The Foundations of Warm-Glow Theory*

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October 22, 2009

Abstract

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1. Introduction

Citizens may endure long lines to vote in large elections even though their single vote is unlikely to change the election outcome. People recycle paper and plastic even though the impact of their individual action is environmentally negligible. Voting in elections, recycling and other activities such as donating money all impose costs on individuals. However, the material consequences of each individual action seems insufficient to motivate the effort. Social scientists often explain such actions by assuming that agents receive a "warm-glow" payoff by taking an action they believe to be virtuous.

The basic warm-glow model is captured by the following simple payoff function. The payoff for choosing an alternative x is

$$\Pi(x) = \begin{cases} \pi(x) + D \quad if & \text{choosing } x \text{ is virtuous} \\ \pi(x) & \text{otherwise} \end{cases}$$

where $\pi(x)$ is the payoff from taking action x whether or not x is understood to be virtuous and $D > 0$ is the *warm-glow* payoff received when choosing x is deemed to be virtuous. A famous example of this basic structure can be found in Riker and Ordeshook (1968). Their model is captured by the equation

$$[pu - C] + D$$

where p is the probability an agent's vote is pivotal, u is the difference in payoff between the favored candidate and his opponent being elected, C is the cost of voting and D is the warm-glow payoff received by voting for the favored candidate. In large elections, since the probability p a vote is pivotal is very small, Riker and Ordeshook's model reduces to predicting a vote if and only if the warm-glow payoff D is greater than the voting cost C.[1]

In spite of the growing popularity of warm-glow models, several economists are

[1] Andreoni (1989) writes a more complicated warm-glow utility function as follows:

$$U = U(y, Y, g)$$

where y is the agent's consumption of private goods, Y is the total supply of the public good and g is the warm-glow the agent experiences by virtue of giving. We will discuss extensions of our basic warm-glow model below.

reluctant to adopt a warm-glow framework and also refer to this approach as ad-hoc. A salient difficulty is that, in this approach, modellers assume a warm-glow payoff. So, by assumption, decision makers are motivated to take actions that the decision makers ascribe as virtuous. However, the observable basis of this assumption are unclear. In particular, it is unclear how a hypothesis of a warm-glow payoff can be tested and rejected.

Like standard economic models, warm-glow models allow that agents have preferences over outcomes. However, warm-glow models make two additional assumptions. First, agents have a (limited) tolerance for taking actions that are suboptimal in terms of their preferences. In the Riker and Ordeshook model this is captured in the D term that the agent receives if he votes. The D term is large enough to motivate some people to vote but not large enough to produce universal turnout. Second, when agents do make suboptimal choices they do so intentionally, in the service of a known goal or aspiration i.e., to help a good cause. In the election context agents receive the warm-glow payoff not simply for the act of voting but as a consequence of voting for a specific candidate.

In this paper we provide a general formal structure that captures these three essential elements of warm-glow decision making. We model a decision maker (Dee) who is endowed with: (1) a preference over a finite set of alternatives; (2) a tolerance threshold function that, for each alternative, identifies the least preferred alternative that might be chosen instead; and (3) aspirations that identify the alternative that is chosen when the most preferred alternative is not. Dee chooses according to her aspirations if that choice is ranked above her tolerance threshold, otherwise she chooses her most preferred alternative.

The following example illustrates our model and highlights the fundamental differences between standard models and warm-glow. Suppose Dee faces a choice about how much money to donate to charity. There are three possible donation levels: 0, 5, 10. Dee prefers to donate the least amount possible but can tolerate donating up to 5 more than the minimum possible donation. While Dee prefers to donate the minimum she aspires to donate the maximum possible. So, given the choice between donating 0 and 5 Dee will donate 5 because she can tolerate the 5 donation and she aspires to contribute the maximum suggested. Similarly, if Dee is given the choice between donating 5 and 10 she will donate 10. However, if Dee is given the choice between donating 0, 5 or 10 Dee will donate 0. Dee prefers to donate 0 and aspires to donate the suggested maximum of 10. However, Dee's aspiration to donate 10 is ranked below her 5 donation threshold and so Dee chooses to donate 0.

The example illustrates that warm-glow produces behavior that is inconsistent with a standard model of preferences. The standard model can only produce choices that are consistent with the Weak Axiom of Revealed Preferences (WARP). The WARP requires that if 5 is chosen over 0 in the binary choice then 0 cannot be chosen in a superset that contains both 0 and 5. Dee's donation behavior violates the WARP.

Warm-glow applications typically make assumptions about preferences, tolerance thresholds, and aspirations analogous to the assumptions about preferences made in standard models. However, standard theory provides an empirical foundation for assumptions about preferences by equating preferences with observable choice behavior. To our knowledge warm-glow models lack a similar empirical foundation. In this paper we define a methodology which can be used to define the unobserved elements of warm-glow in a manner analogous to standard theory.

Where standard theory defines preferences in terms of actual choices we define preferences and aspirations on the basis of actual and hypothetical choices. For a given set of alternatives the decision maker's actual choice is the alternative selected when the choice has consequences. A hypothetical choice is the alternative selected when the decision maker is asked "what would you do if given the choice from a given set of alternatives." For example, suppose there are three alternatives: vote for Bush, vote for Gore or abstain. The actual choice is what the decision maker does when and if she arrives at the voting booth. The hypothetical choice is the answer to a pollster who calls the voter and asks what they intend to do.

As is well known, there is a connection between what people do and what they say they will do. This connection makes opinion polls predictive. However, it is also well-known that simply because someone says they intend to vote in an election does not mean that they will actually vote. For example, overreporting of voting is a topic long studied in political science. Survey researchers typically ask people whether they voted in an election and, if so, for whom. While it is impossible to verify actual vote choice, voting in an election is part of the public record. Sigelman (1982), for example, reports that only about 1% of survey respondents mistakenly report abstaining when they, in fact, voted. On the other hand, around 13% of respondents incorrectly report having voted when the record shows they did not. That respondents are much more likely to falsely report having voted than having abstained is entirely consistent with the warm-glow model. Respondents who aspire to abstain actually do so because abstention is also the least costly action. On the other hand, respondents who aspire to vote do not always do so because they find the personal costs of voting too high relative to the warm-glow satisfaction they get from voting.

In this paper, we propose that aspirations can be defined directly from observed hypothetical choices. We then show two axioms on actual choice behavior are necessary and sufficient to permit definition of preferences and tolerance threshold functions. The first axiom requires that for all sets of alternatives such that the hypothetical choice differs from the actual choice, actual choices must satisfy the Weak Axiom of Revealed Preference (WARP). Since warm-glow decision makers always choose their most preferred alternative when they don't choose their aspiration this axiom is essential to guarantee that revealed preferences will be an order. The second axiom we call the Warm-glow Axiom. If an agent cannot tolerate choosing a suboptimal alternative in the presence of an alternative she prefers in some choice situation then she cannot tolerate that suboptimal alternative in every choice situation in which both alternatives are available.

The paper proceeds as follows. In section 2 we provide a literature review that links our approach to the literature on warm-glow decision making and a broader class of behavioral decision models. In section 3, we present the formal model along with some examples to illustrate the model. In section 4 we present the formal results. In section 5 we discuss how our basic warm-glow model might be extended to allow for multiple tolerance and explain a broader class of phenomena related to aspirational decision-making.

2. Literature Review

Andreoni (1989) surveys the literature on warm-glow giving and develops a model in which agents have a preference for contributing towards a public good. Andreoni shows that adding a warm-glow component to public goods models will explain why government spending does not crowd out private donations as would be predicted in standard economic models. Andreoni does not take a revealed preference approach as we do here. In political science one of the earliest examples of a warm-glow theory is Riker and Ordeshook (1968). They add a payoff term to the voter's utility function such that voters recieve a modest payoff from the act of voting for their preferred candidate independent of the impact voting has on the outcome of the election. While Riker and Ordeshook recognized that they were taking an "ad hoc" approach to explaining turnout they nevertheless demonstrated that the warm-glow model would explain comparative statics on turnout including responsiveness of turnout to costs to vote. Feddersen and Sandroni (2004...) endogenize the warm-glow payoff as part of an ethical voter model. In their models agents reason about the impact of groups of voters acting in consort. If they reason that their group can, by voting, reasonably expect to impact the voting outcome then they determine that voters like them have a duty to vote for their preferred candidate. Coate and Conlin (2007) find support for the ethical voter model in the field. Gailmard, Feddersen and Sandroni (2009) find support for the ethical voter model in laboratory experiments.[2]

In this paper we argue that survey responses may be indicative of aspirational preferences and, therefor, can augment actual choice data. Evidence for systematic over-reporting of voting can be found in Sigelman (1982) along with Wolfinger and Rosenstone (1980, p. 115)). A large literuture in psychology finds a social desirability bias: survey respondents are prone to answer questions about their behavior in ways that are consistent with social expectations. Furnham (1986) provides a review.[3] In the same spirit, Camerer argues that the set of data available to ascertain choice-theoretic primatives can be expanded to include observations of brain activity.[4]

There is a growing literature in decision theory on the topic of agents with multiple preferences. See Cherapanov, Feddersen and Sandroni (2009) for a review of this literature. In particular, Kalai, Rubinstein and Spiegler (2002) consider a basic model of multiple selves, where choice is optimal according to one of the selves. Green and Hojman (2007) develop a multiple-self model that has no empirical content, but allows partial inferences of preferences. A literature review on multiple-self models can also be found in Ambrus and Rozen (2008) who also develop a multiple-self model.

There is a large literature on violations of WARP along with models that generate such violations. See Cherapanov, Feddersen and Sandroni (2009) and Manzini and

[2] Recent work by Levine and Palfrey (2008) find that ethical voter models are unnecessary to explain behavior in the laboratory. See also Merlo and Palfrey (2009).

[3] Furnham, Adrian. "Response Bias, Social Desirability and Dissimulation", Journal of Personality and Individual Difference, Vol. 7, No. 3, pp. 385-400. 1986.

[4] Camerer, Colin., Lowenstein, George., and Drazen Prelec. 2005. "Neuroeconomics: How Neuroscience Can Inform Economics". Journal of Economic Literature, Vol. XLIII (March 2005), pp. 9-64.

Mariotti (2006) for surveys of this literature.

3. Warm-Glow Theory: Basic Concepts

Let A be a finite set of alternatives. A non-empty subset $B \subseteq A$ of alternatives is called an *issue*. Let $\mathcal{B}$ be the set of all issues with at least two alternatives. A *choice function* is a mapping $C : \mathcal{B} \longrightarrow A$ such that $C(B) \in B$ for every $B \in \mathcal{B}$. Hence, a choice function takes an issue as input and returns a feasible alternative (i.e., the choice) as output.

A *preference order* R is an asymmetric, transitive and complete binary relation. By standard convention, $x\, R\, y$ denotes that, x is $R-$preferred to y. So, x $R-$*optimizes* B if $R(B)\ R\ b$, for every $b \in B$, $b \neq R(B)$. That is, x is $R-$preferred to any other feasible alternative. Given an order R and issue B, let $R(B) \in B$ be the alternative that $R-$optimizes it. In the opposite direction, if $C(B) = R(B)$ then issue B is *resolved* by preference order R.

A *utility function* $u : A \longrightarrow \Re$ takes an alternative $a \in A$ as input and returns a value $u(a) \in \Re$ as output. We assume that $u(x) \neq u(y)$ if $x \neq y$ and so, indifference is ruled out. A preference order R is associated with a utility function u whenever x R $y \Leftrightarrow u(x) > u(y)$ for $x \neq y$. So, preferred alternatives are associated with greater utility.

An *aspiration function* is a mapping $\mathcal{E} : \mathcal{B} \longrightarrow A$ such that $\mathcal{E}(B) \in B$ for every $B \in \mathcal{B}$. A decision maker, named Dee, faces actual choices and hypothetical choices. Dee expresses that her choice should be $\mathcal{E}(B)$ *if* she were to choose from issue B. Dee's actual choice is $C(B)$ when she does face issue B. The choice function C is called Dee's *actual choice* function. So, Dee's actual choice need not coincide with what Dee aspires to do.

Given an issue B and aspiration function $\mathcal{E}$, let $1^{\mathcal{E},B} : B \longrightarrow \{0, 1\}$ be an indicator function such that $1^{\mathcal{E},B}(x) = 1$ iff $x = \mathcal{E}(B)$ (and so, $1^{\mathcal{E},B}(x) = 0$ if $x \neq \mathcal{E}(B)$). That is, $1^{\mathcal{E},B}$ indicates the alternative Dee aspires to take. Given an issue B and aspiration function $\mathcal{E}$, let

$$U^{\mathcal{E},B}(x) = u(x) + D.1^{\mathcal{E},B}(x)$$

be Dee's utility function plus a warm-glow payoff for acting as she aspires. Consider now a behavioral model such that for any given issue $B \in \mathcal{B}$, Dee maximizes

$$U^{\mathcal{E},B}(x) \text{ subject to } x \in B. \tag{3.1}$$

{I don't understand this section since aspirations are directly observable} If the aspiration function $\mathcal{E}$ is arbitrary and unobserved then any choice function C can be accommodated by this model. Just assume a near constant utility function, $D = 1$ and $\mathcal{E} = C$. To obtain empirical content, additional assumptions must be made. These assumptions can be over the structure of the aspiration function $\mathcal{E}$ and/or over its observability. Dee may be endowed with an *expressed preference* order R^e that determines the choices she expresses in hypothetical scenarios. Dee's expressive preference reflects the values that she expresses. So, if $x \; R^e \; y$ then Dee expressed a (moral or otherwise) preference for x over y. The main assumption here is that the values that Dee expresses are a product of an order over alternatives. An aspiration function $\mathcal{E}$ is *ordered* if there exists an preference order R^e such that

$$\mathcal{E}(B) = R^e(B)$$

for every issue $B \in \mathcal{B}$.

Alternatively, aspiration functions may be given from the outset. This approach is often taken in the existing literature. In the voting literature, Dee receives a warm-glow payoff for voting. In the public-goods literature, Dee receives a warm-glow payoff for contributing to the public good. In this approach, the aspiration function is determined not by answers to hypothetical questions, but by direct assumptions. If aspiration functions are observed (or taken at the outset) then the model in 3.1 has empirical content even if the aspiration functions are not ordered. However, if the aspiration functions are assumed to be ordered then 3.1 has empirical content even if they are unobserved. We now define warm-glow choice functions.

Definition 1. *Let $\mathcal{E}$ be an aspiration function. A choice function C is a warm-glow choice function if there exists an utility function u and a scalar $D \geq 0$ such that for every issue $B \in \mathcal{B}$,*

$$U^{\mathcal{E},B}(C(B)) > U^{\mathcal{E},B}(x) \text{ for every } x \in B, \; x \neq C(B). \tag{3.2}$$

That is, warm-glow choice functions are produced by optimization of standard utility functions plus a warm-glow payoff for acting as aspired. This definition holds whether or not the aspiration function is ordered, but it takes some aspiration function as given. In the case of unobserved aspiration functions, a choice function C is called *warm-glow* if it is a warm-glow choice function for some ordered aspiration function.

3.1. Warm-Glow Theory: Comment

In warm-glow theory, Dee expresses aspirations on how she should act. As long as her aspirations do not require her to sacrifice utility greater than D, she does act as she aspires. The idea of making small, but intentional, sacrifices makes warm-glow theory appealing in the voting and contributions to public goods literature because individuals repeatedly take some costly actions that, in themselves, have almost no consequence to the decision maker. In addition, these actions are not random or arbitrary, but rather those such as voting or contributing to public goods that individuals often express as their moral obligation. However, the notion of a decision maker willing to make intentional sacrifices (either large or small) cannot be easily accommodated in standard theory because utility is maximized in standard theory.[5] This makes warm-glow theory problematic because it is unclear what is the empirical meaning of sacrificing utility. The main objective in this paper is to show and characterize the empirical content of warm-glow theory.

3.2. Warm-Glow Theory and Ordered Preferences

In this section, we introduce a model of choice that is observationally equivalent to the warm-glow model defined by 3.2. This model delivers a first step in the determination of the empirical content of warm-glow theory.

Dee is endowed with an aspiration choice function $\mathcal{E}$ and a preference order called Dee's *preference order* R. Either Dee chooses as she aspires or as she prefers. If her aspirations are tolerable then she acts as she aspires. However, if her aspirations are excessively costly then chooses as she prefers. In order to determine which options are tolerable, Dee is endowed with a *tolerance function* $\tau : A \longrightarrow A$ that maps any alternative a into another alternative $\tau(a)$ that is Dee's tolerance limit when option

[5] There are some models of choice that relax some of the basic conditions of standard theory and allow for some deviations of utility maximization. These deviations are typically either based on random erros or on bounded rationality. In contrast, deviations from utility are intentional in warm-glow theory.

a is feasible. Any alternative $b \in B$ that Dee prefers to $\tau(a)$ (i.e., $b \;\; R \;\; \tau(a)$) are *tolerable* when a is feasible. The alternative $\tau(a)$ itself is tolerable, but an alternative $b \in B$ that is $R-$worse than $\tau(a)$ (i.e., $\tau(a) \;\; R \;\; b$) are *too costly* when a is feasible. So, if

$$\mathcal{E}(B) \;\; R \;\; \tau(R(B)) \text{ or if } \mathcal{E}(B) = \tau(R(B))$$

then Dee's aspirations are tolerable. If

$$\tau(R(B)) \;\; R \;\; \mathcal{E}(B)$$

then Dee's aspirations are too costly.

Dee's tolerance function $\tau : A \longrightarrow A$ must satisfy

$$\begin{gathered} \text{either } \tau(a) = a \;\text{ or }\; a \;\; R \;\; \tau(a); \text{ and} \\ \text{if } a' \;\; R \;\; a \text{ then either } \tau(a) = \tau(a') \text{ or } \;\tau(a') \;\; R \;\; \tau(a). \end{gathered} \tag{3.3}$$

So, the tolerance limit of a is either a itself or it is ranked (by R) below a. In addition, if R ranks a' above a then the tolerance limit of a' is either the same as the tolerance limit of a or ranked (by R) above the tolerance limit of a.

The conditions in 3.3 are natural requirements. The tolerance limit $\tau(a)$ of an alternative a should not be $R-$preferred to a because $\tau(a)$ marks the least attractive option that Dee can tolerate when a is feasible. In addition, if b is too costly in the presence of a then it should remain so when an even better opportunity (a' st. $a' \; R \; a$) becomes feasible.

Preliminary Result Let $\mathcal{E}$ be an aspiration function. A choice function C is a warm-glow choice function if and only if there exists a preference order R, and a tolerance function τ that satisfies 3.3 such that for any issue $B \in \mathcal{B}$,

$$C(B) = \mathcal{E}(B) \text{ if } \quad \mathcal{E}(B) \;\; R \;\; \tau(R(B)) \text{ or } \mathcal{E}(B) = \tau(R(B)) \tag{3.4}$$

$$C(B) = R(B) \text{ if } \quad \tau(R(B)) \;\; R \;\; \mathcal{E}(B) \tag{3.5}$$

So, in warm-glow choice functions, Dee chooses as she aspires when her aspirations are tolerable and Dee chooses as she prefers when her aspirations are too costly. This result holds whether or not Dee's aspirations are ordered.

The preliminary result allows the warm-glow model of 3.2 to be understood with no reference to cardinal concepts such as utility. Instead, warm-glow theory can now be based on the ordinal concept of preference (and aspiration and tolerance functions).

The preliminary result is quite intuitive. In the warm-glow model of 3.2, Dee maximizes $u(x) + D.1^{\mathcal{E},B}(x)$. So, if the utility of Dee's preferred choice $R(B)$ exceeds, by D, the utility of her aspiration $\mathcal{E}(B)$ then her aspiration is too costly and she resolves the issue by her preferences. On the other hand, if Dee's preferred choice does not exceeds, by D, the utility of her aspiration $\mathcal{E}(B)$ then her aspiration is tolerable and she chooses as she aspires.

3.3. General Warm-Glow Theory

The warm-glow theory of choice given by 3.2 is quite structured. It requires Dee to either act as she aspires or to choose by her preferences. This follows from the assumption that Dee receives a warm-glow payoff for acting exactly as she aspires, but she gets no warm-glow payoff if she takes any other option, even an option that is quite close to her aspiration. In general, one could conceive more flexible models of warm-glow that allow Dee to receive different levels of warm-glow payoffs.

Consider the following model. Let $V : A\text{x}A \longrightarrow \Re$ be a function, called *dual utility function*, that takes a pair of alternatives as input and return a value as output. Given an aspiration function $\mathcal{E}$, a choice function C is a *general warm-glow* choice function if there exists an dual utility function V such that for every issue $B \in \mathcal{B}$,

$$V(C(B), \mathcal{E}(B)) > V(x, \mathcal{E}(B)) \text{ for every } x \in B,\ x \neq C(B). \qquad (3.6)$$

In this model, Dee's aspirations function as a reference point. The payoff of each alternative is evaluated depending upon her aspiration $\mathcal{E}(B)$ for the issue B that she faces. Given Dee's aspiration $\mathcal{E}(B)$, Dee maximizes her utility $V(\ .\ , \mathcal{E}(B))$ subject to the feasibility constriant.

The warm-glow model of 3.2 is a special case of the general warm-glow model of 3.6 where the dual utility function $V(x, \mathcal{E}(B))$ equals $U^{\mathcal{E},B}(x)$. As far as we now, there are no conclusive empirical rejection of the warm-glow model of 3.2 that requires the further generalization of the more flexible model given by 3.6. In addition, the warm-glow model of 3.2 suffices to make a conceptual distinction between preferences and aspirations and, consequentially, to formalize the idea of making intentional sacrifices in order to achieve an aspiration. However, the general model of warm-glow given by

3.6 will turn out to be useful once we provide an axiomatic caractherization of the empirical content of the warm-glow model of 3.2 because it will permit a discussion of which axioms must hold in general and which axioms are associated with the particular form given by 3.2.

4. Empirical Content of Warm-Glow Theory

Recall the standard theory of choice and its testable implications. In standard theory, all issues are resolved by a single order. As is well known, the standard theory of choice holds if and only if the weak axiom of revealed preferences (hereafter WARP) holds. WARP states that if B and $\tilde{B}$ are a pair of issues such that $B \subseteq \tilde{B}$ and the choice in $\tilde{B}$ is feasible in B then choices in $\tilde{B}$ and B must coincide. A violation of WARP is often called a *behavioral anomaly.*

Warm-glow theory can accommodate and make sense of some behavioral anomalies. Consider the contribution to public goods study of Berger and Smith (1997). They find that some potential donors (to universities) elect to make a small solicited contribution (x) over no contribution (y), but if either a small or a large contribution (z) is solicited then many people do not contribute at all. This is an anomaly because x is chosen over y and y is chosen over x and z.

Warm-glow theory can produce this anomaly as follows: Dee prefers to have more money to less, but she aspires to donate as much as requested. The small contribution is tolerable to Dee, but the large one is too costly. So, when only the small contribution is solicited she contributes, but when an option to make a large donation is added she does not contribute at all.

This example shows that behavioral anomalies can be the product of simple and compelling models of choice where agents have well defined preference orders and maximize their payoffs which include their preferences and their desire to act as aspired. This example also shows that warm-glow theory is empirically distinct from standard theory of choice and, hence, the empirical meaning of the payoff for acting as aspired cannot be interpreted within the standard theory of choice.

The contribution to public goods is an example of a behavior anomaly that can be accommodated by warm-glow theory. We now characterize the observable implications of a 3.2 model of warm-glow. Consider the case that Dee's aspiration function is directly observable by her expressed values. If her aspirations are also assumed to be ordered then WARP must hold for her expressed choices. If her aspirations are not assumed to be ordered then no expressed choices, by themselves, will lead to a rejection of the model. Now consider Dee's actual choices. As shown in the contribution to public goods, some violations of WARP can be accommodated by warm-glow theory. However, the warm-glow cannot accommodate any anomaly.

Let $\mathcal{B}^s$ be the set of all issues such that $C(B) \neq \mathcal{E}(B)$. So, $\mathcal{B}^s$ are the issues that Dee's actual choice is not as she aspires. Under the 3.2 model of warm-glow, these issue must be resolved by Dee's $R-$preference order. Hence, restricted to $\mathcal{B}^s$, the WARP must hold for actual choices.

Limited WARP Let $B \in \mathcal{B}^s$ and $\tilde{B} \in \mathcal{B}^s$ be a pair of issues such that $B \subseteq \tilde{B}$.

$$C(\tilde{B}) \in B \Longrightarrow C(B) = C(\tilde{B}).$$

The Limited WARP is a characterization, in terms of actual choice and expressed aspirations, on the assumption that Dee's actual choices, when different from expressed choice, are resolved by her $R-$preferences. So, if the 3.2 model of warm-glow model then the Limited WARP axiom must hold.

Warm-glow theory also requires a condition when Dee acts as she aspires. So, additional axiom are required for a complete characterization of warm-glow theory of choice.

Warm-Glow Axiom Let $B \in \mathcal{B}$ and $\tilde{B} \in \mathcal{B}^s$ be any pair of issues.

$$\mathcal{E}(B) = \mathcal{E}(\tilde{B}) \text{ and } C(\tilde{B}) \in B \implies B \in \mathcal{B}^s.$$

The warm-glow axiom states that if B and $\tilde{B}$ are two issues such that Dee's aspiration is the same and Dee does not act as she aspires in $\tilde{B}$, but instead took an alternative $C(\tilde{B})$ that remains feasible in B then Dee does not act as she aspires in B.

Under warm-glow theory, if Dee does not act as she aspires in $\tilde{B}$ then Dee's aspiration $\mathcal{E}(\tilde{B})$ is too costly in the presence of $C(\tilde{B})$. Hence, $\mathcal{E}(\tilde{B})$ must remain too costly when $C(\tilde{B})$ is feasible because then Dee's $R-$preferences must either also direct her to $C(\tilde{B})$ and to an even more attractive alternative. So, $\mathcal{E}(B) = \mathcal{E}(\tilde{B})$ must be too costly in B. It follows that Dee does not act as she aspires in B. So, the warm-glow axiom must hold under warm-glow theory. Our main result now characterize the empirical content of a model of warm-glow by these two axioms.

Proposition 1. *Let $\mathcal{E}$ be an ordered aspiration function. A choice functions C is a warm-glow choice function (of 3.2) if and only if the limited WARP and the warm-glow axiom are satisfied.*

Proposition 1 demarcates the scope of a warm-glow model. It shows the choice functions that can and cannot be accommodated by the model. Any violation of the limited WARP or of the warm-glow axiom leads to a rejection of the 3.2 model of warm-glow. It follows that warm-glow theory has empirical content and, like the standard theory of choice, warm-glow theory can also be non-parametrically tested.

4.1. Comments on the Proof

The proof of proposition 1 takes a choice function C and an ordered aspiration function $\mathcal{E}$ that satisfies limited WARP and the warm-glow axiom. Then, it constructs a preference order R and a tolerance function τ that satisfies 3.3 such that 3.4 and 3.5 hold. We now provide an informal description of how R and τ are constructed.

If there is any issue B such that Dee's actual choice is not as she aspires then she must prefer her choice $C(B)$ to any other feasible alternative in B. Thus, those issues in which actual choice and expressed aspirations differ reveal some of Dee's preferences. However, there is an another way in which Dee's preferences may be revealed. Consider three different alternatives w, y and z. Assume that Dee expresses an aspiration of z over y, but she actually chooses y over z and w over y. Then, Dee must prefer w over z. The argument is as follows: Given that between y and z, aspiration and choice differ, it follows that Dee prefers y over z and z is too costly in the presence of y. Now assume, by contradiction, that Dee prefers z over w. Then, w is also too costly in the presence of y. But this contradicts the choice of w over y. So, these actual and expressed aspirations reveal that Dee must prefer w over z. Consider the binary relation given by Dee's revealed preferences. With the aid of the limited WARP, the warm-glow axiom (and the assumption of ordered aspirations), it is shown that this binary is acyclic and, hence, extended to an order R.

Take an alternative x and consider all other options y such that, for some issue, Dee expresses an aspiration for y, but instead chooses x. Any such alternative y

has been revealed to be too costly in the presence of x. The tolerance of x, $\tau(x)$, is defined as the option ranked (by R) just above these options that have been revealed too costly in the presence of x. With some analytical effort, it is possible to show that τ that satisfies 3.3 and, for the choice function C aspiration function $\mathcal{E}$, 3.4 and 3.5 hold for (R, τ).

4.2. Comments on the Axioms

The axioms in proposition 1 fully characterize the possible ways in which the structured model of warm-glow (given by 3.2 with ordered aspirations) can be rejected. Now let's consider less structured models. Consider first a model warm-glow given by 3.2, but without any assumptions on the aspiration functions. Then, as shown above, both the limited WARP and the warm-glow axiom must still hold. However, the converse is not true. In the appendix we show an example (example 1) of an non-ordered aspiration function and a choice function that is not a warm-glow choice function, but that satisfies the limited WARP and the warm-glow axiom. Hence, a violation of either the limited WARP or the warm-glow axiom leads to a rejection of the 3.2 warm-glow model regardless of the structure of the aspiration function. However, additional axioms are required to fully characterize the warm-glow model of 3.2 without provisos on aspiration functions.

Now consider the general model of warm-glow given by 3.6 (with dual utility functions and no assumptions on aspirations functions). A simple example in the appendix (example 2) shows that, for this model, the limited WARP axiom need not hold. The limited WARP must hold if Dee resolves any issue by a single preference order whenever she does not act as she aspires, but in the 3.6 general model of warm-glow, Dee may have several preferences order, one for which option that her aspirations directs her to. Hence, an empirical violation of the limited WARP rejects the warm-glow model of 3.2, but not the general model of 3.6.

On other hand, the warm-glow axiom must hold even for the general 3.6 model of warm-glow. Take two issues B and $\tilde{B}$. If Dee's aspirations are the same in both issues (i.e., $\mathcal{E}(B) = \mathcal{E}(\tilde{B})$), Dee does not act as she aspires in $\tilde{B}$ (i.e., $C(\tilde{B}) \neq \mathcal{E}(\tilde{B})$) and $C(\tilde{B})$ is feasible in B (i.e., $C(\tilde{B}) \in B$) then, by 3.6,

$$\begin{aligned} V(C(B), \mathcal{E}(B)) &\geq V(C(\tilde{B}), \mathcal{E}(B)) = V(C(\tilde{B}), \mathcal{E}(\tilde{B})) > \\ V(\mathcal{E}(\tilde{B}), \mathcal{E}(\tilde{B})) &= V(\mathcal{E}(B), \mathcal{E}(B)). \end{aligned}$$

Hence, $C(B) \neq \mathcal{E}(B)$. That is, Dee does not act as she aspires in issue B as well.

The warm-glow axiom is critical to warm-glow theory. It plays a central role in the empirical characterization of the 3.2 model of warm-glow with ordered aspiration functions. In addition, an empirical violation of it rejects the general 3.6 model of warm-glow. However, we do not have, at this moment, a good suggestion for an experiment that seems likely to produce an empirical violation of the warm-glow axiom

4.3. Directions for Future Work

In this paper, the empirical content of a structured model of warm-glow is fully characterized. However, it would be interesting to provide a similar characterization for several variations of this model. These related models include the warm-glow model of 3.2 with observed, but arbitrary aspirations functions; the warm-glow model of 3.2 with unobserved, but ordered, aspiration functions; the general 3.6 model of warm-glow (with ordered and arbitrary, but observed aspiration functions and with unobserved, but ordered, aspiration functions); and finally, intermediary models of warm-glow that are less structured then the 3.2 warm-glow model, but more structured then the general 3.6 model.

5. Conclusion

The warm-glow model of choice has been extensively used in the voting and provision of public goods literatures. The warm-glow model can make sense of phenomena such as high turnout in large elections and a series of comparative statics observations in voting that are difficult to reconcile with standard theory. The warm-glow model can also accommodate phenomena such as the relatively small crowding out effect of tax-funded charity programs on philanthropy and many other patterns of behavior in public goods provisions. However, the warm-glow theory of choice is a significant departure of the standard theory of choice and lacks the well-known foundations of the standard theory. This paper delivers some foundations to warm-glow models of choice. It is shown that warm-glow theory has empirical meaning, is falsifiable and can be non-parametrically tested like the standard theory of the choice. In particular, one axiom (the warm-glow axiom) is central to warm-glow theory and its empirical violation would lead to the rejection of warm-glow theory. This paper also delivers a complete characterization of a structured model of warm-glow and motivates a similar characterization for related models.

6. Appendix

6.1. Examples

Example 1. *There are three different alternatives* x, y *and* z. *The actual choice function* C *and aspiration function* $\mathcal{E}$ *are as follows :*

$$C(x,y,z) = x \text{ and } \mathcal{E}(x,y,z) = z;\; C(y,z) = z \text{ and } \mathcal{E}(y,z) = y;$$

$$C(x,z) = \mathcal{E}(x,z) = x \text{ and } C(x,y) = \mathcal{E}(x,y) = y.$$

Note that $\mathcal{E}$ is not ordered because $\mathcal{E}(x,y,z) = z$ and $\mathcal{E}(y,z) = y$. This is a violation of WARP for the expressed aspirations given by $\mathcal{E}$. The limited WARP holds because WARP is not violated by $C(x,y,z) = x$ and $C(y,z) = z$. The warm-glow axiom also holds because the only two issues leading to the same aspiration (y) are (y,z) and (x,y), but $C(y,z) = z \notin (x,y)$.

Assume, by contradiction, that, given $\mathcal{E}$, the choice function C is a warm-glow choice function. Then, the choices $C(x,y,z) = x$ and $C(y,z) = z$ must have been resolved by a single preference R. Hence, $x\ R\ z\ R\ y$. Then, $C(x,y,z) = x$ and $\mathcal{E}(x,y,z) = z$ imply that $\tau(x) = x$. This (and $x\ R\ y$) contradicts $C(x,y) = y$.

Example 2. *There are three different alternatives* x, y *and* z. *The actual choice function* C *and aspiration function* $\mathcal{E}$ *are as follows :*

$$C(x,y,z) = x \text{ and } \mathcal{E}(x,y,z) = z;\; C(x,y) = y \text{ and } \mathcal{E}(x,y) = x;$$

$$C(x,z) = x \text{ and } \mathcal{E}(x,z) = z;\, C(y,z) = \mathcal{E}(y,z) = z.$$

The aspiration function $\mathcal{E}$ is ordered ($z\ \ R^e\ y\ R^e\ x$) and given $\mathcal{E}$, the choice function C can be accommodated by the general warm-glow model of 3.6. The associated dual utility function V need only to satisfy $V(x,z) > V(z,z) > V(y,z)$ and $V(y,x) > V(x,x)$. However, given the aspiration function $\mathcal{E}$, the choice function C cannot be accommodated by the warm-glow model of 3.2. This follows because in actual choice and expressed aspiration differ in the issues (x,y,z) and (x,y), but the choices $C(x, y, z) = x$ and $C(x, y) = y$ violate WARP. Hence, limited WARP is violated.

6.2. Proof of the Preliminary Result

Let $\mathcal{C}$ be a warm-glow choice function. Let utility function u and scalar $D \geq 0$ be such that 3.2 holds. Let R be a preference order associated with u. For any given alternative $a \in A$, let $\tau(a) \in A$ be the lowest $R-$ranked alternative alternative such that $u(a) - D \leq u(\tau(a))$. So, $u(a) - D > u(b)$ for any alternative $b \in A$ such that $u(b) < u(\tau(a))$.

Now, $u(a) \geq u(\tau(a))$ because if $u(a) < u(\tau(a))$ then $u(a) - D > u(a)$. In addition, if $u(a') > u(a)$ then $u(\tau(a')) \geq u(\tau(a))$. Otherwise, $u(\tau(a')) < u(\tau(a))$ which would imply that $u(a) - D > u(\tau(a'))$ and so, $u(a') - D > u(\tau(a'))$. A contradiction. Hence, 3.3 holds.

Now assume that $u(\mathcal{E}(B)) \geq u(\tau(R(B)))$. Assume, by contradiction, that $\mathcal{C}(B) \neq \mathcal{E}(B)$. Then, $\mathcal{C}(B) = R(B)$ because otherwise, by the definition of $R(B)$, $u(\mathcal{C}(B)) < u(R(B))$ contradicting 3.2. Hence, by 3.2, $u(\mathcal{C}(B)) > u(\mathcal{E}(B)) + D > u(\tau(\mathcal{C}(B))) + D$. This contradicts 3.3. Thus, $\mathcal{C}(B) = \mathcal{E}(B)$. So, 3.4 holds.

Now assume that $u(\mathcal{E}(B)) < u(\tau(R(B)))$. Then, by definition of τ, $u(R(B)) - D > u(\mathcal{E}(B))$. Assume, by contradiction, that $\mathcal{C}(B) \neq R(B)$. Then, $\mathcal{C}(B) = \mathcal{E}(B)$ because otherwise $U^{\mathcal{E},B}(\mathcal{C}(B)) = u(\mathcal{C}(B)) < u(R(B)) \leq U^{\mathcal{E},B}(R(B))$ contradicting 3.2. Hence, $U^{\mathcal{E},B}(\mathcal{C}(B)) = u(\mathcal{C}(B)) + D = u(\mathcal{E}(B)) + D < u(R(B)) \leq U^{\mathcal{E},B}(R(B))$ contradicting 3.2. So, 3.5 holds.

Let $\mathcal{C}$ be a choice function that satisfies 3.4 and 3.5. Let $D = 1$. We now show that there exists a utility function u that is associated with preference R and such that 3.2 holds. The proof is by induction on the size of A. Assume that $\sharp(A) = 2$ so that A has only two elements. So, let $A = \{\mathcal{C}(A), b\}$, $b \neq \mathcal{C}(A)$. Consider first the case $\mathcal{C}(A)\ R\ b$. Then, by definition, $\mathcal{C}(A) = R(A)$. We define $u(\mathcal{C}(A)) = 0.5$ and $u(b) = 0$ if $\tau(\mathcal{C}(A)) = b$ and $u(\mathcal{C}(A)) = 2$ and $u(b) = 0$ if $\tau(\mathcal{C}(A)) = \mathcal{C}(A)$. By definition, u is associated with R and $u(a) < u(\tau(a)) + 1$ for $a \in A$ (note that, by 3.3, $\tau(b) = b$). In addition, 3.2 holds because if $\tau(\mathcal{C}(A)) = \mathcal{C}(A)$ then $U^{\mathcal{E},A}(\mathcal{C}(A)) \geq u(\mathcal{C}(A)) > u(b) + 1 \geq U^{\mathcal{E},A}(b)$. If $\tau(\mathcal{C}(A)) = b$ then $\mathcal{E}(A) = \mathcal{C}(A)$. Otherwise, $\mathcal{E}(A) = b = \tau(R(A))$ and, by 3.4, $\mathcal{C}(A) = \mathcal{E}(A)$. Thus, $U^{\mathcal{E},A}(\mathcal{C}(A)) = u(\mathcal{C}(A)) + 1 > u(b) = U^{\mathcal{E},A}(b)$.

Now consider the case $b\ R\ \mathcal{C}(A)$. Then, $\mathcal{E}(A) = \mathcal{C}(A)$. Otherwise, $\mathcal{E}(A) = R(A) = b$. Then, either $\mathcal{E}(A)\ \ R\ \ \tau(R(A))$ (when $\tau(b) = \mathcal{C}(A)$) or $\mathcal{E}(A) = \tau(R(A))$ (when

$\tau(b) = b$). By 3.4, $\mathcal{E}(A) = \mathcal{C}(A)$. Let $u(b) = 0.5$ and $u(\mathcal{C}(A)) = 0$. By definition, u is associated with R and $u(a) < u(\tau(a)) + 1$ for $a \in A$. In addition, 3.2 holds because $U^{\mathcal{E},A}(\mathcal{C}(A)) = u(\mathcal{C}(A)) + 1 > u(b) = U^{\mathcal{E},A}(b)$.

The induction assumption is that whenever when $\sharp(A) = n$ then there exists a utility function u associated with R such that $u(a) < u(\tau(a)) + 1$ for all $a \in A$ and 3.2 holds. Now assume that $\sharp(A) = n + 1$. Let $\bar{a} \in A$ be the highest $R-$ranked alternative. So, $\bar{a}\ R\ a$ for every $a \in A$ such that $a \neq \bar{a}$. Let $\tilde{A}$ be $A/\{\bar{a}\}$, i.e., $\tilde{A}$ is A without $\bar{a}$. So, $\sharp(\tilde{A}) = n$ and by the induction assumption there exists a utility function $\tilde{u} : \tilde{A} \longrightarrow \Re$ that, restricted to $\tilde{A}$, is associated with R, $\tilde{u}(a) < \tilde{u}(\tau(a)) + 1$ for all $a \in \tilde{A}$ and such that 3.2 holds for any issue $B \subseteq \tilde{A}$. If $\tau(\bar{a})$ is not $R-$ranked lowest then let $\hat{a} \in A$ be the lowest $R-$ranked option such that $\tau(\bar{a})\ R\ \hat{a}$. So, $\hat{a}\ R$ b for any alternative b such that $\tau(\bar{a})\ R\ b$. Let $\vec{a}$ be the second highest $R-$ranked alternative. So, $\vec{a}\ R\ a$ for every $a \notin \{\vec{a}, \bar{a}\}$.

Let $u : A \longrightarrow \Re$ be such that $u(a) = \tilde{u}(a)$ for any $a \in \tilde{A}$; $u(\bar{a}) \in (\max\{\tilde{u}(\vec{a}), \tilde{u}\,(\hat{a}) + 1\}, \tilde{u}(\tau(\bar{a})) + 1)$ if $\tau(\bar{a}) \neq \bar{a}$ and $\tau(\bar{a})$ is not $R-$ranked lowest); $u(\bar{a}) > \tilde{u}(\vec{a}) + 1$ if $\tau(\bar{a}) = \bar{a}$; and $u(\bar{a}) \in (\tilde{u}(\vec{a}), \tilde{u}(\tau(\bar{a})) + 1)$ if $\tau(\bar{a})$ is $R-$ranked lowest.

We first show that u is well-defined. If $\tau(\bar{a}) \neq \bar{a}$ then, by 3.3, $\tau(\vec{a}) = \tau(\bar{a}) \in \tilde{A}$. So, by induction assumption, $\tilde{u}(\vec{a}) < \tilde{u}(\tau(\bar{a})) + 1$. In addition, by definition, $\tau(\bar{a})\ R$ $\hat{a}$. So, by induction assumption, $\tilde{u}\,(\hat{a}) < \tilde{u}(\tau(\bar{a}))$. Hence, u is well-defined.

By definition, $u(\bar{a}) < u(\tau(\bar{a})) + 1$ and $u(\bar{a}) > u(\vec{a})$. So, by induction assumption, $u\,(a) < u(\tau(a)) + 1$ for all $a \in A$ and u is associated with R.

We now show that 3.2 holds. Let B be an issue such that $\bar{a} \in B$. So, by definition, $R(B) = \bar{a}$. Assume that $\mathcal{E}(B)\ \ R\ \ \tau(\bar{a})$ or $\mathcal{E}(B) = \tau(\bar{a})$. Then, by 3.4, $C(B) = \mathcal{E}(B)$. It follows that $U^{\mathcal{E},B}(\mathcal{C}(B)) = u(\mathcal{C}(B)) + 1 \geq u(\tau(\bar{a})) + 1 > u(\bar{a}) \geq u(x) = U^{\mathcal{E},B}(x)$ for every $x \in B$, $x \neq \mathcal{C}(B)$. Now assume that $\tau(\bar{a})\ \ R\ \ \ \mathcal{E}(B)$. Then, by 3.5, $C(B) = R(B) = \bar{a}$ (and $\tau(\bar{a})$ is not $R-$ranked lowest). If $\tau(\bar{a}) \neq \bar{a}$ then $U^{\mathcal{E},B}(\mathcal{C}(B)) = u(\bar{a}) \geq u\,(\hat{a}) + 1 \geq u(\mathcal{E}(B)) + 1 = U^{\mathcal{E},B}(\mathcal{E}(B))$. In addition, $U^{\mathcal{E},B}(\mathcal{C}(B)) = u(\bar{a}) \geq u\,(x) = U^{\mathcal{E},B}(x)$ for all $x \notin \{\mathcal{E}(B), \bar{a}\}$.■

6.3. Proof of the Proposition 1

The proof that Limited WARP and the warm-glow axiom are satisfied under warm-glow theory is immediate and made in the body of the paper. So, assume that a choice function C is such that the Limited WARP and the warm-glow axiom are satisfied.

Let R^e be a preference order such that for any $B \in \mathcal{B}$,

$$\mathcal{E}(B) = R^e(B).$$

Step 0. If $B \in \mathcal{B}^s$, $B \subset \tilde{B}$ and $R^e\left(\tilde{B}\right) \in B$ then $\tilde{B} \in \mathcal{B}^s$.

Given that $B \subset \tilde{B}$ and $R^e\left(\tilde{B}\right) \in B$ it follows that $R^e\left(\tilde{B}\right) = R^e(B)$. In addition, $C(B) \in B \subset \tilde{B}$. So, $C(B) \in \tilde{B}$. The conclusion now follows from Limited WARP.■

Corollary to Step 0. If $B_1 \in \mathcal{B}^s$, and $B_2 \in \mathcal{B}^s$ then $B_1 \bigcup B_2 \in \mathcal{B}^s$.

Let $B = B_1 \bigcup B_2$. Then, either $R^e(B) \in B_1$ or $R^e(B_2) \in B_2$. In either case, the conclusion follows from step 0.■

Given two different alternatives $w \in A$ and $z \in A$, we define the binary relation $\succ$ as follows : $w \succ z$ if and only if for some $B \in \mathcal{B}^s$,

$$w = C(B) \text{ and } z \in B. \tag{6.1}$$

Lemma 1. *The binary relation $\succ$ is asymmetric and transitive.*

Assume, by contradiction, that $x \neq y$, $x \succ y$ and $y \succ x$. Then, there exists $B_1 \in \mathcal{B}^s$ and $B_2 \in \mathcal{B}^s$ such that $\{x, y\} \subseteq B_1$, $\{x, y\} \subseteq B_2$, $x = C(B_1)$ and $y = C(B_2)$. By the corollary to step 0, $B = B_1 \bigcup B_2 \in \mathcal{B}^s$. So, either $C(B) \in B_1$ or $C(B) \in B_2$. Assume that $C(B) \in B_1$. Then, by Limited WARP, $C(B) = x$. So, $C(B) \in \{x, y\} \subseteq B_2$. Hence, by Limited WARP, $C(B) = C(B_2) = y$. A contradiction. The proof for the case $C(B) \in B_2$ is analogous.

Now assume that $x \succ y$ and $y \succ z$. Then, there exists $B_1 \in \mathcal{B}^s$ and $B_2 \in \mathcal{B}^s$ such that $\{x, y\} \subseteq B_1$, $\{y, z\} \subseteq B_2$, $x = C(B_1)$ and $y = C(B_2)$. By the corollary to step 0, $B = B_1 \bigcup B_2 \in \mathcal{B}^s$. So, either $C(B) \in B_1$ or $C(B) \in B_2$. If $C(B) \in B_2$ then, by Limited WARP, $C(B) = C(B_2) = y$. So, $y \succ x$ (because $x \in B$). This contradicts $x \succ y$ and the asymmetry of $\succ$. Hence, $C(B) \in B_1$. By Limited WARP, $C(B) = x$. So, $x \succ z$ (because $z \in B$).■

We define the binary relation $\succ^*$ as follows : Given two different alternatives w and z,

$$w \succ^* z$$

if there exists an alternative $y \notin \{w, z\}$ such that

$$R^e(y, z) = z,\ C(y, z) = y,\ R^e(y, w) = w,\ C(y, w) = w. \tag{6.2}$$

Lemma 2. *The binary relation $\succ^*$ is acyclic.*

Step 1. The binary relation $\succ^*$ is asymmetric.

Assume, by contradiction, that $x \succ^* j$ and $j \succ^* x$. Then, there are alternatives y_1 and y_2 such that

$$R^e(y_1, j) = j,\ C(y_1, j) = y_1,\ R^e(y_1, x) = x,\ C(y_1, x) = x$$

and

$$R^e(y_2, x) = x,\ C(y_2, j) = y_2,\ R^e(y_2, j) = j,\ C(y_2, j) = j.$$

So, $\{y_1, j\} \in \mathcal{B}^s$, $\{y_2, x\} \in \mathcal{B}^s$, $j \ R^e\ y_1$, $j \ R^e\ y_2$, $x \ R^e\ y_1$, $x \ R^e\ y_2$. In particular, $R^e(y_1, y_2, j) = j \in \{y_1, j\}$ and $\{y_1, j\} \in \mathcal{B}^s$. By step 0, $\{y_1, y_2, j\} \in \mathcal{B}^s$. If $C(y_1, y_2, j) = y_2$ then, by the warm-glow axiom, $\{y_2, j\} \in \mathcal{B}^s$. This contradicts $R^e(y_2, j) = C(y_2, j) = j$. So, $C(y_1, y_2, j) = y_1$. Thus, $y_1 \succ y_2$.

Analogously, $R^e\{y_1, y_2, x\} = x \in \{y_2, x\}$ and $\{y_2, x\} \in \mathcal{B}^s$. By step 0, $\{y_1, y_2, x\} \in \mathcal{B}^s$. Thus, $C(y_1, y_2, x) = y_1$ (otherwise $y_2 \succ y_1$). By the warm-glow axiom, $(y_1, x) \in \mathcal{B}^s$. This contradicts $R^e(y_1, x) = x$, $C(y_1, x) = x$.■

Step 2. If $x \succ^* k$, $k \succ^* j$ and $j \ R^e\ k$ then $x \succ^* j$.

By definition, there are alternatives y_1 and y_2 such that

$$R^e(y_1, k) = k,\ C(y_1, k) = y_1,\ R^e(y_1, x) = x,\ C(y_1, x) = x,$$

and

$$R^e(y_2, j) = j,\ C(y_2, j) = y_2,\ R^e(y_2, k) = k,\ C(y_2, k) = k.$$

So, $\{y_1, k\} \in \mathcal{B}^s$, $\{y_2, j\} \in \mathcal{B}^s$, $j \ R^e\ y_2$, $k \ R^e\ y_2$, $k \ R^e\ y_1$. Now, given that $j \ R^e\ k$ it follows that $j \ R^e\ y_1$.

So, $R^e(y_1, y_2, k) = k \in \{y_1, k\}$ and $\{y_1, k\} \in \mathcal{B}^s$. By step 0, $\{y_1, y_2, k\} \in \mathcal{B}^s$. If $C(y_1, y_2, k) = y_2$ then, by the warm-glow axiom, $\{y_2, k\} \in \mathcal{B}^s$. This contradicts $R^e(y_2, k) = C(y_2, k) = k$. So, $C(y_1, y_2, k) = y_1$. Thus, $y_1 \succ y_2$.

Now $R^e(y_1, y_2, j) = j \in \{y_2, j\}$ and $\{y_2, j\} \in \mathcal{B}^s$. By step 0, $\{y_1, y_2, j\} \in \mathcal{B}^s$. If $C(y_1, y_2, j) = y_2$ then $y_2 \succ y_1$ contradicting $y_1 \succ y_2$. So, $C(y_1, y_2, j) = y_1$. Thus, by the warm-glow axiom, $(y_1, j) \in \mathcal{B}^s$. In addition, $R^e(y_1, j) = j$. So, $C(y_1, j) = y_1$. Thus,

$$R^e(y_1, j) = j,\ C(y_1, j) = y_1,\ R^e(y_1, x) = x,\ C(y_1, x) = x$$

So, $x \ \succ^* j$.■

Proof of Lemma 2. Step 1 shows that there are no cycles with 2 alternatives. Assume, by induction, that there are no cycles with $n-1$ (or less) alternatives. Also assume, by contradiction, that $\{x_1, ..., x_n\}$ is a $n-$cycle. So, $x_i \ \succ^* x_{i+1}$, $i = 1, ..., n-1$, and $x_n \ \succ^* x_1$.

If $x_1 \ \ R^e \ x_2$ then, by step 2, $x_n \ \succ^* x_2$. If $x_n \ \ R^e \ x_1$ then, by step 2, $x_{n-1} \ \succ^* x_1$. If $x_i \ \ R^e \ x_{i+1}$ $i = 2, ..., n-1$ then, by step 2, $x_{i-1} \ \succ^* x_{i+1}$. Any of these cases produce a cycle with at most $n-1$ alternatives. This violates the induction hypothesis. Hence, $x_1 \ \ R^e \ x_n$, and $x_{i+1} R^e \ x_i$, $i = 1, ..., n-1$. Therefore, R^e is cyclic. A contradiction.■

We define the binary relation $\succ^s$ as follows : Given two different alternatives w and z,

$$w \ \succ^s z$$

if either $w \ \succ^* z$ or $w \ \succ z$ (or both).

Lemma 3. *The binary relation $\succ^s$ is acyclic.*

Step 1. If $x \ \succ^* k$, $k \ \succ j$ and $j \ \ R^e \ k$ then $x \ \succ^* j$.

By definition there is an alternative y_1 such that

$$R^e (y_1, k) = k, \ C(y_1, k) = y_1, \ R^e (y_1, x) = x, \ C(y_1, x) = x.$$

So, $\{y_1, k\} \in \mathcal{B}^s$, $y_1 \succ k$ and $k \ \ R^e \ y_1$. Now, given that $j \ \ R^e \ k$ then $j \ \ R^e \ y_1$.

Now $R^e(y_1, k, j) = j \in \{y_1, k\}$ and $\{y_1, k\} \in \mathcal{B}^s$. By step 0, $\{y_1, k, j\} \in \mathcal{B}^s$. If $C(y_1, k, j) = k$ then $k \succ y_1$ contradicting $y_1 \succ k$. So, $C(y_1, y_2, j) = y_1$. Thus, by the warm-glow axiom, $(y_1, j) \in \mathcal{B}^s$. In addition, $R^e(y_1, j) = j$. So, $C(y_1, j) = y_1$. Thus,

$$R^e(y_1, j) = j, \ C(y_1, j) = y_1, \ R^e (y_1, x) = x, \ C(y_1, x) = x$$

So, $x \ \succ^* j$.■

Step 2. If $k \ \succ^* j$ and $j \ \ R^e \ k$ then $k \ \succ j$.

By definition there is an alternative y_2 such that

$$R^e (y_2, j) = j, \ C(y_2, j) = y_2, \ R^e (y_2, k) = k, \ C(y_2, k) = k.$$

So, $\{y_2, j\} \in \mathcal{B}^s$, $y_2 \succ j$ and $j \ \ R^e \ y_2$. Now, given that $j \ \ R^e \ k$ then $R^e(y_2, k, j) = j \in \{y_2, j\}$ and $\{y_2, j\} \in \mathcal{B}^s$. By step 0, $\{y_2, k, j\} \in \mathcal{B}^s$. By the warm-glow axiom,

$(k,j) \in \mathcal{B}^s$. In addition, $R^e(k,j) = j$. So, $C(k,j) = j$. Thus, $k \succ j$.■

Corollary of step 2. If $x \succ k$, $k \succ^* j$ and j R^e k then $x \succ j$.

Step 3. The binary relation $\succ^s$ is asymmetric.

Assume, by contradiction, that $w \succ^s y$ and $w \succ^s y$. Then, there are four cases to consider. But if $w \succ y$ and $y \succ w$ or if $w \succ^* y$ and $y \succ^* w$ then a contradiction is immediately obtained given that $\succ$ and $\succ^*$are asymmetric. Assume that $w \succ y$ and $y \succ^* w$. If w R^e y then, by step 2, $y \succ w$. This contradicts $w \succ y$. If y R^e w then, by step 1, $y \succ^* y$. A contradiction. The case $w \succ^* y$ and $y \succ w$ is analogous.■

Step 4. If $x \succ^s k$, $k \succ^s j$ and j R^e k then $x \succ^s j$.

Step 4 follows directly from step 1, the corollary of step 2, step 2 in the proof of Lemma 2 and the transitivity of $\succ$.■

Proof of Lemma 3. Step 3 shows that there are no cycles with 2 alternatives. Assume, by induction, that there are no cycles with $n-1$ (or less) alternatives. Also assume, by contradiction, that $\{x_1, ..., x_n\}$ is a $n-$cycle. So, $x_i \succ^s x_{i+1}$, $i = 1, ..., n-1$, and $x_n \succ^s x_1$.

If x_1 R^e x_2 then, by step 4, x_n R^e x_2. If x_n R^e x_1 then, by step 4, x_{n-1} R^e x_1. If x_i R^e x_{i+1} $i = 2, ..., n-1$ then, by step 4, x_{i-1} R^e x_{i+1}. Any of these cases produce a cycle with at most $n-1$ alternatives. This violates the induction hypothesis. Hence, x_1 R^e x_n, and x_{i+1} R^e x_i, $i = 1, ..., n-1$. Therefore, R^e is cyclic. A contradiction.■

By topological ordering, an acyclical binary relation may be extended (not necessarily uniquely) to an order (see Cormen et al. (2001, pp.549–552)). Let R be any extension of $\succ^s$. So, R is a preference order such that if

$$w \succ^s y \Longrightarrow w \; R \; y.$$

Given $x \in A$, let $\mathcal{D}^x$ be the set of all alternatives $z \neq x$ such that for some issue $B \in \mathcal{B}$,

$$x = C(B) \text{ and } z = R^e(B). \tag{6.3}$$

That is, $\mathcal{D}^x$ are the alternatives $z \neq x$ such that $x \succ z$.

Let $d(x) \in \mathcal{D}^x$ be the element such that $d(x)$ R z for any $z \in \mathcal{D}^x$, $d(x) \neq z$. If $\mathcal{D}^x$ is empty then $d(x)$ is not defined. Let $\mathcal{L}^x$ be the set of all alternatives $d(y)$ where y is either x or any alternative such that x R y. Let $\tau(x)$ be the alternative such that $\tau(x)$ R z for any $z \in \mathcal{L}^x$, and if $w \neq \tau(x)$ is such that w R z for any $z \in \mathcal{L}^x$ then w R $\tau(x)$. If $\mathcal{L}^x$ is empty then $\tau(x)$ is such that a R $\tau(x)$ for every $a \in A$, $a \neq \tau(x)$.

We have now defined R^e, R, and τ. We now show that (R, τ) is a warm-glow model that produce choice functions C.

Proof of proposition 1 (Conclusion).

Step $\bar{0}$. If $B \in \mathcal{B}^s$ then $R(B) = C(B)$. If $B \notin \mathcal{B}^s$ then $R^e(B) = C(B)$. So, $C(B) \in \{ R(B), R^e(B)\}$.

Step $\bar{0}$ follows immediately from 6.1.■

Step 1. If $z \in \mathcal{D}^x$ then $R^e(x,z) = z$, $C(x,z) = x$ and $x \succ z$.

By definition, there exists some issue $B \in \mathcal{B}^s$ such that $x = C(B)$ and $z = R^e(B)$. So, 6.1, $x \succ z$. Let $\tilde{B} = \{x, z\}$. So, $C(B) \in \tilde{B}$ and $\tilde{B} \subseteq B \Rightarrow R^e(B) = R^e(\tilde{B})$. By the warm-glow axiom, $\tilde{B} \in \mathcal{B}^s$. Hence, $C(\tilde{B}) \neq z = R^e(\tilde{B}) \Rightarrow C(\tilde{B}) = x$.■

Step 2. For any $x \in A$, if $\mathcal{D}^x \neq \varnothing$ then $x \succ d(x)$.

Step 2 follows because by 6.1, $x \succ z$ for any $z \in \mathcal{D}^x$.■

Step 3. For any $x \in A$, $x\ R\ d(y)$ for any alternative y such that either $y = x$ and $\mathcal{D}^x \neq \varnothing$ or $\mathcal{D}^y \neq \varnothing$ and $x\ R\ y$.

From step 2, if $y = x$ and $\mathcal{D}^x \neq \varnothing$ then $x \succ d(x) \Rightarrow x\ R\ d(x)$. If $\mathcal{D}^y \neq \varnothing$ and $x\ R\ y$ then $y\ R\ d(y)$. The conclusion now follows from the transitivity of R.■

Step 4. Given three different alternatives x, y, and z. If $z \in \mathcal{D}^y$, $x\ R\ y$ and $z\ R^e\ x$ then $z \in \mathcal{D}^x$.

It follows from $z \in \mathcal{D}^y$ that $z\ R^e\ y$. Let $\hat{B} = \{x, y, z\}$. Then, $R^e(\hat{B}) = z$. Let $B = \{y, z\}$. By definition, $B \in \mathcal{B}^s$, $B \subseteq \hat{B}$ and $R^e(B) = R^e(\hat{B})$. So, by the warm-glow axiom, $\hat{B} \in \mathcal{B}^s$. Thus, $C(\hat{B}) \neq z$. Assume that $C(\hat{B}) = y$. Then, $y \succ x$. This contradicts $x\ R\ y$. So, $C(\hat{B}) = x$.■

Step 5. For any $B \in \mathcal{B}$, if $R^e(B)\ R\ d(R(B))$ then $C(B) = R^e(B)$.

Assume, by contradiction, that $R^e(B) \neq C(B)$. Then, $B \in \mathcal{B}^s$ and, by step $\bar{0}$, $C(B) = R(B)$. Let $x = C(B)$ and $z = R^e(B)$. By step $\bar{0}$, $z \in \mathcal{D}^x$. If $R^e(B) \neq d(R(B))$ then $z \neq d(x)$. So, $z \in \mathcal{D}^x$ and $d(x) \neq z$. Hence, $d(x)\ R\ z$. If $R^e(B)\ R\ d(R(B))$ then $z\ R\ d(x)$. It now follows that $d(x) \neq z$, $d(x)\ R\ z$ and $z\ R\ d(x)$. This contradicts the asymmetry of R.■

Step 6. For any $B \in \mathcal{B}$, if $d(R(B))\ R\ R^e(B)$ and $R^e(B) \neq d(R(B))$ then $C(B) = R(B)$.

Let $x = R(B)$, $\eta = R^e(B)$ and $\xi = d(x)$. We can assume, without loss of generality, that $x \neq \eta$ (Otherwise, by step 0, $C(B) = R(B) = R^e(B)$). By definition, $d(x) \in \mathcal{D}^x$ and so, $\xi \neq x$. From $x \in B$ it follows that $R^e(x, \eta) = \eta$. Now either $C(x, \eta) = x$ or $C(x, \eta) = \eta$. Assume that $C(x, \eta) = \eta$. By definition, $\xi \in \mathcal{D}^x$. By step 1, $R^e(x, \xi) = \xi$,

$C(x,\xi)=x$. So, $R^e(x,\xi)=\xi, C(x,\xi)=x$, $R^e(x,\eta)=\eta$, $C(x,\eta)=\eta$. By 6.2, $\eta \succ^s \xi$. This contradicts $\xi\ R\ \eta$. So, $C(x,\eta)=x$. Then, $\{x,\eta\}\in\mathcal{B}^s$. Moreover, $R^e(\{x,\eta\})=R^e(B)=\eta$ and $\{x,\eta\}\subseteq B$. By step 0, $B\in\mathcal{B}^s$. By step $\bar{0}$, $C(B)=R(B)$.■

Step 6B. For any $B\in\mathcal{B}$, if $R^e(B)=d(R(B))$ then $C(B)=R(B)$.

Let $x=R(B)$ and $y=d(R(B))$. Let $\hat{B}=\{x,y\}$. By definition, $y\in\mathcal{D}^x$. So, $\hat{B}\in\mathcal{B}$. In addition, by step 1, $C(\hat{B})=x\in B$ and $R^e(B)=R^e(\hat{B})=y$. So, $B\in\mathcal{B}^s$. By step $\bar{0}$, $C(B)=R(B)$.■

Step 7. The limit function $\tau : A\longrightarrow A$ (as defined in this proof) satisfies 3.3.

Consider an element $x\in A$. By definition, if $z\in\mathcal{L}^x$ then $z=d(y)$ for some alternative y that is either x (and so, $\mathcal{D}^x\neq\varnothing$) or such that $x\ R\ y$ (and so, $\mathcal{D}^y\neq\varnothing$). So, by step 3, if $z\in\mathcal{L}^x$ then $x\ R\ z$. So, if $z=\tau(x)$ then $x\ R\ \tau(x)$. If $z\neq\tau(x)$ then, by definition, $x\ R\ \tau(x)$.

Now assume that $a'\ R\ a$. Then, $\mathcal{L}^a\subseteq\mathcal{L}^{a'}$. This follows because if $z\in\mathcal{L}^a$ then $z=d(y)$ for some y that is either a or such that $a\ R\ y$. By the transitivity of R, $a'\ R\ y$. So, $z\in\mathcal{L}^{a'}$. By definition, $\tau(a')\ R\ z$ for any $z\in\mathcal{L}^{a'}$. So, $\tau(a')\ R\ z$ for any $z\in\mathcal{L}^a$. Thus, if $\tau(a')\neq\tau(a)$ then, by definition, $\tau(a')\ R\ \tau(a)$.■

Step 8. If $R^e(B)\ R\ \tau(R(B))$ or if $R^e(B)=\tau(R(B))$ for some issue $B\in\mathcal{B}$ then $C(B)=R^e(B)$.

Assume that $R^e(B)\ R\ \tau(R(B))$. Then, by definition, for any issue $B\in\mathcal{B}$, either $\tau(R(B))\ R\ d(R(B))$ or $\tau(R(B))=d(R(B))$. The conclusion now follows from the transitivity of R and step 5.

Now assume that $R^e(B)=\tau(R(B))$. Also, assume, by contradiction, that $B\in\mathcal{B}^s$. Let $x=R(B)$ and $y=\tau(R(B))$. If $x=y$ then $R(B)=R^e(B)$. So, by step 0, $C(B)=R^e(B)$. Now assume that $x\neq y$. Let $\hat{B}=\{x,y\}$. By step 0, $C(B)\in\{x,y\}$. In addition, given that $x\in B$, $y=R^e(B)=R^e(\hat{B})$. So, by the warm-glow axiom, $\hat{B}\in\mathcal{B}^s$. It follows that $y\in\mathcal{D}^x$. So, $\tau(x)\ R\ \tau(x)$. A contradiction.■

Step 9. If $\tau(R(B))\ R\ R^e(B)$ for some issue $B\in\mathcal{B}$ then $C(B)=R(B)$.

Let $z=R^e(B)$, and $x=R(B)$. We can assume that $\mathcal{L}^x$ is not empty. Otherwise, $\tau(R(B))\ R\ R^e(B)$ cannot hold. It follows from $\tau(R(B))\ R\ R^e(B)$ and $\mathcal{L}^x\neq\varnothing$ that either $\tau(R(B))=d(x)$ or $d(x)\ R\ R^e(B)$ or there exists an alternative y such that $x\ R\ y$ and either $d(y)=R^e(B)$ or $d(y)\ R\ R^e(B)$ (if $d(x)$ does nor exit then, because $\mathcal{L}^x\neq\varnothing$, there must exist the alternative y with the stated properties). If either $d(x)\ R\ R^e(B)$ or $\tau(R(B))=d(x)$ then the conclusion follows from steps 6 and 6B.

So, we can assume, without loss of generality, that there exists an alternative

y such that $x\ R\ y$ and either $d(y) = R^e(B)$ or $d(y)\ R\ R^e(B)$. Consider the case $d(y) = R^e(B)$. Then, $z \in \mathcal{D}^y$, $x\ R\ y$ and $z\ R^e\ x$. By step 4, $z \in \mathcal{D}^x$. The conclusion now follows from steps 6 and $6B$.

So, we can assume, without loss of generality, that exists an alternative y such that $x\ R\ y$ and $d(y)\ R\ R^e(B)$. Now, either $x\ R^e\ d(y)$ or $d(y)\ R^e\ x$. Let's consider first the case $x\ R^e\ d(y)$. Then $z\ R^e\ d(y)$ (because $z\ R^e\ x$). So, $z\ R^e\ y$ (because $d(y)\ R^e\ y$). Now, it follows from 6.2 that $C(y, z) = y$. To see this assume, by contradiction, that $C(y, z) = z$. But, $R^e(z, y) = z$ and, by step 1, $C(y, d(y)) = y$, $R^e(y, d(y)) = d(y)$. So, by 6.2, $d(y) \succ^s z$. This contradicts $d(y)\ R\ z$. Now from $C(y, z) = y$ and $R^e(z, y) = z$ it follows that $z \in \mathcal{D}^y$. Then, as above, $z \in \mathcal{D}^y$, $x\ R\ y$ and $z\ R^e\ x$. By step 4, $z \in \mathcal{D}^x$. The conclusion now follows from steps 6 and $6B$.

Now consider the remaining case in which $d(y)\ R^e\ x$. Let $B = \{y, d(y)\}$ and $\hat{B} = \{x, y, d(y)\}$. By definition, $B \in \mathcal{B}^s$. Because $d(y)\ R^e\ y$ it follows that $R^e(B) = R^e(\hat{B})$. This, combined with $B \subseteq \hat{B}$, implies, by the warm glow axiom, that $\hat{B} \in \mathcal{B}^s$. So, $C(\hat{B}) \neq d(y)$. Assume, by contradiction, that $C(\hat{B}) = y$. Then, $y \succ x$. This contradicts $x\ R\ y$. So, $C(\hat{B}) = x$. Hence, $d(y) \in \mathcal{D}^x$. It now follows that either $d(x) = d(y)$ or $d(x)\ R\ d(y)$. But, $d(y)\ R\ z$. So, $d(x)\ R\ z$. The conclusion now follows from step 6.■

The proof of Proposition 1 now follows directly from steps 7, 8, 9 and the preliminary result.■

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