Problem Set 5: Solutions

ECON 301: Intermediate Microeconomics
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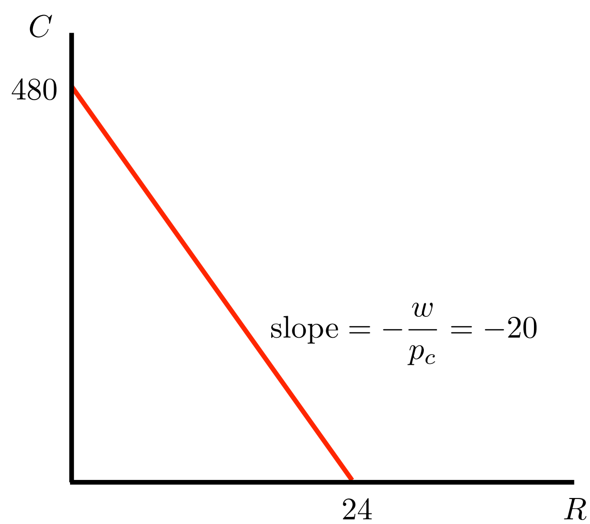
Problem 1

(a) Kate's real wage in terms of bananas is 20 pounds of bananas per hour. (She can purchase $\frac{w}{p_c} = \frac{100}{5} = 20$ pounds per hour of work.)

(b) Her budget set is shown in the consumption-leisure space below:



(c) To find her optimal time spent at work and consumption of bananas, notice that she has Cobb-Douglas utility with $a = 1$ and $b = 1$ (just replace x_1 with R and x_2 with C). Then her demand for hours of relaxation is:

$R = \frac{1}{1+1}\frac{m}{w} = \frac{1}{2}\frac{24w}{w} = 12$ (She is endowed with 24 hours which is worth $24w$ on the market.)

Her demand for the consumption good bananas is:

$$C = \frac{1}{1+1}\frac{24w}{p_c} = \frac{12w}{p_c} = 240$$

Hours of labor supplied is $LS = 24 - R = 24 - 12 = 12$. (So she consumes 12 hours of relaxation and supplies 12 hours of work with her endowment of 24 hours.)

(d) With $w = 200$, we have

$$R = 12 \quad \text{and} \quad C = 480\,.$$

Her labor supply is unchanged because, in this case, the substitution effect (where a higher wage encourages a higher number of hours of labor supplied) is exactly offset by the income effect.

Problem 2 (Intertemporal Choice)

(a) The present value of Gerald's stream of income over the two periods is

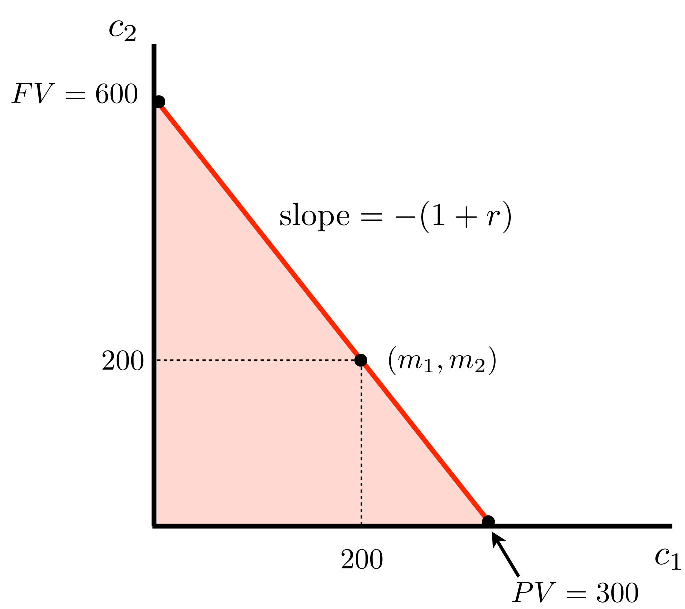
$$PV = \omega_1 + \frac{\omega_2}{1+r} \quad \Longrightarrow \quad PV = 200 + \frac{200}{1+1} = 300$$

and in terms of future value the stream of income is worth

$$FV = (1+r)\omega_1 + \omega_2 \quad \Longrightarrow \quad FV = (1+1)200 + 200 = 600\,.$$

Note that $PV = \frac{FV}{1+r}$ and $FV = (1+r)PV$.

(b) Gerald's budget set in (c_1,c_2)-space is shown below, with PV and FV points labeled. The PV point coincides with consuming (the present value of) everything in period one and nothing in period two. The FV point represents the point where Gerald consumes nothing in period one and (the future value of) everything in period two.



The slope of the budget line is $-(1+r)$. (The slope of the budget line is $-\frac{p_1}{p_2}$; here $p_1 = 1$ and $p_2 = \frac{1}{1+r}$.)

(c) The two "extreme" consumption points (which are where the budget line intercepts the c_1- and c_2-axes) have the following interpretations:

Point $(300, 0)$ on the c_1-axis represents a situation in which Gerald consumes all of his period one income of $\omega_1 = \$200$ *plus* and additional \$100, the maximum amount he could borrow, which must be payed back in period two at the $r = 100\%$ rate. All of his period two income goes to repaying this and he consumes nothing in the second period ($\omega_2 = \$200$ and he uses this to repay the $100 \times (1+1) = 200$ borrowed in the first period).

Point $(0, 600)$ on the c_2-axis represents the situation where Gerald consumes nothing in the first period and saves all of his ω_1 income, earning a return of $r = 100\%$ in the second period. In the second period he consumes all of his $\omega_2 = \$200$ income and the amount saved with interest from the first period, $\omega_1(1+r) = \$200 \times (1+1) = \$400$ for a total consumption of \$600 in the second period.

(d) We can always take a monotonic transformation of the utility function to get one that is in Cobb-Douglas form with $a = 1$ and $b = 1$ and use the "magic formulas" to find demand for c_1 and c_2 ($c_1 = \frac{a}{a+b}\frac{m}{p_1}$ and $c_2 = \frac{b}{a+b}\frac{m}{p_1}$). Alternatively, we could derive demand from the utility function given using the two "secrets of happiness" for well-behaved preferences.

In any case we will have (since $p_1 = 1$, $p_2 = \frac{1}{1+r}$, and $m = PV = 300$):

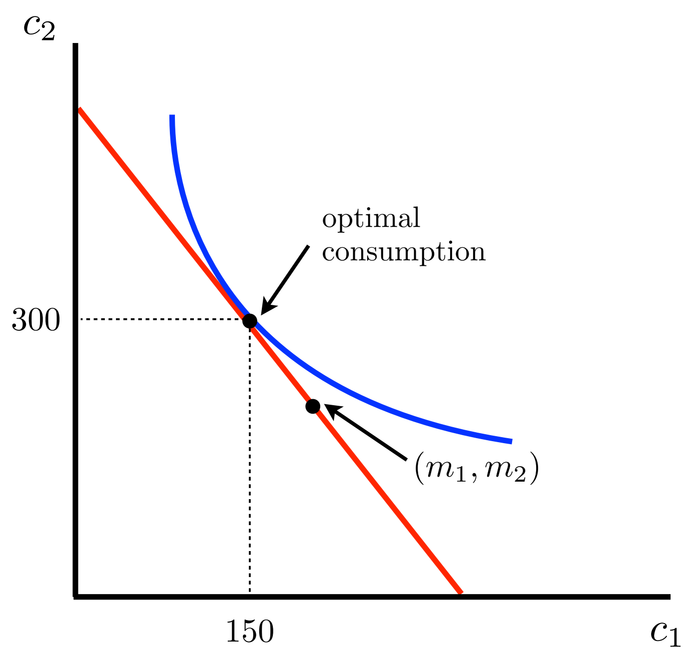
$$c_1 = \frac{1}{2}\frac{300}{1} = 150 \quad \text{and} \quad c_2 = \frac{1}{2}\frac{300}{\left(\frac{1}{1+1}\right)} = 300\,.$$

Here, Gerald is consuming \$150 saving \$50 in period one (consumes \$50 less than what his income in period one is: $S = \omega_1 - c_1 = \$200 - \$150 = \$50$). The \$50 he saves gives him $\$50 \times (1+1) = \$100$ in period two, and he consumes $\omega_2 + \$100 = \$300$ in period two.

Problem 3 (Intertemporal Choice)

(a) The coefficient δ is the discount rate which signifies how impatient the agent is. The greater δ the more impatient the agent, giving smaller weight to utility from future consumption (high $\delta \implies$ low $\frac{1}{1+\delta}$).

(b) If the income stream for the manager is $\omega_1 = 0$ and $\omega_2 = 3{,}000$, then the present value of the stream is $PV = 0 + \frac{3{,}000}{1+1} = 1{,}500 = m$. As in Problem 1, we can use the Cobb-Douglas



magic formulas for demand with $a = 1$ and $b = \frac{1}{1+\delta}$ to get

$$c_1 = \frac{1}{1 + \frac{1}{1+\delta}} \cdot \frac{m}{p_1} = \frac{1+\delta}{2+\delta} \cdot 1,500 = 1,000$$

and

$$c_2 = \frac{\frac{1}{1+\delta}}{1 + \frac{1}{1+\delta}} \cdot \frac{m}{p_2} = \frac{1+r}{2+\delta} \cdot 1,500 = 1,000\,.$$

His savings in the first period is $S = \omega_1 - c_1 = 0 - 1,000 = -1,000$. So he borrows \$1,000 the first period (and repays $(1 + r) \times 1,000 = \$2,000$ in period two).

(c) If the income stream for the sportsman is $\omega_1 = 1,500$ and $\omega_2 = 0$, we have $PV = 1,500 = m$ and the same formulas we used in part **(b)** give

$$c_1 = \frac{1}{1 + \frac{1}{1+\delta}} \cdot \frac{m}{p_1} = \frac{1+\delta}{2+\delta} \cdot 1,500 = 1,000$$

and

$$c_2 = \frac{\frac{1}{1+\delta}}{1 + \frac{1}{1+\delta}} \cdot \frac{m}{p_2} = \frac{1+r}{2+\delta} \cdot 1,500 = 1,000\,.$$

Savings in the first period is now $S = \omega_1 - c_1 = 1,500 - 1,000 = 500$.

(d) Both the manager and the sportsman consume such that $c_1 = c_2$ and hence perfectly smooth their consumption over the two periods.

(e) From the second secret of happiness for well-behaved preferences:

$$MRS = -\frac{p_1}{p_2} \implies \frac{1/c_1}{\frac{1}{1+\delta}(1/c_2)} = -\frac{1}{\left(\frac{1}{1+r}\right)}$$

Rearranging this equation to solve for c_1, we have

$$c_2 = \frac{(1+r)}{(1+\delta)} c_1 \,.$$

When $r < \delta$, we have that $\frac{(1+r)}{(1+\delta)} < 1$, which implies that $c_2 < c_1$ (consumption is decreasing over time).