• Now $x_{t+1}, r_{t+1}$ are random, unknown at $t$.
• Agents form expectations of future income, maximize expected utility.
• Can derive an Euler equation of the same form, but now must have expectations over $c_{t+1}$ and $r_{t+1}$:

$$u'(c_t) = \beta E_t \left[ u'(c_{t+1})(1 + r_{t+1}) \right]$$

• Here $E_t(\cdot)$ represents the agent’s expectations, conditional on all information available at date $t$. 
Suppose again that $r_t = r$ and $\beta(1 + r) = 1$, so the Euler equation is:

$$u'(c_t) = E_t u'(c_{t+1})$$

Also suppose that agents have quadratic preferences, where $a > 0$ is a constant:

$$u(c_t) = c_t - \frac{a}{2} c_t^2,$$

So $u'(c_t) = 1 - ac_t$ and the Euler equation becomes:

$$c_t = E_t c_{t+1}$$

Also by the law of iterated expectations:

$$c_t = E_t c_{t+1} = E_t (E_{t+1} c_{t+2}) = E_t c_{t+2}$$
Implications

- With these preferences consumption is a random walk:

\[ c_{t+1} = c_t + \varepsilon_{t+1}, \quad E_t\varepsilon_{t+1} = 0 \]

- The best predictor of consumption one period ahead is current consumption. No other variables which are known at date \( t \) help predict consumption at \( t + 1 \).

- To express this another way, note that the present value budget constraint holds for any date \( t \):

\[
\sum_{s=0}^{\infty} \frac{E_t c_{t+s}}{(1 + r)^s} = \sum_{s=0}^{\infty} \frac{E_t x_{t+s}}{(1 + r)^s} + a_t(1 + r)
\]
• Then note that $E_t c_{t+s} = c_t$ for all $s$. So then we have:

$$c_t \sum_{s=0}^{\infty} \frac{1}{(1 + r)^s} = \sum_{s=0}^{\infty} \frac{E_t x_{t+s}}{(1 + r)^s} + a_t(1 + r)$$

$$c_t = \frac{r}{1 + r} \sum_{s=0}^{\infty} \frac{E_t x_{t+s}}{(1 + r)^s} + r a_t$$

Consumption depends on expectations of all future income.

• Changes in consumption over time are driven by changes in expectations of future income. Information revealed about future income is the driver of consumption.

$$c_{t-1} = E_{t-1} c_t = \frac{r}{1 + r} \sum_{s=0}^{\infty} \frac{E_{t-1} x_{t+s}}{(1 + r)^s} + r a_t$$
A pure transitory income shock reveals at date $t$ that $x_t > E_{t-1}x_t$ is higher than anticipated, but $E_t x_{t+s}$ is unaffected for $s \geq 1$. Example: $x_t = x_{t-1} + v_t$, $x_{t+s} = x_{t-1}$.

$$c_t = c_{t-1} + \frac{r}{1+r} v_t$$

A permanent income shock reveals at date $t$ that $x_t > E_{t-1}x_t$ is higher than anticipated, and $E_t x_{t+s}$ is also higher for $s \geq 1$. Example: $x_{t+s} = x_{t-1} + x^p$

$$c_t = c_{t-1} + x^p$$
Extensions of Permanent Income Theory

- With quadratic utility, uncertainty in income does not affect decisions:
  \[ c_t = \frac{r}{1 + r} \sum_{s=0}^{\infty} \frac{E_t x_{t+s}}{(1 + r)^s} + ra_t \]

- This is a property known as certainty equivalence. Decisions are the same as if \( x_t \) took on its expected value with certainty.

- With more general preferences, variability of income would matter.

- Suppose again that \( r_t = r \) and \( \beta(1 + r) = 1 \), so the Euler equation is:
  \[ u'(c_t) = E_t u'(c_{t+1}) \]

- If \( u'(c) \) is convex (\( u'''(c) > 0 \)), then more uncertain income will lead to lower consumption today, more savings: precautionary savings.
- Quadratic utility: $u'(c) = 1 - ac$, $u''(c) = -a < 0$, $u'''(c) = 0$.

$$u'(c_t) = E_t u'(c_{t+1}) \Rightarrow c_t = E_t c_{t+1}$$

- Power utility: $u'(c) = c^{-\gamma}$, $u''(c) = -\gamma c^{-\gamma - 1} < 0$, $u'''(c) = -\gamma(-\gamma - 1)c^{-\gamma - 2} > 0$.

$$u'(c_t) = E_t u'(c_{t+1}) > u'(E_t c_{t+1}) \Rightarrow c_t < E_t c_{t+1}$$
Uncertainty about future income will lead to more savings, to allow households to smooth potential consumption fluctuations.

Periods of increased uncertainty will be characterized by reductions in household consumption.

Another complication we’ve abstracted from is borrowing constraints. These affect consumption in two ways:

1. When household is constrained, consumption will closely follow income. Unable to smooth.
2. Household will build up stock of assets to diminish the impact of the constraint.

There is significant micro evidence for these effects on household consumption. Macro effects are less clear.
Retirement saving is an important component of household saving decisions.

In US, main government program to support retired is Social Security which is a “pay-as-you-go” system, as opposed to a fully-funded one.

Use simple two-period life-cycle model to analyze the impact of Social Security on saving, welfare.

Assume $y' = 0$ and $A = 0$, for simplicity. So $y^{PV} = y$.

Consider the parametric example from before $u(c) = \log c$. 

Without social security. Euler Equation:

\[
\frac{1}{c} = \beta (1 + r) \frac{1}{c'} \Rightarrow c' = \beta (1 + r) c
\]

Note that

\[
c = y - \frac{c'}{1 + r} = y - \beta c
\]

So that:

\[
c = \frac{y}{1 + \beta} \quad c' = \frac{\beta (1 + r) y}{1 + \beta} \quad s = \frac{\beta y}{1 + \beta}
\]
Introduce a pay as-you-go social security system: currently working generation pays payroll taxes, whose proceeds are used to pay the pensions of the currently retired generation.

Payroll taxes at rate $\tau$ in first period. After tax wage is $(1 - \tau)y$. Currently in US $\tau = 15.3\%$

Social security payments $b$ in second period: assume that population grows at rate $n$ and pre-tax-income grows at rate $g$.

Social security system balances its budget:

\[ b = (1 + g)(1 + n)\tau y \]

Household’s budget constraints

\[ c + s = (1 - \tau)y \]
\[ c' = (1 + r)s + b \]
Figure 8.18  Pay-As-You-Go Social Security for Consumers Who Are Old in Period T
Figure 8.19 Pay-As-You-Go Social Security for Consumers Born in Period T and Later
- Present value budget constraint

\[ c + \frac{c'}{1 + r} = (1 - \tau)y + \frac{b}{1 + r} \equiv y^{PV} \]

- Maximizing utility subject to the budget constraint again yields

\[ c = \frac{y^{PV}}{1 + \beta} \]

\[ c' = \frac{\beta (1 + r) y^{PV}}{1 + \beta} \]

But now for new \( y^{PV} \).
Since \( b = (1 + g)(1 + n)\tau y \):

\[
y^{PV} = (1 - \tau)y + \frac{b}{1 + r}
\]

\[
= (1 - \tau)y + \frac{(1 + g)(1 + n)\tau y}{1 + r}
\]

\[
= y - \left(1 - \frac{(1 + g)(1 + n)}{1 + r}\right)\tau y
\]

\[
= y + \left(\frac{(1 + g)(1 + n)}{1 + r} - 1\right)\tau y \equiv \tilde{y}
\]

Hence consumption in both periods is higher with social security than without if and only if \( \tilde{y} > y \). So people are better off with social security if:

\[
(1 + g)(1 + n) > 1 + r
\]
Intuition: If people save by themselves for retirement, return on their savings equals $1 + r$. If they save via a social security system, return equals $(1 + n)(1 + g)$

Rough US numbers: $n = 1\%$, $g = 2\%$, $r = 7\%$ (avg. stock returns). Suggests reform of the social security system desired.

In 2005, Pres. Bush proposed a transition to a fully funded private system. Went nowhere in Congress.

Proposal was controversial, to say the least. Social security is also a redistributive program. That role would be lessened or eliminated with private accounts.

Private retirement accounts also subject to greater risk due to market fluctuations.
Even in this basic model there is one main problem: Costly transition from pay-as-you-go to full funding.

Problem: one missing generation: at the introduction of the system there was one generation that received social security but never paid taxes.

Dilemma:

1. Currently young pay double, or
2. Default on the promises for the old, or
3. Increase government debt, financed by higher taxes in the future, i.e. by currently young and future generations. (Tax those who would benefit from switch.)
What are the effects of government deficits in the economy?
A first answer: none (Ricardo (1817) and Barro (1974)).
How can this be?
All that matters is present value of government expenditures and taxation. Timing does not matter. Deficits today imply higher taxes in future.
The answer outside our simple model is not as clear.
Lump-sum taxes. Government debt $B$, borrows at rate $r$.

Government budget constraints:

\[ G = T + B \]
\[ G' + (1 + r) B = T' \]

Consolidating to present value govt. BC:

\[ G + \frac{G'}{1 + r} = T + \frac{T'}{1 + r} \]

Now suppose that the government changes timing of taxes but (PV of) spending unchanged. Example: Cuts taxes today by $\Delta$, runs a deficit $B = \Delta$, pays back next period. So current taxes now $T - \Delta$, future taxes $T' + (1 + r)\Delta$. 
Household’s Problem

- Original problem:

\[
\max_{c,c'} u(c) + \beta u(c')
\]

\[
\text{s.t. } c + \frac{c'}{1+r} + T + \frac{T'}{1+r} = y^{PV}
\]

- Tax cut changes budget constraint to:

\[
c + \frac{c'}{1+r} + T - \Delta + \frac{T' + (1+r)\Delta}{1+r} = y^{PV}
\]

\[
\Rightarrow c + \frac{c'}{1+r} + T + \frac{T'}{1+r} = y^{PV}
\]

- Problem of the consumer is same as before.
Figure 8.17 Ricardian Equivalence with a Cut in Current Taxes for a Borrower
Comments on Ricardian Equivalence

- Consumer spend same amount, but current income increases by $\Delta$ to savings increases by $\Delta$. Individuals save their tax cut by buying government debt.

<table>
<thead>
<tr>
<th></th>
<th>Before Cut</th>
<th>After Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>$c + s = y + A - T$</td>
<td>$c + s = y + A - (T - \Delta)$</td>
</tr>
<tr>
<td>Period 2</td>
<td>$c' = y' + Rs - T'$</td>
<td>$c' = y' + Rs - (T' + R\Delta)$</td>
</tr>
<tr>
<td>Savings</td>
<td>$s$</td>
<td>$s + \Delta$</td>
</tr>
</tbody>
</table>

- Does not say fiscal policy is irrelevant. Here level of spending was held constant. (Compare to Lect. 5 on WWII.)

- Some argue that deficits “starve the beast”: cut taxes today, run deficit, force reduction in future govt spending.
Deviations from Ricardian Equivalence

Exact Ricardian equivalence depends on some key assumptions:

1. Taxes are nondistortionary (lump-sum).
2. The tax change has no redistributive consequences.
3. Current taxpayers are alive to pay for future increases. (Or they care about their children.)
4. Credit markets are perfect. Consumers and government face same interest rate.
Empirical Evidence

- Many instances of temporary tax cuts.
- President George W. Bush (2007-08): Stimulus rebate. Seems to have mostly led to increased saving, modest consumption increase.
- In its exact form, Ricardian equivalence fails. Evidence that consumption does respond to temporary tax cuts, but effects not substantial.
Using questions expressly added to the Consumer Expenditure Survey, we estimate the change in consumption expenditures caused by the 2001 federal income tax rebates and test the permanent income hypothesis. We exploit the unique, randomized timing of rebate receipt across households. Households spent 20 to 40 percent of their rebates on nondurable goods during the three-month period in which their rebates arrived, and roughly two-thirds of their rebates cumulatively during this period and the subsequent three-month period. The implied effects on aggregate consumption demand are substantial. Consistent with liquidity constraints, responses are larger for households with low liquid wealth or low income.
Figure 1. Growth Rates of Personal Consumption Expenditures from 2001:Q1 to 2002:Q4
Impact of 2007-08 Rebate on Savings

Personal Saving Rate, July 2007 to June 2008

- Saving Rate
- Saving Rate excluding Rebate
Dynamic Equilibrium

- We learned how to think about a household that makes dynamic decisions.
- We learned how to think about the intertemporal implications government policy.
- Now we want to introduce investment and capital accumulation.
- With these, and our previous static considerations on the labor market, we put everything together in a Dynamic General Equilibrium.
- We have seen implications of optimal dynamic allocation, now look at equilibrium.
- Will start with two-period model, then extend to infinite horizon.
A representative household maximizes $u(c, l) + \beta u(c, l')$

Its preferences satisfy the usual assumptions.

It faces two intertemporal budget constraints:

$$c + s = w(h - l) + \pi - T$$
$$c' = w'(h - l') + \pi' - T' + (1 + r)s$$

As before, we can combine these into **PV budget constraint**:

$$c + \frac{c'}{1 + r} = w(h - l) + \frac{w'(h - l')}{1 + r} + \pi + \frac{\pi'}{1 + r} - T - \frac{T'}{1 + r}$$
The Household’s Problem

- We can write the choice problem as a Lagrangian:

\[
L = u(c, l) + \beta u(c', l') + \\
\lambda \left( w(h-l) + \frac{w'(h-l')}{1+r} + \pi + \frac{\pi'}{1+r} - T - \frac{T'}{1+r} - c - \frac{c'}{1+r} \right)
\]

- There are four first order conditions:

\[
c : \quad u_c(c, l) - \lambda = 0
\]
\[
l : \quad u_l(c, l) - \lambda w = 0
\]
\[
c' : \quad \beta u_c(c', l') - \frac{\lambda}{1+r} = 0
\]
\[
l' : \quad \beta u_l(c', l') - \frac{\lambda w'}{1+r} = 0
\]
First order conditions for $c$ and $l$ imply

$$MRS_{l,c} = \frac{u_l(c, l)}{u_c(c, l)} = w$$

First order conditions for $C'$ and $l'$ imply

$$MRS_{l',c'} = \frac{u_l(c', l')}{u_c(c', l')} = w'$$

First order conditions for $c$ and $c'$ imply Euler equation:

$$MRS_{c,c'} = \frac{u_c(c, l)}{u_c(c', l')} = \beta(1 + r)$$

Combining these equations gives

$$\frac{MRS_{l,c}MRS_{c,c'}}{MRS_{l',c'}} = MRS_{l',c'} = \frac{w}{w'} \beta(1 + r)$$
Determinants of current labor supply $N = h - l$

$$MRS_{l,w} = \frac{w}{w'} \beta (1 + r)$$

- Higher current wage $w$ raises labor supply.
- Higher future wage $w'$ lowers labor supply.
- Higher interest rate $r$ raises labor supply.
- Higher lifetime wealth reduces labor supply.
- The labor supply curve is the relationship between $w$ and $N$, and so is upward sloping.
- The other factors shift the labor supply curve.
Figure 9.2  An Increase in the Real Interest Rate Shifts the Current Labor Supply Curve to the Right
Determinants of current consumption $c$

\[ MRS_{c,c'} = \beta(1 + r) \]

- Higher interest rates reduce consumption.
- Higher current (wage or profit) income raises consumption.
- Higher future (wage or profit) income raises consumption.
- The consumption demand curve plots aggregate consumption as a function of current aggregate income, and so is upward sloping.
- The other factors shift the consumption demand curve.
Figure 9.6 An Increase in Lifetime Wealth for the Consumer Shifts Up the Demand for Consumption Goods.