

Problem Set 10: Solutions

ECON 301: Intermediate Microeconomics
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Problem 1 (Equilibrium with N Firms)

(a) First note that marginal cost is $MC(y) = c'(y) = 8y$. Using the condition that $p = MC(y)$, we have $p = 8y \implies y = \frac{1}{8}p$. For prices equal to above ATC^{MES} , the supply function is $y = \frac{1}{8}p$, for prices below ATC^{MES} , the firm would be operating at a loss if producing and so supply is $y = 0$.

We can find the minimum of ATC , which is what ATC^{MES} is, either by (1) setting ATC' equal to 0 and solving for y , or (2) equating $ATC = MC$ and solving for y (since the minimum of ATC corresponds to the point at which $ATC = MC$). Both give us $y^{MES} = 1$ and $ATC^{MES} = ATC(y^{MES}) = 8$.

The individual supply is then:

$$y(p) = \begin{cases} 0 & \text{for } p < 8 \\ \frac{1}{8}p & \text{for } p \geq 8 \end{cases}$$

(b) The aggregate supply with the three identical cost structures is $y^{AGG}(p) = 3y(p)$ (the supply curves are added horizontally over the y -axis). Letting $S(p) = y^{AGG}(p)$:

$$S(p) = \begin{cases} 0 & \text{for } p < 8 \\ \frac{3}{8}p & \text{for } p \geq 8 \end{cases}$$

(c) We'll first find the equilibrium price and aggregate level of production by equating $S(p) = D(p)$:

$$S(p) = D(p) \implies \frac{3}{8}p = 8 - \frac{1}{8}p \implies p = 16 \text{ (which is } \geq 8)$$

and the aggregate level of production is $S(16) = D(16) = 6$.

The production of each firm is $y(p) = 2$ (since $S(p) = 3y(p)$, $6 = 3y(p)$), which gives profit $\pi = TR - TC = 16 \cdot 2 - (4 \cdot 2^2 + 4) = 12$ for each factory.

(d) The highest amount a firm would pay for the license is 12 (its profit when producing in

this market, leaving it with 0 economic profit after paying the license fee).

Problem 2 (Free Entry and Market Structure)

(a) We'll first solve this for a general fixed cost of F , as we saw in Problem Set 8. The price in equilibrium with free entry must be $p = ATC^{MES} = 4\sqrt{F}$ where F is fixed cost with $y^{MES} = \frac{1}{2}\sqrt{F}$. At $p = ATC^{MES} = 4\sqrt{F}$, quantity demanded is

$$D(4\sqrt{F}) = 8 - \frac{1}{8}(4\sqrt{F})$$

and aggregate supply with N firms is

$$S(4\sqrt{F}) = N\frac{1}{2}\sqrt{F}.$$

Equating the two, we get

$$S(4\sqrt{F}) = D(4\sqrt{F}) \implies N\frac{1}{2}\sqrt{F} = 8 - \frac{1}{8}(4\sqrt{F}) \implies N = \frac{16}{\sqrt{F}} - 1.$$

So for fixed cost $F = 4$, the number of firms in the market will be $N = 7$.

(b) The number of firms in the market for various fixed costs are shown below, using the formula we found above, $N = \frac{16}{\sqrt{F}} - 1$:

Fixed Cost F :	64	16	4	$\frac{1}{4}$	$\frac{1}{16}$
Firms N :	1	3	7	31	63

(c) The market structures are:

- Monopoly at $F = 64$
- Oligopoly at $F = 16, F = 4$
- Nearly perfect competition at $F = \frac{1}{4}, F = \frac{1}{16}$

Problem 3 (Reasons why Monopolies Exist)

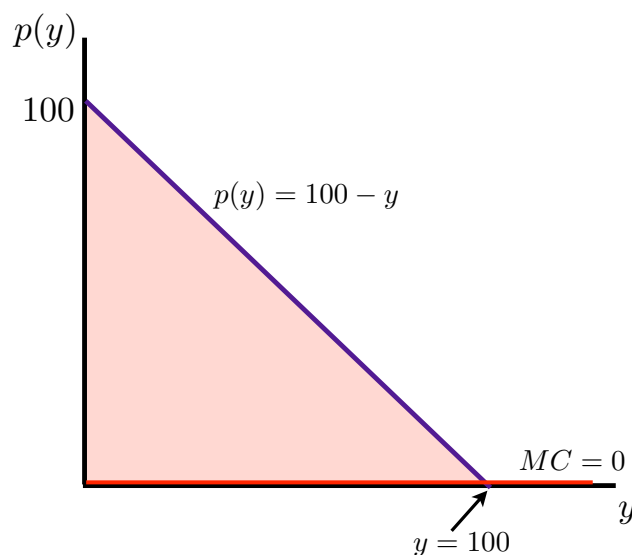
- (1) Nuclear power plant: Large fixed costs.
- (2) Microsoft's Vista operating system: Patent.
- (3) Casinos, U.S. Post Office: Legal fiat.

(4) Niagara Falls State Park: Sole owner of waterfalls.

Problem 4 (Monopoly, Uniform Price)

(a) The total gains to trade is the pink area in the figure below *minus* the fixed cost of $F = \$1,000$ and is calculated as

$$TS = \frac{1}{2} \times 100 \times 100 - F = 5,000 - 1,000 = 4,000.$$



If this were a competitive market and Microsoft were a price taker, output would occur where $p = MC$. Since $MC = 0$, we get output $y = 100$. Consumer surplus is the area under the inverse demand curve and above price ($p = 0$ here), which corresponds to the shaded pink area above and is:

$$CS = \frac{1}{2} \times 100 \times 100 = 5,000.$$

At output $y = 100$ and price $p = 0$, producer surplus is

$$PS = -1,000.$$

Note that with price being equal to zero the firm has a negative profit and should exit the industry.

An answer where fixed cost is not subtracted (so $TS = 5,000$ and $PS = 1,000$) would also be correct; in such a case F would be considered a sunk cost.

(b) If Microsoft cannot discriminate among customers' willingness to pay, its profit-maximizing output is where $MR = MC$.

Marginal revenue is the derivative of total revenue, which is

$$TR(y) = p(y) \cdot y = (100 - y)y = 100y - y^2,$$

and so total revenue is

$$MR(y) = TR'(y) = 100 - 2y.$$

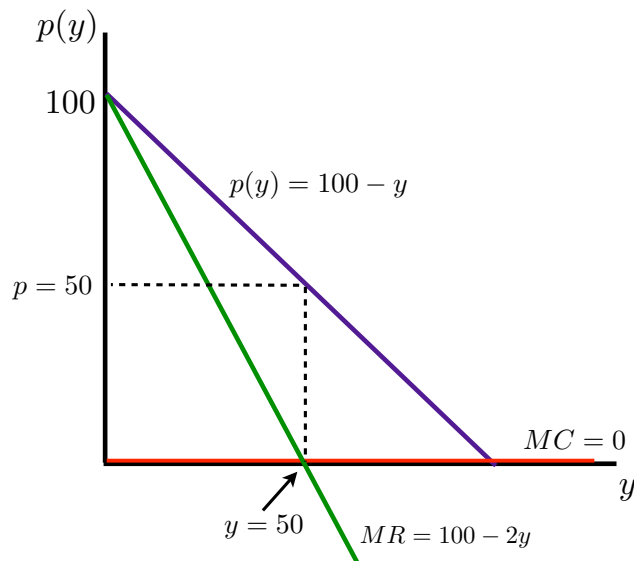
Since $MC(y) = 0$,

$$MR(y) = MC(y) \implies 100 - 2y = 0 \implies y = 50$$

giving price

$$p(50) = 100 - (50) = 50.$$

Profit is $\pi = TR(50) - TC(50) = 50 \cdot 50 - 1,000 = 1,500$.



(c) The outcome in part (b) is not Pareto efficient; there are only 50 trades made while in the competitive market (which is Pareto efficient) there are 100 trades. The deadweight loss is the gray shaded area seen in the figure below:

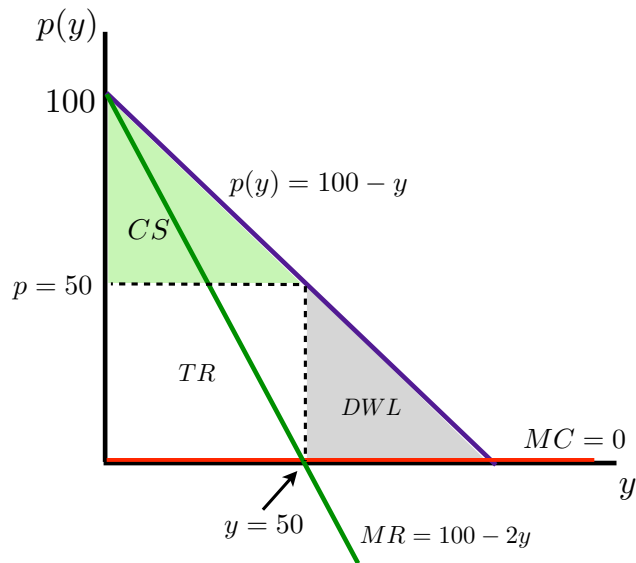
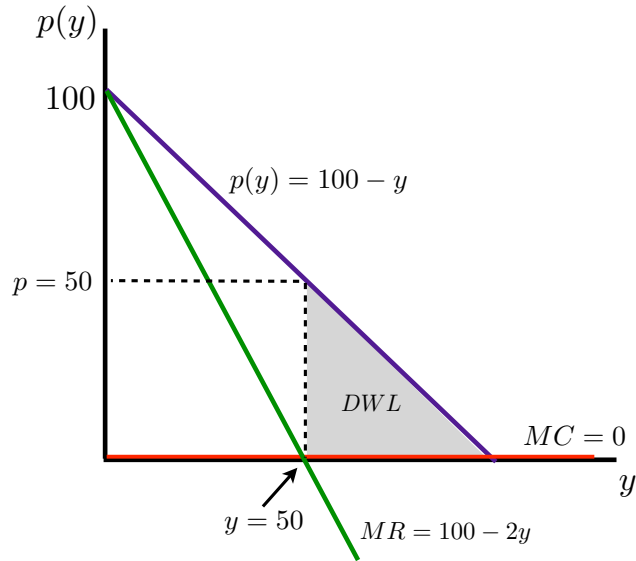
Analytically, the deadweight loss is

$$DWL = \frac{1}{2} \times 50 \times (100 - 50) = 1,250.$$

(d) The consumer surplus is the area under the demand curve and above the price:

$$CS = \frac{1}{2} \times (100 - 50) \times 50$$

This is the green shaded area in the figure below:



Producer surplus is the total revenue (TR) area *minus* the fixed cost of $F = 1,000$:

$$PS = 50 \times 50 - 1,000 = 1,500$$

Notice that consumer and producer surplus add up to less than the total surplus we found in part (a); the difference is the deadweight loss of $DWL = 1,250$.

(e) Elasticity is defined as

$$\varepsilon = \frac{\Delta y/y}{\Delta p/p} = \frac{\Delta y}{\Delta p} \times \frac{p}{y}$$

where $\frac{\Delta y}{\Delta p}$ is the slope of the demand function $y(p)$ (not inverse demand $p(y)$, though they happen to have the same slope in this problem). Here, $y(p) = 100 - p$, and since $p = 50$ and

$y = 50$,

$$\varepsilon = y'(p) \times \frac{p}{y} = (-1) \times \frac{50}{50} = -1.$$

Since $|\varepsilon| = 1$, the firm is operating at a point along the demand curve that is neither elastic ($|\varepsilon| > 1$) nor inelastic ($|\varepsilon| < 1$); it's at the threshold between the two regions along the demand curve.

(f) Markup is p/MC . Since we are operating where $MR = MC$, we can write markup as p/MR . Since $MR = p[1 + 1/\varepsilon]$, we have

$$\frac{p}{MC} = \frac{p}{MR} = \frac{p}{p[1 + 1/\varepsilon]} = \frac{1}{1 + 1/\varepsilon}$$

since $\varepsilon = -1$, we have $\frac{p}{MC} = \infty$. Markup is infinite with marginal cost of zero.