Problem Set 12: Solutions

ECON 301: Intermediate Microeconomics
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Problem 1 (Externalities)

(a) Examples of some market interactions with positive externalities: participation in a cellular network, education, pleasant scents arising from the production of baked goods, R&D, etc.

Examples of some market interactions with negative externalities: neighbor singing in the shower, congestion in city traffic, loud noises and dirt due to road work, etc.

(b) The outcome is most likely to be Pareto inefficient: In the case of a positive externality there are "too few" trades or activity in what is Pareto optimal, while with negative externalities there are "too many" trades relative to the Pareto efficient number. This is because prices in the markets to not capture the full benefits and costs (i.e., the utility and profit are not internalized by all affected parties).

(c) A subsidy would encourage more activity (in the case of a positive externality); taxes encourage less activity (in the case of negative externalities).

Problem 2 (Positive Externality)

(a) Positive externality: The ammonia, which is a byproduct of the nitrogen used to may the dynamite, lowers the cost of tomato production (or increases the output for a given cost).

(b) Finding the quantity d and x that maximizes profit in dynamite production:

The profit of the dynamite producer is

$$\pi_d = p_d d - TC_d(d, x) = d - \left[\frac{1}{2}d^2 + (x-2)^2\right] ,$$

Recalling that $p_d = 1$. Profit-maximizing production d is such that $\frac{\partial \pi_d}{\partial d} = 0$:

$$\frac{\partial \pi_d}{\partial d} = 0 \implies 1 - d = 0 \implies d = 1 .$$

The optimal use of nitrogen x is such that $\frac{\partial \pi_d}{\partial x} = 0$:

$$\frac{\partial \pi_d}{\partial x} = 0 \implies -2(x-2) = 0 \implies x = 2 .$$

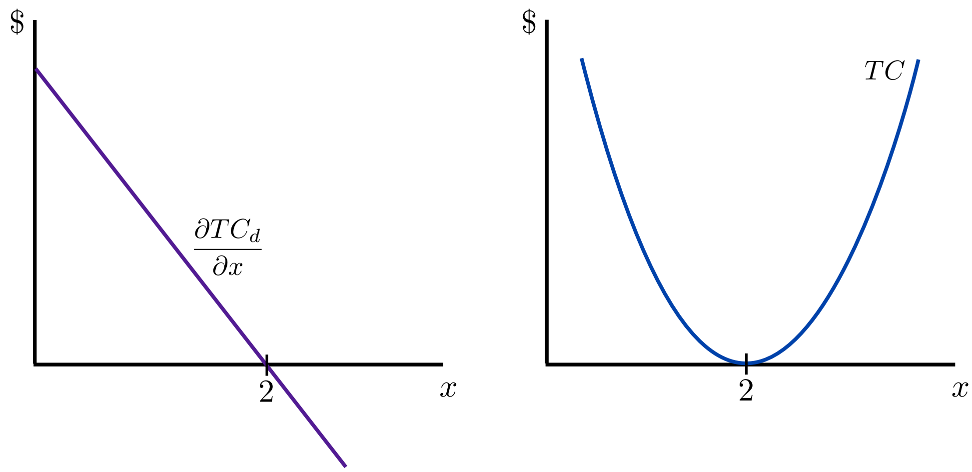
So $d = 1$ is the profit-maximizing output, using $x = 2$ units of nitrogen. This gives a profit of $\pi_d = \frac{1}{2}$.

(c) The marginal benefit MB or the negative of the marginal cost is

$$MB = -\frac{\partial TC_d(d, x)}{\partial x} = -(x - 2)$$

which is 0 at the optimum of $x = 2$.

The graph on the left shows MB as a function of x, which crosses the x-axis at $x = 2$. Notice that this corresponds to the minimum of the total cost function $TC(x)$ on the right (which is the maximum of the function $-TC(x)$).



(d) The farmer maximizes the profit equation

$$\pi_t = p_t t - \left[\frac{1}{2}t^2 + 2t - xt\right] .$$

Since $p_t = 1$ and the dynamite producer produces using $x = 2$ units of nitrogen, this becomes

$$\pi_t = t - \left[\frac{1}{2}t^2 + 2t - 2t\right] .$$

Our first-order condition for profit maximization gives us

$$\frac{\partial \pi_t}{\partial 1} = 0 \implies 1 - (t + 2 - 2) = 0 \implies t = 1 .$$

Then $t = 1$ tomato unit will be be produced with an associated profit of $\pi_t = \frac{1}{2}$.

(e) Their joint profit is $\pi_d + \pi_t = 1$.

(f) To find the Pareto efficient level of production of d and t as well as use of x, we can solve the problem as if the two firms are one firm maximizing profit $\pi_p = \pi_d + \pi_t$ or

$$\pi_p = d - \left[\frac{1}{2}d^2 + (x-2)^2\right] + t - \left[\frac{1}{2}t^2 + 2t - xt\right]$$

(This is sometimes referred to as the *planner's problem.*)

With the variables we have three first-order conditions to give us the optimal levels:

$$\frac{\partial \pi}{\partial d} = 0 \implies d = 1 \quad (1)$$

$$\frac{\partial \pi}{\partial t} = 0 \implies 1 - t - 2 + x = 0 \quad (2)$$

$$\frac{\partial \pi}{\partial x} = 0 \implies 4 - 2x + t = 0 \quad (3)$$

Plugging $t = 2x - 4$ from (3) into (2), we get

$$1 + 4 - 2x - 2 + x = 0 \implies x = 3$$

and plugging this into (1) we get $t = 2$.In summary, we have:

- $d = 1$ units of dynamite (before it was $d = 1$ as well)
- $t = 2$ units of tomatoes (used to be $t = 1$)
- $x = 3$ nitrogen units (used to be $x = 2$)

The optimal or efficient usage is higher than the market outcome, as is the production. The reason nitrogen usage was lower in the first part of this problem (which also led to a lower level of tomato production) was because the dynamite producer does not internalize the positive benefits to the tomato farmer. (It does not show up in the dynamite profit function.) The cost of using nitrogen *as well as the benefit to the third-party tomato farmer* is accounted for in the planner's problem, which is why we get different outcomes.

(g) Now the marginal benefit for the dynamite producer from the usage of $x = 3$ is

$$-\frac{\partial TC_d(d,x)}{\partial x} = -2(x-2) = -2\,.$$

The reason for this is that the negative marginal benefit for the dynamite producer is compensated for (in the planner's problem) by the benefits to tomato production: $\frac{\partial TC_t(t,x)}{\partial x} = t$ and $t = 2$ as we found in part **(f)**.

(h) Yes, the results we found here are consistent with that statement (since we found that there was "too little" nitrogen usage).

Problem 3 (Non-excludable and Non-rival Goods)

(a) A *non-excludable* good is a good for which it is difficult to enforce property rights and hence prevent others from using it. A *non-rival* good is one for which the consumption of it by one agent does not affect or prohibit the consumption of it by another agent.

(b) (For examples, see class slides.)

(c) A pure "public good" is one that is both *non-rival* and *non-excludable.*

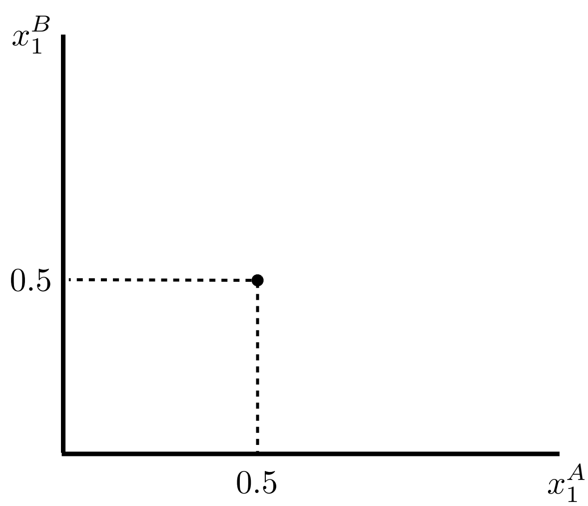
Problem 4 (Provision of a Public Good)

(a) Firm A maximizes profit function

$$\begin{aligned}\pi^A &= v^A(x_1, x_2^A) - x_1^A - x_2^A \\ &= \ln(x_1^A + x_1^B) + \ln(x_2^A) - x_1^A - x_2^A \\ &= \ln(x_1^A + 0.5) + \ln(x_2^A) - x_1^A - x_2^A \quad \text{when } x_1^B = 0.5\,.\end{aligned}$$

First-order condition $\frac{\partial \pi^A}{\partial x_1^A} = 0$ gives us

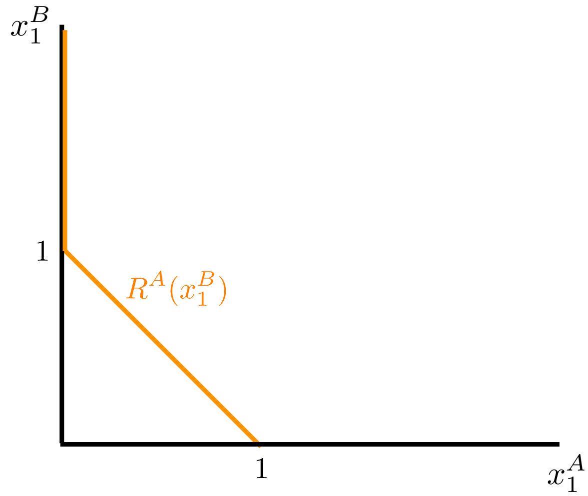
$$\frac{1}{x_1^A + 0.5} - 1 = 0 \quad \Longrightarrow \quad x_1^A = 0.5\,.$$



(b) To find Firm A's best response function, $R^A(x_1^B)$, we again use the first order condition $\frac{\partial \pi^A}{\partial x_1^A} = 0$ as we did above (except we do not substitute in $x_1^B = 0.5$):

$$\begin{aligned}\frac{\partial \pi^A}{\partial x_1^A} = 0 \quad &\Longrightarrow \quad \frac{1}{x_1^A + x_1^B} - 1 = 0 \\ &\Longrightarrow \quad x_1^A = 1 - x_1^B\end{aligned}$$

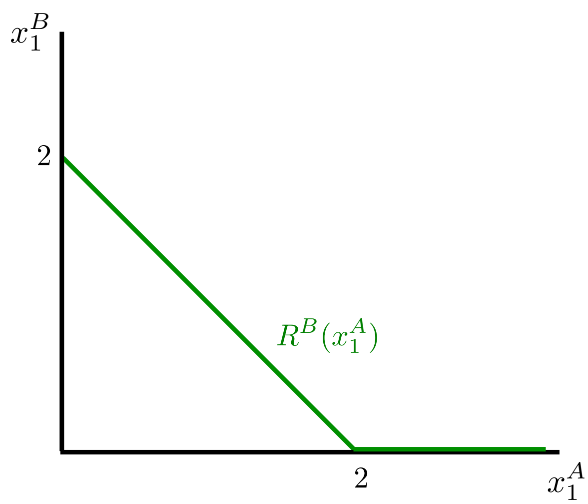
so Firm A's best response function is $R^A(x_1^B) = 1 - x_1^B$ (for $x_1^B \leq 1$). When $x_1^B > 1$, Firm A's best response is $R^A(x_1^B) = 0$.



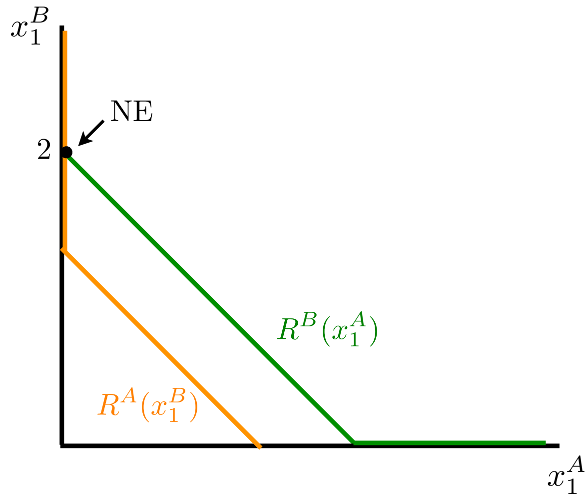
(c) To find Firm A's best response function, $R^B(x_1^A)$, we use the first order condition $\frac{\partial \pi^B}{\partial x_1^B} = 0$ as we did above (except we do not substitute in $x_1^B = 0.5$):

$$\frac{\partial \pi^B}{\partial x_1^B} = 0 \quad \Longrightarrow \quad \frac{2}{x_1^A + x_1^B} - 1 = 0$$
$$\Longrightarrow \quad x_1^B = 2 - x_1^A$$

so Firm B's best response function is $R^B(x_1^A) = 2 - x_1^A$ for $x_1^A \leq 2$. When $x_1^A > 2$, Firm B's best response is $R^B(x_1^A) = 0$.



(d) The Nash equilibrium is where the two best-response functions intersect, i.e., (x_1^{*A}, x_1^{*B}) such that both contributions are mutual best responses. There is only one such point: $(0, 2)$. The total amount spent on the common area is then $0 + 2 = 2$.



(e) Yes, Firm A is free riding on Firm B's investment since it benefits from Firm B's investment in the area—it's customers enjoy the space and Firm A's property is worth more—but it does not contribute to it.

(f) The Pareto efficient investment can be determined by maximizing the joint/total profit of both firms:

$$\pi^{A+B} = \pi^A + \pi^B = \left[\ln(x_1^A + x_1^B) + \ln(x_2^A) - x_1^A - x_2^A\right] + \left[2\ln(x_1^A + x_1^B) + \ln(x_2^B) - x_1^B - x_2^B\right] .$$

Substituting in $x_1 = x_1^A + x_1^B$ and combining terms we have

$$\pi^{A+B} = 3\ln(x_1) + \ln(x_1^A) + \ln(x_1^B) - x_1 - x_2^A - x_2^B .$$

The first order condition we use to find the optimal level of x_1 invested is

$$\frac{\partial \pi^{A+B}}{\partial x_1} = 0 \implies \frac{3}{x_1} - 1 = 0 \implies x_1 = 3 .$$

This is greater than the total we found in part **(d)**, which was $x_1 = 2$. This is because in the market interaction, the agents do not account for all of the positive side effects of their investment and hence underinvest.

Problem 5 (Adverse Selection)

(a) The total gains to trade are 30 for lemons and 20 for plums. If the probability of a car being a lemon or a plum is 1/2, for example, then the *expected* gains to trade are $\frac{1}{2} \cdot 20 + \frac{1}{2} \cdot 30 = 25$.

(b) If there is perfect information as to whether any given car is a lemon or a plum, the prices $p_L = 15$ and $p_P = 110$ would split the gains to trade in both the lemon and plum markets, respectively. The allocation is Pareto efficient as both lemons and plums end up in the hands of the trader valuing them the most.

(c) If the potential buyers cannot distinguish between a lemon and a plum, then the expected value from buying a car is:

$$EVB = \frac{1}{3} \cdot 30 + \frac{2}{3} \cdot 20 = 90\,.$$

In this case a maximum selling price for a car (which is like a lottery) cannot exceed 90.

(d) With the probability of a lemon being sold at $\frac{1}{3}$, no plums are sold since the sellers value them at 100 (which is more than the 90 they could get selling it). Only lemons will be sold, with $p = 15$.

(e) Outcome is not Pareto efficient as the plums are not traded and hence the gains to trade of 20 are lost.

(f) The probability can be found from the condition $EVB \geq 100$. Since

$$EVB = \pi \times 30 + (1 - \pi) \times 120$$

condition $EVB \geq 100$ is

$$\pi \times 30 + (1 - \pi) \times 120 \geq 100 \quad \Longrightarrow \quad \pi \leq \frac{2}{9}\,.$$

So as long as the probability of any given car being a lemon is less than $\frac{2}{9}$, we will observe a pooling equilibrium.

(g) The allocation where there is a pooling equilibrium is Pareto efficient as cars end up with the trader valuing them the most.

(h) The owners of plums could offer a warranty on the car.

Problem 6 (Signaling)

(a) In a pooling equilibrium, with the probability of a worker being either type equal to $\frac{1}{2}$, the expected productivity of a worker is

$$EV = \frac{1}{2} \times 10 + \frac{1}{2} \times 4 = 7$$

and this is the wage offered to the worker (now the workers productivity is a lottery from the perspective of the firm). Both types of workers accept the job (we'll assume they both have the same reservation wage and that it is less than 7).

(b) Two passed tests is not a credible signal. This is because in the separating equilibrium, the additional benefit from being known as a workaholic as opposed to a lazybones is

$$w^W - w^L = 10 - 4 = 6\,.$$

Since to pass two tests, it costs the lazybones only $c^L(2) = 2 \cdot 2 = 4$, so the lazybones take the tests (as do the workaholics) and therefore only two tests passed is not a credible signal.

(c) The minimum credible number of passed tests is the one that just prevents the lazybones from taking the tests (and obtaining wage w^W instead of w^L). It is the e such that $c^L(e) \geq 6$, so

$$c^L(e) \geq 6 \implies 2e \geq 6 \implies e \geq 3\,.$$

So $e = 3$ is a sufficient number of tests to lead to a separating equilibrium. (We'll assume that at $e = 3$, the lazybones will not take the tests as he is indifferent at that point, so we can assume only the workaholics are the ones taking the tests. Note that the highest number of tests the workaholic would take anyway is 6.)

(d) In a separating equilibrium, the workaholics will be forced to take three tests. Because such tests do not improve the productivity of the workers, the tests are a waste of resources and Pareto inefficient. (However this is inefficient only compared to a world with perfect information!)