Problem Set 4: Solutions

ECON 301: Intermediate Microeconomics
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Problem 1

Note that for this problem, we can just use the formulas for demand with Cobb-Douglas utility:

$$x_1 = \frac{a}{a+b} \cdot \frac{m}{p_1} = \frac{4m}{5p_1} \quad \text{and} \quad x_2 = \frac{b}{a+b} \cdot \frac{m}{p_2} = \frac{m}{5p_2}$$

While the utility function we're given, $U(x_1, x_2) = 4 \ln x_1 + \ln x_2$, is not Cobb-Douglas, we can always take a monotonic transformation of it and obtain the same answers. Here, let $f(u) = e^u$ be the transformation so we're working instead with $U(x_1, x_2) = x_1^4 x_2$.

(a) With initial prices $p_1 = 10$, $p_2 = 1$, and income $m = 100$, we get demand

$$x_1(p_1, p_2, m) = \frac{4m}{5p_1} = 8 \quad \text{and} \quad x_2(p_1, p_2, m) = \frac{m}{5p_2} = 20 \ .$$

With new price $p_1' = 5$ we have that

$$x_1(p_1', p_2, m) = \frac{4m}{5p_1'} = 16 \quad \text{and} \quad x_2(p_1', p_2, m) = \frac{m}{5p_2} = 20 \ .$$

So the change in the number of movies (x_1) watched is

$$\Delta x_1 = x_1(p_1', p_2, m) - x_1(p_1, p_2, m) = 16 - 8 = 8 \ .$$

(b) Movies are ordinary goods since the number demanded increases when its price decreased (demand is downward sloping).

(c) To find the substitution effect, we first need to find m', which is the amount of income that would be required to purchase the $(x_1, x_2) = (8, 20)$ bundle (which was optimal at the original prices) at the new price p_1':

$$m' = p_1' x_1 + p_2 x_2 = 5 \cdot 8 + 1 \cdot 20 = 60 \, .$$

So to purchase the original bundle at the new prices would require an income of $m' = \$60$ (rather than \$100). To find what amount of the total change in demand can be attributed to the substitution effect, we need to find the quantity demanded with income $m' = 60$ and price $p_1 = 5$:

$$x_1(p_1', p_2, m') = \frac{4m'}{5p_1'} = 9\frac{3}{5} \, .$$

So we have that the substitution effect is

$$\Delta x_1^s = x_1(p_1', p_2, m') - x_1(p_1, p_2, m) = 9\frac{3}{5} - 8 = 1\frac{3}{5} \ .$$

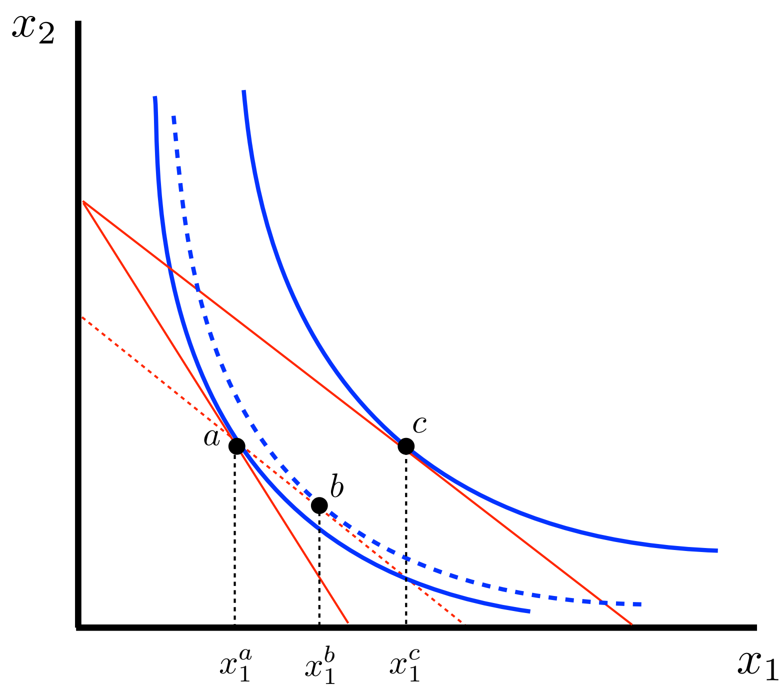
(d) The income effect is

$$\Delta x_1^n = x_1(p_1', p_2, m) - x_1(p_1', p_2, m') = 16 - 9\frac{3}{5} = 6\frac{2}{5} \ .$$

(Of course, since $\Delta x_1 = \Delta x_1^s + \Delta x_1^n$ we can get this from $8 = 1\frac{3}{5} + \Delta x_1^n \implies \Delta x_1^n = 6\frac{2}{5}$ as well.)

(e) The income effect is positive (with prices p_1' and p_2, less of x_1 was demanded with the lower hypothetical m' than was demanded at income m). This means preferences over these goods are normal, which is true with Cobb-Douglas preferences.

(f) These effects are shown graphically below. Point a represents the initial bundle, where demand for x_1 is $x_1(p_1, p_2, m) = 8$. Point c represents demand after the price change where we had $x_1(p_1', p_2, m) = 16$. Point b represents the optimal (and hypothetical) value with p_1' and m', with x_1 demand $x_1(p_1', p_2, m') = 9\frac{3}{5}$. The difference between a and c is the total effect; between a and b is the substitution effect; between b and c is the income effect.



Problem 2

Recall that Trevor's preferences are represented by the utility function $U(x_1, x_2) = \min\{5x_1, x_2\}$. (He consumes five times as many strawberries (x_2) as he does milk (x_1) in making the perfect strawberry milkshake.) We found that his demand functions are given by:

$$x_1 = \frac{m}{p_1 + 5p_2} \quad \text{and} \quad x_2 = \frac{5m}{p_1 + 5p_2}\,.$$

(a) With $m = 200$, $p_1 = 15$, and $p_2 = 1$, his demands for milk (x_1) and strawberries (x_2) are:

$$x_1(p_1, p_2, m) = \frac{m}{p_1 + 5p_2} = \frac{200}{15 + 5 \cdot 1} = 10 \quad \text{and} \quad x_2(p_1, p_2, m) = \frac{5m}{p_1 + 5p_2} = \frac{5 \cdot 200}{15 + 5 \cdot 1} = 50\,.$$

With the price of milk decreasing to $p_1' = 5$, his new quantities demanded become:

$$x_1(p_1', p_2, m) = \frac{m}{p_1' + 5p_2} = \frac{200}{5 + 5 \cdot 1} = 20 \quad \text{and} \quad x_2(p_1', p_2, m) = \frac{5m}{p_1' + 5p_2} = \frac{5 \cdot 200}{5 + 5 \cdot 1} = 100\,.$$

The total change in the demand for milk is

$$\Delta x_1 = x_1(p_1', p_2, m) - x_1(p_1, p_2, m) = 20 - 10 = 10.$$

(b) The substitution effect, Δx_1^s, for complementary goods is always zero. Let's verify this.

First, we need to find the m' income that would make the original bundle just affordable with the price change:

$$m' = p_1' x_1 + p_2 x_2 = 5 \cdot 10 + 1 \cdot 50 = 100\,.$$

Next we find the quantity of x_1 demanded with $m' = 100$, $p_1' = 5$ and $p_2 = 1$:

$$x_1(p_1', p_2, m') = \frac{m'}{p_1' + 5p_2} = \frac{100}{5 + 5 \cdot 1} = 10\,.$$

So we have that the substitution effect is

$$\Delta x_1^s = x_1(p_1', p_2, m') - x_1(p_1, p_2, m) = 10 - 10 = 0\,.$$

(c) The income effect is

$$\Delta x_1^n = x_1(p_1', p_2, m) - x_1(p_1', p_2, m') = 20 - 10 = 10\,.$$

(Again, since $\Delta x_1 = \Delta x_1^s + \Delta x_1^n$ we can get this from $10 = 0 + \Delta x_1^n \implies \Delta x_1^n = 10$ as well.)

Problem 3

(a) The MRS for these preferences is $MRS(x_1, x_2) = -\frac{5}{x_1}$. (Notice that this does not depend on x_2, over which utility is linear.) The two secrets of happiness for an interior solution (with $x_1 > 0$ and $x_2 > 0$) are:

- **(1)** $p_1 x_1 + p_2 x_2 = m$
- **(2)** $MRS(x_1, x_2) = -\frac{p_1}{p_2} \implies -\frac{5}{x_1} = -\frac{p_1}{p_2}$

(b) Solving for the two above equations for x_1 and x_2 we get demand functions

$$x_1 = \frac{5p_2}{p_1} \quad \text{and} \quad x_2 = \frac{m - 5p_2}{p_2}$$

as long as $m \geq 5p_2$. (If $m < 5p_2$ we have a corner solution with $x_1 = \frac{m}{p_1}$ and $x_2 = 0$.)

(c) Given that $m = 10$ and $p_2 = 1$, at price $p_1 = 5$, our Miriam's demand for coffee x_1 is $x_1(p_1, p_2, m) = \frac{5 \cdot 1}{5} = 1$. (Also, $x_2(p_1, p_2, m) = \frac{10 - 5 \cdot 1}{1} = 5$.)

With a price of $p_1' = 1$, her demand is $x_1(p_1', p_2, m) = \frac{5 \cdot 1}{1} = 5$.

The total effect on demand for x_1 resulting from the price change is $\Delta x_1 = x_1(p_1', p_2, m) - x_1(p_1, p_2, m) = 5 - 1 = 4$.

To find the substitution effect, we first find $m' = p_1' x_1 + p_2 x_2$, the hypothetical income that would be necessary for her to buy her initial bundle with the price change, which is $m' = 1 \cdot 1 + 1 \cdot 5 = 6$.

Next we need to find the bundle Miriam would actually choose if her income were $m' = 6$. (We know she can afford the initial bundle, but that's not necessarily what she buys given the new relative prices.) At $p_1' = 1$, $p_2 = 1$ and $m' = 6$, we have that $x_1(p_1', p_2, m') = \frac{5 \cdot 1}{1} = 5$.

The substitution effect is then

$$\Delta x_1^s = x_1(p_1', p_2, m') - x_1(p_1, p_2, m) = 5 - 1 = 4\ .$$

(d) The income effect on the demand for x_1 is

$$\Delta x_1^n = x_1(p_1', p_2, m) - x_1(p_1', p_2, m') = 5 - 5 = 0\ .$$

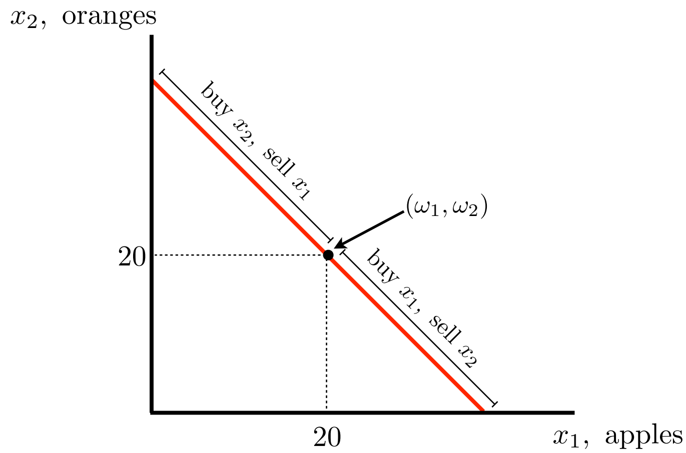
(Again, since $\Delta x_1 = \Delta x_1^s + \Delta x_1^n$ we can get this from $4 = 4 + \Delta x_1^n \implies \Delta x_1^n = 0$ as well.) With quasilinear preferences, the good in which utility is linear (here, x_2) "absorbs all income effects."

Problem 4

(a) Let x_1 be the consumption of apples and x_2 be the consumption of oranges. Dave's budget constraint is given by

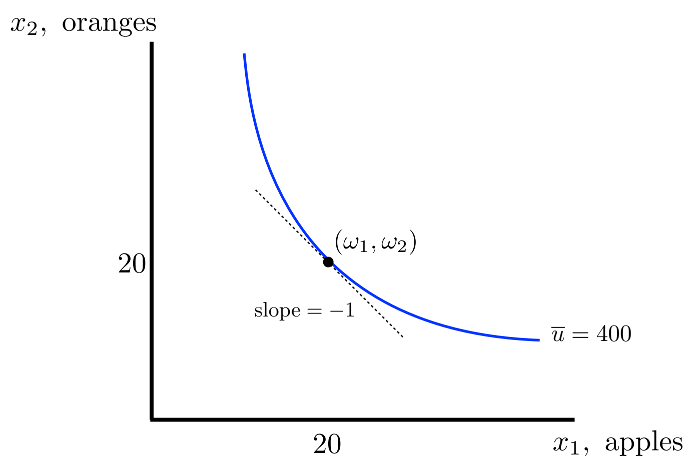
$$p_1 x_1 + p_2 x_2 \le p_1 \omega_1 + p_2 \omega_2 \quad \Longrightarrow \quad 2x_1 + 2x_2 \le 80\,.$$

The bundles for which Dave would be selling apples and buying oranges is shown below:



(b) First, the MRS of $U(x_1, x_2) = x_1 x_2$ is $MRS(x_1, x_2) = -\frac{x_2}{x_1}$. At his endowment point $x_1 = \omega_1 = 20$ and $x_2 = \omega_2 = 20$, we have $MRS(20, 20) = -\frac{20}{20} = -1$.

The utility associated with this bundle is $U(20, 20) = 20 \cdot 20 = 400$. Dave is indifferent then among his endowment and all the other bundles for which $x_1 x_2 = 400$ so the indifference curve passing through his endowment is, analytically, $x_2 = 400/x_1$. This is shown graphically below:



(c) Demand with these preferences is

$$x_1(p_1, p_2, m) = \frac{m}{2p_1} \quad \text{and} \quad x_2(p_1, p_2, m) = \frac{m}{2p_2}.$$

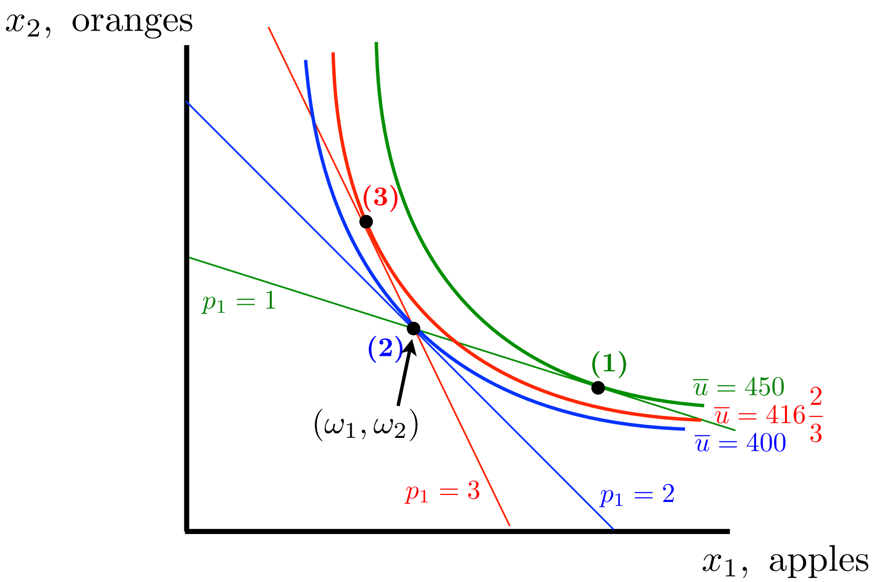
Holding $p_2 = 2$ fixed, for the three different p_1 prices (and resulting m changes), we have the following demand:

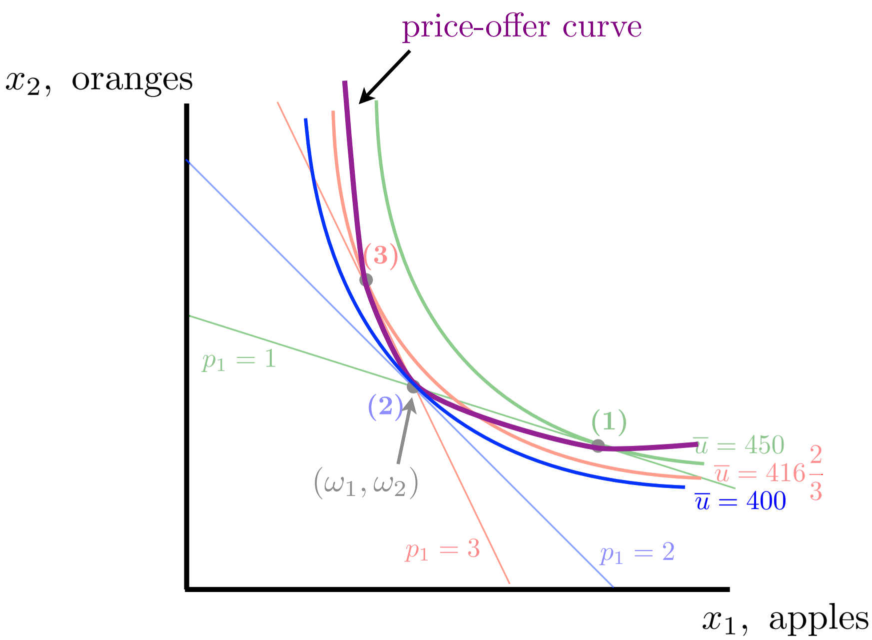
- **(1)** $p_1 = 1$, $m = 1 \cdot 20 + 2 \cdot 20 = 60$: $x_1(1, 2, 60) = \frac{60}{2 \cdot 1} = 30$ and $x_2(1, 2, 60) = \frac{60}{2 \cdot 2} = 15$
  - Net demand for x_1 is given by $x_1(1, 2, 60) - \omega_1 = 30 - 20 = 10$
  - Net demand for x_2 is zero since $x_2(1, 2, 60) = 15 < 20 = \omega_2$. (He will be a net supplier of x_2.)
- **(2)** $p_1 = 2$, $m = 2 \cdot 20 + 2 \cdot 20 = 80$: $x_1(2, 2, 80) = \frac{80}{2 \cdot 2} = 20$ and $x_2(2, 2, 80) = \frac{80}{2 \cdot 2} = 20$
  - Net demand for x_1 is given by $x_1(2, 2, 80) - \omega_1 = 20 - 20 = 0$. (He is neither a net supplier for nor a net demander of x_1.)
  - Net demand for x_2 is given by $x_2(2, 2, 80) - \omega_2 = 20 - 20 = 0$. (He is neither a net supplier for nor a net demander of x_2.)
- **(3)** $p_1 = 3$, $m = 3 \cdot 20 + 2 \cdot 20 = 100$: $x_1(3, 2, 100) = \frac{100}{2 \cdot 3} = \frac{50}{3}$ and $x_2(3, 2, 100) = \frac{100}{2 \cdot 2} = 25$
  - Net demand for x_1 is zero since $x_1(3, 2, 100) = \frac{50}{3} > 20 = \omega_1$. (He will be a net supplier of x_1.)
  - Net demand for x_2 is given by $x_2(3, 2, 100) - \omega_1 = 25 - 20 = 5$

(d) The magnitude of the slope of the indifference curve at the endowment point is $|MRS(20, 20)| = |-\frac{20}{20}| = 1$.

- **(1)** For $p_1 = 1$, $|MRS(20, 20)| = 1 > \frac{1}{2} = |-\frac{p_1}{p_2}|$. He is better off buying more apples (x_1) and selling some of his oranges (x_2).
- **(2)** At $p_1 = 2$, $|MRS(20, 20)| = 1 = 1 = |-\frac{p_1}{p_2}|$. He can not get any greater utility from buying or selling apples and oranges; he is best off with his endowment.
- **(3)** At $p_1 = 3$, $|MRS(20, 20)| = 1 < \frac{3}{2} = |-\frac{p_1}{p_2}|$. He is better of buying more oranges (x_2) and selling some of his apples (x_1).

(e) Dave's optimal bundles for the three price combinations found above analytically are shown graphically below with bundles **(1)**, **(2)**, and **(3)** labeled. The price-offer curve must pass through these points, and is shown in the second graph. You will see that at all but one point (the endowment, point **(2)**=(ω_1, ω_2)), the price offer curve lies strictly above the indifference curve passing through the endowment (where $\overline{u} = 400$). This is significant: It means that one is always at least as well off trading as he is just consuming his endowment!

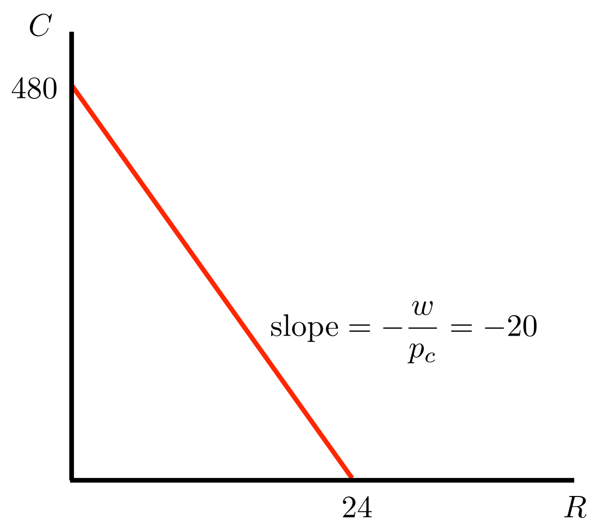

x_2, oranges
(3)
$p_1 = 1$
(2)
(1)
$\overline{u} = 450$
$\overline{u} = 416\frac{2}{3}$
$\overline{u} = 400$
(ω_1, ω_2)
$p_1 = 3$
$p_1 = 2$
x_1, apples


price-offer curve
x_2, oranges
(3)
$p_1 = 1$
(2)
(1)
$\overline{u} = 450$
$\overline{u} = 416\frac{2}{3}$
$\overline{u} = 400$
(ω_1, ω_2)
$p_1 = 3$
$p_1 = 2$
x_1, apples

Problem 5

(a) Kate's real wage in terms of bananas is 20 pounds of bananas per hour. (She can purchase $\frac{w}{p_c} = \frac{100}{5} = 20$ pounds per hour of work.)

(b) Her budget set is shown in the consumption-leisure space below:



(c) To find her optimal time spent at work and consumption of bananas, notice that she has Cobb-Douglas utility with $a = 1$ and $b = 1$ (just replace x_1 with R and x_2 with C). Then her demand for hours of relaxation is:

$R = \frac{1}{1+1}\frac{m}{w} = \frac{1}{2}\frac{24w}{w} = 12$ (She is endowed with 24 hours which is worth $24w$ on the market.)

Her demand for the consumption good bananas is:

$$C = \frac{1}{1+1}\frac{24w}{p_c} = \frac{12w}{p_c} = 240$$

Hours of labor supplied is $LS = 24 - R = 24 - 12 = 12$. (So she consumes 12 hours of relaxation and supplies 12 hours of work with her endowment of 24 hours.)

(d) With $w = 200$, we have

$$R = 12 \quad \text{and} \quad C = 480\,.$$

Her labor supply is unchanged because, in this case, the substitution effect (where a higher wage encourages a higher number of hours of labor supplied) is exactly offset by the income effect.