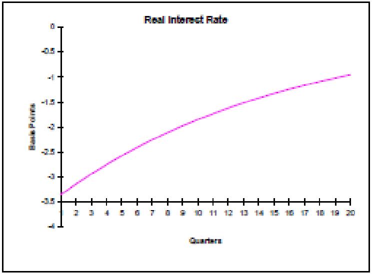
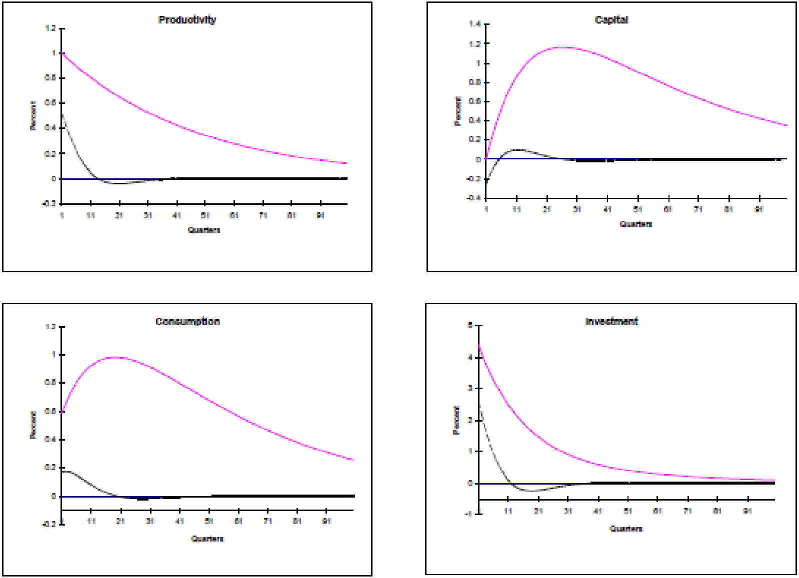
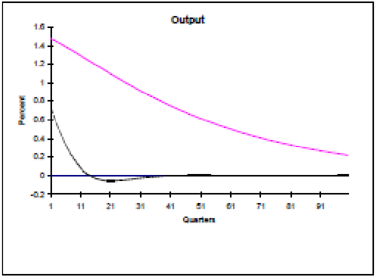
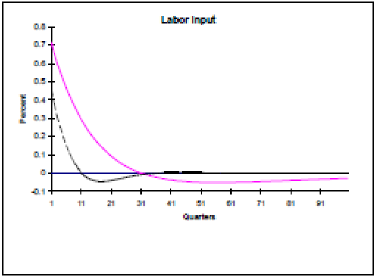
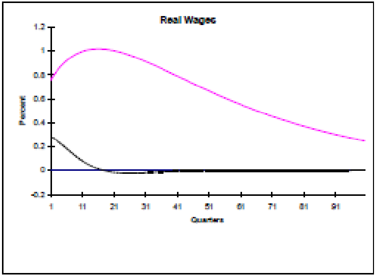

Real Interest Rate
Basis Points
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Quarters

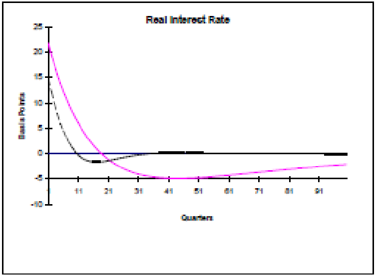
Figure 10:
Impulse Responses to a More Persistent Shock (rho=.979)




Output
Percent
Quarters


Labor Input
Percent
Quarters


Real Wages
Percent
Quarters


Real Interest Rate
Basis Points
Quarters

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w	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	-0.35
A	0.98	0.54	0.74	0.78

Table 3
Business Cycle Statistics for Basic RBC Model[35]

	Standard Deviation	Relative Standard Deviation	First Order Auto-correlation	Contemporaneous Correlation with Output
Y	1.39	1.00	0.72	1.00
C	0.61	0.44	0.79	0.94
I	4.09	2.95	0.71	0.99
N	0.67	0.48	0.71	0.97
Y/N	0.75	0.54	0.76	0.98
w	0.75	0.54	0.76	0.98
r	0.05	0.04	0.71	0.95
A	0.94	0.68	0.72	1.00

Note: All variables have been logged (with the exception of the real interest rate)and detrended with the HP filter.

Assessment of the Basic Real Business Model

- It accounts for a substantial amount of the observed fluctuations. Accounts for the covariances among a number of variables. Has some problems accounting for hours worked, consumption volatility.
- Are fluctuations in TFP really productivity fluctuations?
- Factor utilization rates vary over the business cycle. During recessions, firms reduce the number of shifts. Similarly, firms are reluctant to fire trained workers.
- Neither is well-measured – show up in the Solow residual.
- There is no direct evidence of technology fluctuations.
- Is intertemporal labor supply really so elastic?
- All employment variation in the model is voluntary, driven by intertemporal substitution.
- Deliberate monetary policy changes appear to have real effects.