

## Problem Set 11: Solutions

ECON 301: Intermediate Microeconomics  
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### Problem 1 (Price Discrimination)

(a) If Microsoft can perfectly price discriminate, its profit (and the producer surplus  $PS$ ) is the total surplus area (accounting for the fixed costs of  $F = 1,000$ ) that we found in **Problem 2 (a)**:

$$PS = \pi = TS = \frac{1}{2} \times 100 \times 100 - F = 5,000 - 1,000 = 4,000.$$

Consumer surplus is zero and the outcome is Pareto efficient since there is no deadweight loss.

(b) To find the aggregate demand with these two segments, we sum over quantity  $y$ . (Note that if the firm does not price discriminate, it must charge the same price so  $p = p^I = p^F$ .)

$$y^{AGG}(p) = y^I(p) + y^F(p) = (50 - \frac{4}{5}p) + (50 - \frac{1}{5}p) = 100 - p$$

which gives us the inverse aggregate demand we had in Problem 2:

$$p^{AGG}(y) = 100 - y.$$

(c) With third-degree price discrimination, Microsoft charges price  $p^I$  to the individual buyers market segment and price  $p^F$  to the firm buyers. We can simply solve the profit maximizing problem for each segment separately.

*Individual Buyers:* The inverse demand curve in this market is

$$p^I(y) = \frac{125}{2} - \frac{5}{4}y^I$$

and so the marginal revenue in the individual buyers market is

$$MR^I(y) = \frac{125}{2} - \frac{10}{4}y^I.$$

Profit maximizing condition  $MR(y^I) = MC(y^I) \implies \frac{125}{2} - \frac{10}{4}y^I = 0 \implies y^I = 25$ .

Price is then  $p^I(25) = 31\frac{1}{4}$ . Total revenue in the individual segment is then  $TR^I = 25 \times \frac{1}{4} = 1,781\frac{1}{4}$ , and consumer surplus is  $CS^I = \frac{1}{2} \times (\frac{125}{2} - 31\frac{1}{4}) \times 25 = 390\frac{5}{8}$ .

*Firm Buyers:* The inverse demand curve in this market is

$$p^F(y) = 250 - 5y^F$$

and so the marginal revenue in the firm buyers market is

$$MR^F(y) = 250 - 10y^F.$$

Profit maximizing condition  $MR(y^F) = MC(y^F) \implies 250 - 10y^F = 0 \implies y^F = 25$ .

Price is then  $p^F(25) = 125$ . Total revenue in the firm segment is then  $TR^F = 25 \times 125 = 3,125$ , and consumer surplus is  $CS^F = \frac{1}{2} \times (250 - 125) \times 25 = 1,562\frac{1}{2}$ .

To find Microsoft's total profit (which corresponds to producer surplus here), we add the total revenue from each segment and subtract the total cost of  $F = 1,000$ :

$$\pi = TR^I - TR^F - TC = 1,781\frac{1}{4} + 4,125 - 1,000 = 3,906\frac{1}{4}$$

(It would also be fine to assume that the fixed cost of  $F = 1,000$  is incurred in each market, giving instead profit  $\pi = 2,906\frac{1}{4}$ .)

(d) Here we compare consumer and producer surplus in the three scenarios.

*Uniform pricing:*

- CS=1,250
- PS=1,500

*Perfect price discrimination (first-degree):*

- CS=0
- PS=4,000

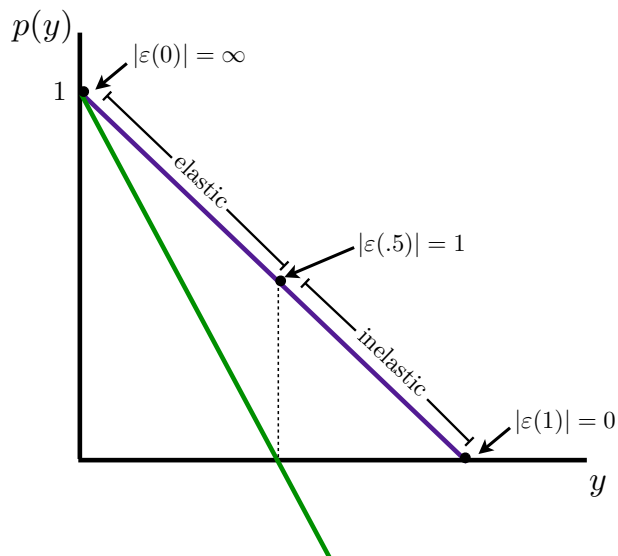
*Third-degree price discrimination:*

- CS=  $390\frac{5}{8} + 1,562\frac{1}{2} = 1,953\frac{1}{8}$
- PS= $3,906\frac{1}{4}$  (or  $2,906\frac{1}{4}$ )

Notice that the producer surplus is the greatest with first-degree price discrimination and lowest with uniform pricing.

## Problem 2 (Demand Elasticity)

(a) The inverse demand curve,  $p(y) = 1 - y$ , is shown below:



(b) For this demand function, we have  $\varepsilon = y'(p) \times \frac{p(y)}{y} = (-1)\frac{1-y}{y}$ , so

- $\varepsilon(0) = -\frac{1-0}{0} = -\infty$
- $\varepsilon(.5) = -\frac{1-.5}{.5} = -1$
- $\varepsilon(1) = -\frac{1-1}{1} = 0$

These points are marked on the graph in part (a).

(c) When  $TC(y) = cy$ , we have constant marginal cost of  $MC(y) = TC'(y) = c$ . Total revenue is  $TR(y) = p(y) \cdot y$  and so marginal revenue is  $MR(y) = TR'(y) = p(y) + y \cdot p'(y)$  (product rule). Since the profit-maximizing firm chooses output such that  $MR(y) = MC(y)$ , we have

$$MR(y) = MC(y) \implies p(y) + y \cdot p'(y) = c.$$

Changing notation so that  $p(y) = p$  and  $p'(y) = \frac{dp}{dy}$ , we have

$$p + y \frac{dp}{dy} = c \implies p \left[ 1 + \frac{y}{p} \frac{dp}{dy} \right] = c.$$

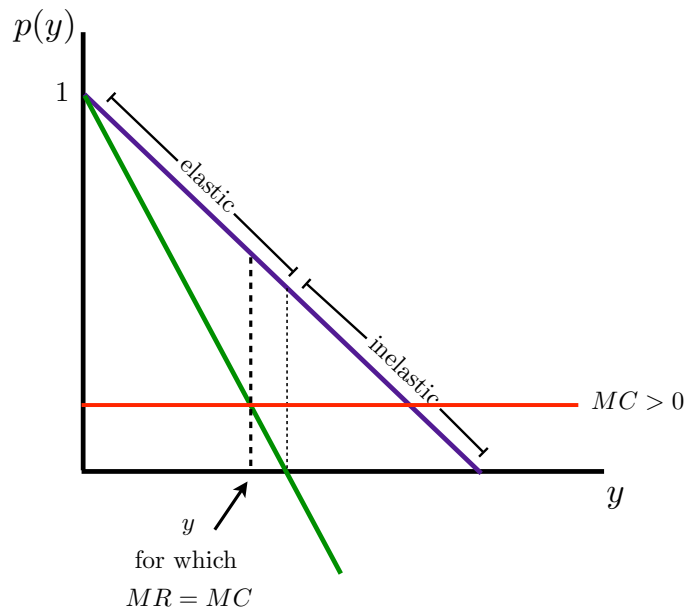
Recognizing that  $1/\varepsilon = \frac{dp}{dy} \frac{y}{p}$ , we have

$$c = p \left[ 1 + \frac{1}{\varepsilon} \right].$$

Then since  $c > 0$ , we must have  $p \left[1 + \frac{1}{\varepsilon}\right] > 0$ . Since  $p > 0$ , we can solve for  $\varepsilon$  to get

$$p \left[1 + \frac{1}{\varepsilon}\right] > 0 \implies 1 + \frac{1}{\varepsilon} > 0 \implies \frac{1}{\varepsilon} > -1 \implies \varepsilon < -1 \implies \varepsilon > 1$$

and so the optimal output is on the elastic portion of the demand curve for any  $MC = c > 0$ .



As we can see in the figure, whenever  $MC > 0$ , output occurs where  $MR=MC$  which is necessarily on the elastic region of the demand curve.

(d) Equating  $MR(y) = MC(y)$ , we get

$$p \left[1 + \frac{1}{\varepsilon}\right] = MC \implies p = \left[\frac{1}{1 + 1/\varepsilon}\right] MC$$

so the price markup over marginal cost is  $\left[\frac{1}{1+1/\varepsilon}\right]$ .

### Problem 3 (Oligopolistic Industry)

(a) The “big four” concentration ratio of the light beer industry in the U.S. is

$$CR = 36.8\% + 19.1\% + 18.5\% + 9.2\% = 83.6\%.$$

(b) The HHI is given by

$$HHI = 36.8^2 + 19.1^2 + 18.5^2 + 9.2^2 + 6.1^2 + 3.3^2 + 2.6^2 + 2.3^2 + 0.8^2 + 0.7^2 \approx 2,117.$$

(c) This industry can be considered highly concentrated since  $HHI > 1,800$ .

(d) The FTC would likely attempt to block the merger as a monopoly would result from it.

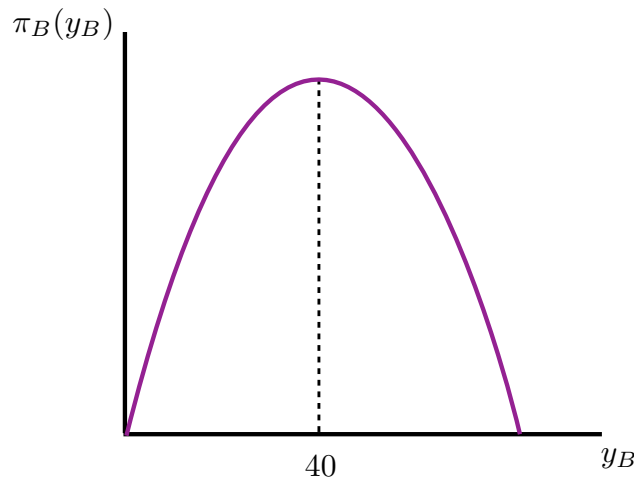
#### Problem 4 (Aircraft Industry)

(a) Boeing's profit function is

$$\pi_B(y_B) = TR - TC = (200 - y_A - y_B)y_B - 20y_B$$

and given that Airbus is production at  $y_A = 100$  this reduces to

$$\pi_B(y_B) = 80y_B - (y_B)^2$$



This attains an optimum at  $y_B = 40$  (using first-order condition  $\pi'_B(y_B) = 0$ ).

(b) No, we found in part (a) that Boeing's best response to Airbus producing  $y_A = 100$  is  $y_B = 40$  (not  $y_B = 100$ ). For  $y_A = 50$ , Boeing's profit-maximizing output is  $y_B = 65$ ; when  $y_A = 0$ , we have  $y_B = 90$ . (You can verify this by going through the steps in part (a).)

(c) The best response function for Boeing, which tells us their optimal output for any level of production Airbus might choose, we take the first-order condition of Boeing's profit function

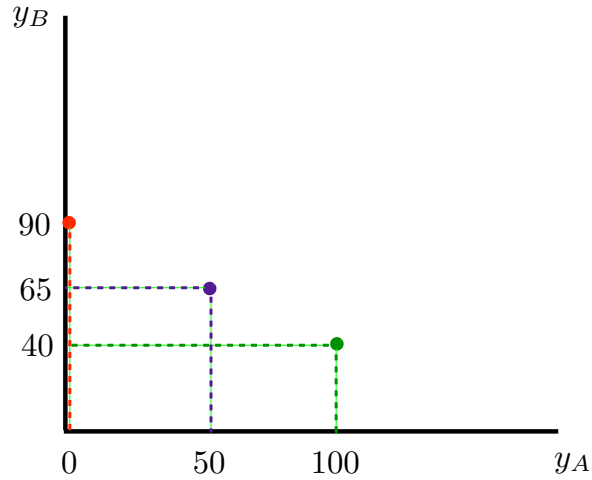
$$\pi_B(y_B) = (200 - y_A - y_B)y_B - 20y_B,$$

which is

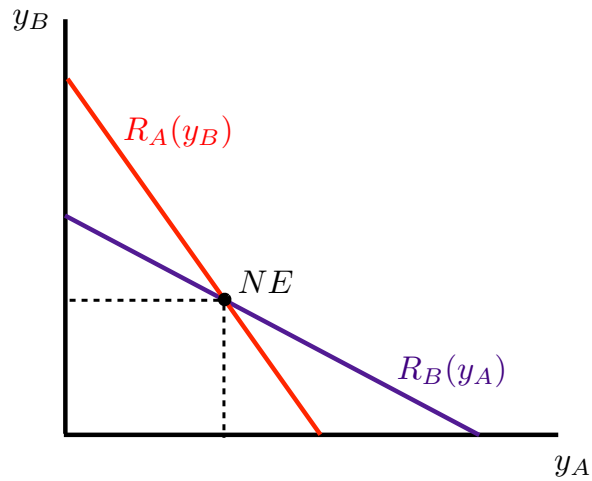
$$\frac{\partial \pi_B}{\partial y_B} = 0 \implies 200 - y_A - 2y_B - 20 = 0.$$

Solving this for  $y_B$ , we find  $y_B = \frac{180 - y_A}{2}$ , which is what gives us the best response function, so

$$R_B(y_A) = \frac{180 - y_A}{2}.$$



This is shown in the graph below:



(d) The best response function for Airbus is symmetric (they have the same cost functions). To find Airbus' best response function in general though we take the same steps as we did to find Boeing's. We take the profit function

$$\pi_A(y_A) = (200 - y_A - y_B)y_A - 20y_A$$

and the first-order condition

$$\frac{\partial \pi_A}{\partial y_A} = 0 \implies 200 - 2y_A - y_B - 20 = 0.$$

Solving this for  $y_A$ , we find  $y_A = \frac{180 - y_B}{2}$ , which is what gives us the best response function

$$R_A(y_B) = \frac{180 - y_B}{2}.$$

This is shown on the graph above along with Boeing's best response function  $R_B(y_A)$ .

(e) The Cournot-Nash equilibrium level of output produced by each firm solves the best response functions simultaneously. You can set  $y_A = R_B(y_A)$  and  $y_B = R_A(y_B)$  to get

$$y_A = y_B = 60$$

so total output in the market is  $y = y_A + y_B = 120$ .

Turning to the inverse demand curve, this output is associated with a market price of

$$p(120) = 200 - 120 = 80$$

and the profits for each firm are

$$\pi_A = \pi_B = 80 \times 60 - 20 \times 60 = 3,600.$$

(f) To find the DWL, we first need to find the Pareto efficient output (where  $p = MC$ ): In this problem  $MC = 20$  and  $p = MC \implies 200 - y = 20 \implies y = 180$ .

The DWL then is the triangular area

$$DWL^{Duopoly} = \frac{1}{2}(80 - 20)(180 - 120) = 1,800.$$

(g) If the two firms were to form a cartel, they would determine output as a monopolist, maximizing cartel profit

$$\pi_{Cartel} = (200 - y)y - 20y.$$

Using the first order condition we get a profit-maximizing output of  $y = 90$  and a cartel price of  $p(90) = 200 - 90 = 110$ . Each of them produce half of the optimal output, so  $y_A = y_B = 45$ , and they split the profits:

$$\pi_A = \pi_B = \frac{\pi_{Cartel}}{2} = \frac{110 \times 90 - 20 \times 90}{2} = 4,050.$$

This gives a higher level of profit than what we found with Cournot competition, so the cartel would be beneficial to both firms.

(h) The deadweight loss here is now the triangular area

$$DWL^{Cartel} = \frac{1}{2}(110 - 20)(180 - 90) = 4,050 > DWL^{Duopoly}.$$

(i) If the interactions are only in a short run, cartel is not sustainable as each firm has incentives to “cheat” the other firm by unilaterally increasing a production above 45 jets; you can see this by checking what their best response is to the other firm producing 45. This

is, because given the other firm produces 45 jets, producing more leads to the firm that is even higher than in the case of the cartel.

When interactions are repeated however, the loss of reputation associated with cheating “today” may make the cooperation of the two firms tomorrow possible. If the firm cheats today, it gains some profit today but loses out on all of the cartel profit it could make in the future if it cooperates (which is always higher than the duopoly profit it gets if it cheats). So such costs of cheating (i.e., the loss of future cartel profits), might make the cooperation sustainable.

### Problem 5 (Accounting and Audit Services in the U.S.)

(a) *Monopoly*: Monopoly profit is

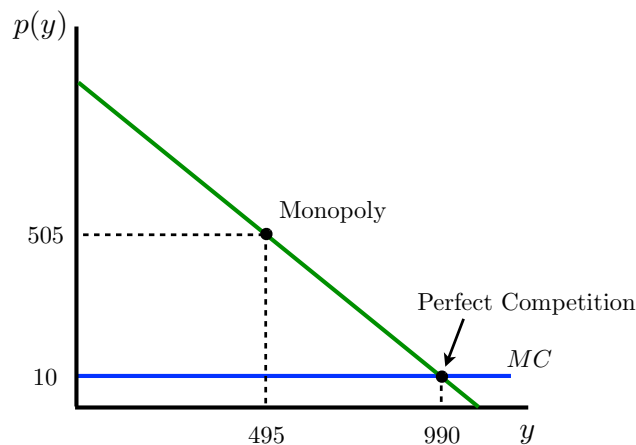
$$\pi_{Monopoly} = (1,000 - y)y - 10y$$

and taking the first-order condition we get optimal output  $y = 495$  and price

$$p(495) = 1,000 - 495 = 505.$$

*Perfect Competition*: With perfect competition, we will have that  $p = MC = 10$  and so output is (turning to demand function  $y(p) = 1000 - p$ ) then  $y(10) = 990$

(b) The two points from part (a) are shown below along the inverse demand curve:



(c) Let  $\sum_{j \neq i} y_j$  represent the output of all of the  $N - 1$  firms other than firm  $i$ . (So that  $\sum_{j \neq i} y_j = y_1 + y_2 + \dots + y_{i-1} + y_{i+1} + \dots + y_N$ .) In Cournot-Nash competition, profit for firm  $i$  is

$$\pi^i = (1,000 - y_i - \sum_{j \neq i} y_j)y_i - 10y_i.$$



The first-order optimality condition gives

$$1,000 - 2y_i - \sum_{j \neq i} y_j - 10 = 0$$

or

$$y_i = \frac{990 - \sum_{j \neq i} y_j}{2}$$

Since the firms are symmetric,  $y_i = y_j$  and so we can write this as

$$y_i = \frac{990 - (N - 1)y_i}{2}$$

and solving for  $y_i$  we get

$$y_i = \frac{990}{N + 1}$$

Aggregate production with  $N$  firms is

$$y = Ny_i = \frac{N}{N + 1}990.$$

This gives price

$$p(y) = 1,000 - \frac{N}{N + 1}990.$$

The DWL will depend on the number of firms in the market and is given by

$$DWL = \frac{1}{2} \times \left[ \left( 1,000 - \frac{N}{N + 1}990 \right) - 10 \right] \times \left[ 990 - \left( \frac{N}{N + 1}990 \right) \right] = \frac{990^2}{2} \times \left( \frac{1}{N + 1} \right)^2$$

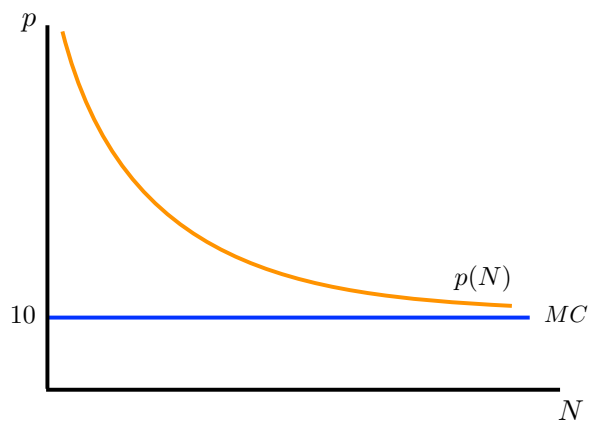
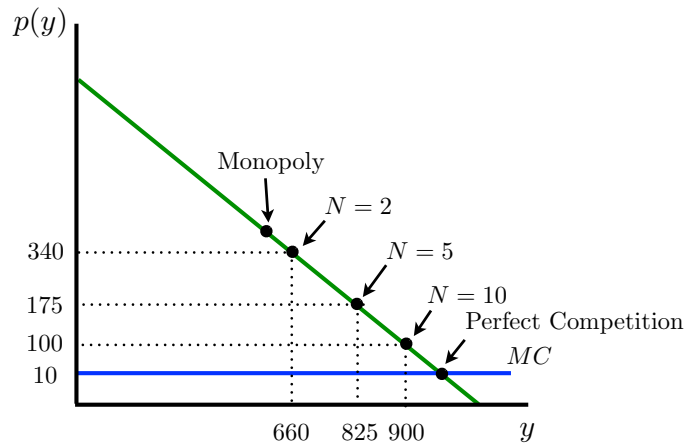
**(d)** We have the following aggregate output  $y$  and prices  $p$  for  $N = 2, 5,$  and  $10$ :

$N$	$y$	$p$
2	660	340
5	825	175
10	900	100

These are shown on the graph below:

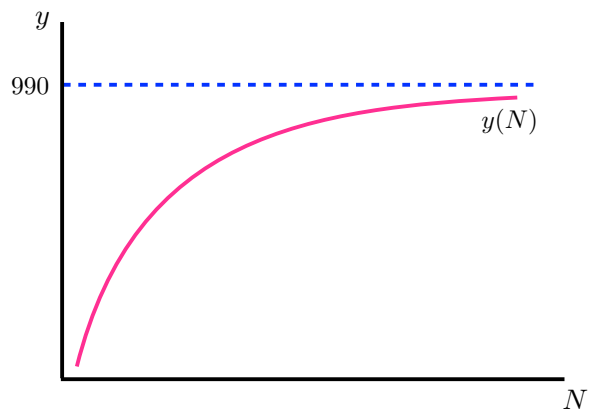
**(e)** As the number of firms  $N$  becomes large, the market looks more like a perfectly competitive market. Since  $\lim_{N \rightarrow \infty} \frac{N}{N + 1} = 1$ , we can see that

$$\lim_{N \rightarrow \infty} p = \lim_{N \rightarrow \infty} \left[ 1,000 - \frac{N}{N + 1}990 \right] = 1,000 - 990 \left( \lim_{N \rightarrow \infty} \frac{N}{N + 1} \right) = 1,000 - 990(1) = 10.$$



(f) As  $N$  increases, we will get that output also converges to its competitive limit:

$$\lim_{N \rightarrow \infty} y = \lim_{N \rightarrow \infty} \frac{N}{N+1} 990 = 990.$$



(g) Again, here we have that as the number of firms becomes very large, the DWL converges

to its competitive limit. Since  $\lim_{N \rightarrow \infty} \frac{1}{N+1} = 0$ , we have that

$$\lim_{N \rightarrow \infty} DWL = \frac{990^2}{2} \times \left( \lim_{N \rightarrow \infty} \frac{1}{N+1} \right)^2 = \frac{990^2}{2} \times (0)^2 = 0.$$