Excess Liquidity against Predation

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December 2, 2009

Abstract

In this paper, first we show that the cash-in-advance constraint on the entrant creates threat of predation and endogenous demand for excess precautionary liquidity. Further, we prove that when the incumbent's strategy is unverifiable, the entrant with small start-up internal capital and less valuable asset cannot raise adequate level of precautionary liquidity; so he shrinks his business so as to avoid the entrant's predation (complete exclusion from the market). While we induce this result by presuming the truth-telling (quasi-)direct mechanism on the loan contract, we generalize it by proving the revelation principle for a sequential equilibrium. This means we select the equilibrium and contract by imposing robustness to strategic uncertainty. After discussing the structural assumptions in detail, we make some suggestions for policy makers to make anti-predatory market environment.

Keywords: Predation, Excess liquidity, Revelation Principle, Sequential Equilibrium, Limited-liability Constraint

*E-mail: `zusai@wisc.edu`. I gratefully acknowledge Noriyuki Yanagawa, Toshihiro Matsumura, Munetomo Ando, Daisuke Oyama, Ikuo Ishibashi, and William H. Sandholm. I also thank to the seminar participants at University of Tokyo, Hitotsubashi, Kyoto, Japan Fair Trade Committee, Japan Economic Association, Hakone Microeconomic Conference, and the graduate seminar in University of Wisconsin-Madison for helpful comments and discussions. Nevertheless, all errors are of course mine.

> The firm keeps a portion of its capital in the form of liquid assets to provide a reserve for unforeseen contingencies. These liquid assets together with the credit lines readily available to the firm play a key role in the analysis of predatory pricing. (Telser, 1966, pp. 261–2)

1 Introduction

It has been long argued in antitrust policy and industrial economics whether predation by a "long-purse" incumbent to a low-capitalized entrant distorts some economic outcome in equilibrium. A negative answer is suggested by Telser (1966) . He emphasizes the role of the entrant's precautionary liquidity as a barrier to predation. In his reasoning, the entrant's liquidity, which enables the entrant to endure predation, raises the incumbent's cost of predation, and restrains the incumbent from predation. Telser concludes that predation does not evolve in equilibrium if firms are rational. However, he leaves out the question of whether the entrant can attain sufficient liquidity to avoid predation. That is, he puts aside the difficulty in financing liquidity in entry. Among empirical studies, Lerner (1995) observes in 1980s disk drive industry that the slump in capital market triggers predatory price cut against entrants with small internal capital. So we can expect a link between difficulty in financing and vulnerability to predation.

We show that, when the rival's strategy is unverifiable, the low-capitalized entrant shrinks his capacity under threat of predation, because he faces financial difficulty in raising precautionary liquidity. To our knowledge, all literature assumes ad hoc threat of predation and considers the entrant who has to borrow a loan to pay some fixed cost for production, not to avoid the predation. In our model, the threat of predation and the need for liquidity sustain each other: the entrant needs precautionary liquidity to avoid predation at the time of entry, though he can borrow loan for production cost just before production. Our concept of predation means just the incumbent's off-path too aggressive strategy to exclude the entrant from the market,[1] which is not an 'empty threat' and affects the equilibrium capacity decision. It has effects because the threat limits the entrant's ability to raise precautionary liquidity under unverifiable predation.

Sketch of Our Model

We see how the threat of predation affects equilibrium outcome in the product market where the entrant and the incumbent make some precommitment, say 'capacity' investment. The incumbent can prey on the entrant by *excess predatory capacity*, which cuts the future profit and drives out the entrant who does not have sufficient liquidity to pay his capacity cost. The entrant borrows precautionary liquidity to protect himself from the predation. The entrant's liquidity holding reduces the incumbent's net benefit of predation, because he has to expand capacity further more so as to succeed in the predation and this eventually reduces his own profit if predation succeeds. Hence the entrant holds *excess precautionary liquidity*, which is kept in the entrant's safe so as to restrain the incumbent from the predation, and is *never spent* in equilibrium. If the states of the world is wholly verifiable, the entrant can always borrow enough external loan and the threat of predation does not affect equilibrium outcome in the product market (Proposition 1).

[1] In our model, the incumbent preys on the entrant to monopolize the current market (forcing the entrant to quit the current production), not the future (forcing him to go bankrupt after the current product is sold out. Bolton and Scharfstein (1990) and Poitevin (1989) consider the latter type of predation, assuming the exogenous predatory benefit. In our model, the incumbent is interested in the amount of the entrant's total liquidity holding, not the amount of loans.

Nevertheless, if the rival's capacity and thus the entrant's profit are unverifiable, the threat of predation brings to the entrant difficulty in financing precautionary liquidity. Under the unverifiability of the rival's capacity, it is unverifiable whether the entrant indeed suffers predatory loss, even though we assume that the demand and cost structure is verifiable and involves no uncertainty. Because of the entrant's limited liability, the repayment of the loan must be reduced in the case where the entrant suffers operating loss. Under the unverifiability of the entrant's profit, the entrant is willing to pretend to suffer operating loss, to avoid repayment.[2]

Therefore the financial contract must induce the entrant's truth telling about the entrant's profit. We prove the Revelation Principle (Theorem 2), focusing on sequential equilibria of the subgame after the loan contract is written. Any of these equilibria is reduced to equilibrium using a "quasi"-direct mechanism, where the entrant announces the message of 'exit' or the rival's capacity (i.e. the unverified information directly). Then, we impose the limited-liability constraint: the entrant's liquidity holding cannot be negative in the end of the game, provided that his announcement is taken as its face value. Given this, the entrant might pretend to suffer predation by the incumbent's excess capacity; a valid contract still has to give the entrant an incentive to tell the truth.

In conflict between the entrant's incentive compatibility on the announcement and the lender's participation condition, the feasibility of the incentive compatible contract is constrained by the amount of internal capital and the private value of the entrant's business. The incentive compatibility does matter, because the repayment is reduced up to the limited liability when the entrant announces predatory loss. As a result, under the unverifiability of the incumbent's predation, the threat of predation restricts the low-capitalized entrant in financing excess liquidity against predation (Proposition 2).

The low-capitalized entrant's capacity shrinks in equilibrium due to this financial difficulty (Corollary 1). The incumbent's net benefit of predation (the monopoly profit after predation succeeds minus the duopoly one) increases as the entrant's capacity becomes larger. As the entrant sets larger capacity level, the incumbent's profit decreases as long as the entrant stays in the market, while the entrant's capacity does not affect the incumbent's profit once the entrant exits. As the incumbent has more incentive of predation, the entrant needs more excess liquidity to avoid predation. Hence the low-capitalized entrant must shrink his capacity so as to reduce the incumbent's incentive for predation and the requirement of excess liquidity to avoid predation.

Review of Literature on Financial Theory of Predation

To our knowledge, this is the first paper that considers endogenously the incumbent's predation and the entrant's financial difficulty: all previous literature assumes either that the long-purse incumbent can prey on the entrant with some exogenous devices, or that the entrant faces need to borrow money for some exogenous reasons.

Bolton and Scharfstein (1990) is the seminal paper that investigates the optimal financial contract. Their model has two production periods. The financial contract uses threat of liquidation at the end of the first period to force the borrower to truthfully report his productivity. This threat of liquidation in turn invites the rival's predation in the first period. Thus the lender increases the threat of liquidation so as to reduce the incumbent's

[2]The entrant's intention to hide his profit is serious in staging finance by venture capital, though we do not put it into our model. First, the entrant would lie that bad sales on the entry is the result of temporal predation, not of his poor fundamentals. Besides, he would preserve his profit for investment of other business or for cases where the venture capital quits financing him.

net benefit of predation. They focus on the decision whether to proceed with the second-period production, and the effect of predation on capacity levels is not considered.

Furthermore Bolton and Scharfstein (1990) emphasize a different aspect of the contracting problem than we do: they focus on the probability of liquidation in the truth-telling financial contract, while we see whether or not an adequate amount of loan is available with a valid contract for a low-capitalized entrant. Besides in our model the variety of the possible states comes endogenously from predation itself, not some exogenous shock. So we find the origin of the distortion due to predation in "endogenous incompleteness" of contracts (Tirole, 1999, p.763).

We respect Bolton and Scharfstein (1990) in that it clarifies threat of predation caused by imperfect finance in a simple structure as is typical in contract theory, which invites some extension.[3] But this simplicity is criticized in practice and could be the reason that the 'strategic theory' of predation has not yet affected the antitrust court (Elzinga and Mills, 2001). In particular, the entrant has no counter-strategy against predation other than increasing probability to liquidate (exiting the market), which is because they simplify the product market and takes as given the impact of predation on the entrant's business and the incumbent's profit from it.

Poitevin (1989) considers predatory excess supply in a Cournot model as we do. In his model, the high-productivity entrant finances his fixed cost by debt, not by equity, because this financial choice signals high productivity and confirms to the investor that the entrant deserves to enter the product market. Debt financing, however, entails the possibility of bankruptcy, which is assumed to give the incumbent some exogenous (monopoly) profit. Hence, the debt invites the incumbent's predatory excess supply.

In equilibrium of Poitevin's model, the entrant holds excess liquidity as in ours, but the reason is quite different. In his model, the excess liquidity raised by debt just increases the threat of bankruptcy and stimulates the incumbent's predation. This is what the high-productivity entrant himself wants. He raises the debt level so high that the low-productivity entrant cannot bear the intensified predatory excess supply, which works as a signal of his high productivity. In contrast, the incumbent whose productivity is known publicly does not need such a signal and finances his fixed cost by equity. This enables him to exercise predation free from risk of bankruptcy. Poitevin thereby presents "a formal justification for Telser (1966)'s deep-pocket argument." (Poitevin, 1989, 38). Although his model induces the shrinking of the entrant's capacity level, as does ours, this comes from only a response to the incumbent's excess supply. Accordingly, Poitevin does not induce excess liquidity *as a barrier to predation*, which Telser mentions the entrant's countermeasure against the "long-purse" ("deep-pocket") incumbent's predation.

Finally, both Bolton and Scharfstein (1990) (and its successors) and Poitevin (1989) put exogeneity on the link between the product market and the entrant's financing: the entrant's demand for loan comes from exogenous costs. We establish the endogeneity in this link and present the existence of excess liquidity as a barrier of predation. So our model would be the first model that fully captures the classic "long-purse" theory of predation from modern strategic perspective, though we further show distortion in the product market via endogenous imperfect financing on contrary to the classic view posed by Telser (1966).

The paper proceeds as follows. The next section describes the economy. Section 3 presents us the benchmark where the entrant does not face the cash-in-advance constraint

[3]Snyder (1996) introduces into Bolton and Scharfstein (1990)'s model the renegotiation between the entrant and the lender at the beginning of the second period; in each period, the entrant needs to borrow the loan to pay some exogenous cost. He finds that the renegotiation makes it more difficult for the entrant to avoid the predation. See also Fernández-Ruiz (2004).

as well as the incumbent. In Section 4, we see the case where the rival's capacity is verifiable and prove the existence of threat of predation and of the entrant's demand for excess precautionary liquidity. In Section 5, we find that the unverifiablity results in distortion in the product market. We find that presuming the truth-telling (quasi-)direct mechanism, the threat of predation prevents the low-capitalized entrant from raising sufficient excess liquidity (Section 5.1) and forces him to shrink his capacity (Section 5.2); finally, we generalize imperfect financing and the distortion to arbitrary form of the financial contract by arguing the revelation principle (Section 5.3). In Section 6 we discuss the structural assumptions in the model and in each proposition, which guides us to the policy implication in Section 7. We summarize our propositions and the implication in the last section. The formal proof of the revelation principle is given in the Appendix.

2 The Economy

Before giving a formal description of our model, let us share the basic story behind the model. We imagine an entrant (firm 1) and an incumbent (firm 2) who compete in a product market. So long before finishing the production and opening the product market, each firm makes some precommitment that defines competence in the product market, e.g. capacity of production, advertisement of the product. The entrant has to pay these costs before completing the production and gaining the sales, because he is new to this business and thus has no reputation to defer the payment unconditionally.

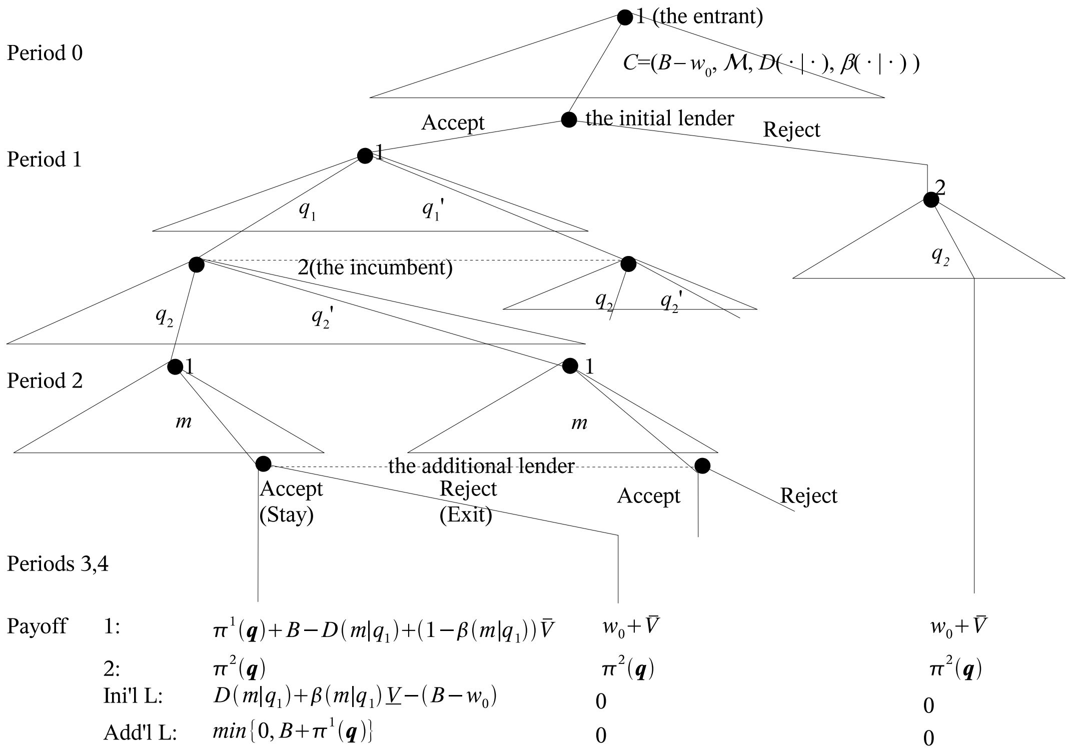
So the entrant faces the cash-in-advance constraint to continue the production and stay in the market. Basically he can borrow these costs by putting his assets as collateral when he enters the market. Besides, after both firms make these commitment, the entrant can borrow the additional loan, as we do not introduce exogenous uncertainty (cost and demand shock) into the market structure and the future profit can be correctly predicted.

On the other hand, the incumbent wants to exclude (prey on) the entrant from the market so as to raise his own profit. He tries to prevent the entrant from getting an adequate additional loan by making aggressive precommitment, which decreases the entrant's future profit and makes it hard for the entrant to obtain the additional loan. Still the entrant can protect himself from such predation if he can obtain sufficient liquidity on the entry; then, even if the incumbent makes aggressive predatory precommitment, the entrant who does not rely on the additional loan can stay in the market and thus the incumbent's predation fails. As Telser (1966) argues, a rational incumbent who predicts it does not try predation. Therefore, availability of sufficient initial loan does matter, even if we allow second chance to finance after the predatory precommitment.

This is the basic story, though the model encompasses more broader situation, which will be revealed when we will discuss the structural assumptions underlying this model in Section 6. Now we formalize the model. The economy starts in period 0 and ends in period 4. Here we separately describe the product market and the financial structure of the entrant. In Fig.1, we summarize all the events in this model sorting them by time.

The Product Market

The prominent assumption on our product market is that both firm must commit some variables and the entrant has to pay the cost for those committed variables so long before they get the sales of their products. That makes the possibility that even after the entrant enters the market he may exit before the product is sold and the incumbent can enjoy large profit by eliminating the rival's product from the market.



Period 0. The entrant with internal capital w_0 borrows *the initial loan* $B - w_0$, under the financial contract $C = \{B - w_0, M, D(\cdot|\cdot), \beta(\cdot|\cdot)\}$. Let B be the entrant's liquidity holding at this period (precautionary liquidity).

Period 1. The entrant and the incumbent simultaneously *determine their capacity levels* q_i. The entrant's capacity q_1 is verifiable. The incumbent's capacity q_2 and thus the gross profit $R^1(q_1, q_2)$ are assumed to be verifiable in Section 3 and to be unverifiable in Section 4.

Period 2. The entrant announces a message $m \in M$. Following the initial financial contract, the lender determines the amount of (monetary) repayment $D(m|q_1)$ and the liquidation policy $\beta(m|q_1)$ based on the message. The entrant must pay his capacity cost $c^1(\mathbf{q})$ to accomplish the production. If he can't, he has to abandon the production due to the cash-in-advance constraint and only the incumbent will supply the product to the market in period 3. To pay this capacity cost, the entrant may borrow *an additional loan*, which is not guaranteed until this period.

Period 3. The entrant (if he stays in the market) and the incumbent sell their products and earn revenues R^1 and R^2. Then the entrant repays the additional loan.

Period 4. The entrant repays $D(m|q_1)$ to the initial lender. With probability $\beta(m|q_1)$, the lender gains control of the business and gets the liquidation value $\underline{V}$. Otherwise, the entrant remains in control and gets the private benefit $\bar{V}$.

N.B. The dotted line on the additional lender's nodes in period 2 means that the two distinct outcomes are unverifiable, which does not necessarily means unobservability for the lender. On the other hand, the dotted line on the firm 2's nodes in period 1 means unobservability for the firm 2 in this period, as the decisions of q_1 and of q_2 are made simultaneously.

Figure 1: The game tree when the incumbent's capacity is unverifiable. When it is verifiable, the two nodes of the additional lender after (C, Accept, q_1, q_2, m) and (C, Accept, q_1, q_2', m) are separated; then the game is solvable by backward induction.

This is modeled as follows. In period 1, each firm $i = 1, 2$ commits strategy, say 'capacity', $q_i \in Q_i \subset \mathbb{R}_+$.[4] We assume that the entrant's capacity q_1 is verifiable in the court by the entrant himself or by the lenders; on the contrary we consider both cases where the incumbent's capacity q_2 is verifiable and where it is unverifiable by these entrant's party.

In period 2, the entrant must pay the capacity cost $C^1(\mathbf{q})$ (**the cash-in-advance constraint**), while the "long-purse" incumbent can delay to pay $C^2(\mathbf{q})$. Here $\mathbf{q} := (q_1, q_2) \in Q_1 \times Q_2 =: Q$. When the entrant does not have enough money, he is forced to exit the market.

In period 3, both firms produce and sell the products.[5] We reduce the outcome in the period-3 market competition into a gross profit function $R^i : \mathbb{R}^2_+ \to \mathbb{R}$. When the entrant proceeds with production, each firm i earns the gross profit (or revenue) $R^i(\mathbf{q})$. When the entrant exits, only the incumbent gets the gross profit $R^2(0, q_2)$.

We denote by $\pi^i(\mathbf{q})$ the firm i's net profit: $\pi^i(\mathbf{q}) = R^i(\mathbf{q}) - C^i(\mathbf{q})$. We assume that each $\pi^i : \mathbb{R}^2_+ \to \mathbb{R}$ is continuously differentiable and concave, i.e. $\pi^i_{ii} < 0, \pi^i_{jj} \leq 0$, as well as $\pi^i_j < 0, \pi^i_{ij} < 0$ for each $i = 1, 2$ and $j \neq i$. (Here $\pi^i_j := \partial\pi^i/\partial q_j, \pi^i_{ij} := \partial^2\pi^i/\partial q_i \partial q_j$.) Hence we assume that larger capacity q_i decreases the rival's net profit π^j ($j \neq i$). Though the source of such substitutability may come from substitution in the factor market (increasing the capacity cost C^j) and/or from substitution in the product market (decreasing the gross profit R^j), our argument applies to both cases. These functions C, R and π are also verifiable and common knowledge. So *the actual revenue is verified if both firms' capacity are verified.*

The Financial Contract

Basically we will argue the condition that the entrant can borrow sufficient precautionary liquidity on entry (initial loan), looking at the optimal financial contract between the entrant and the (initial) lender and the constraints on the contract. Yet, as we will discuss, we keep the flexibility in the entrant's finance by allowing him to borrow the additional loan *after* the entry, i.e. in period 3 when the capacity $\mathbf{q}$ has already been committed and the entrant faces the cash-in-advance constraint.

The entrant's financial schedule goes as follows. In period 0, the entrant appears in the market with the the start-up money (liquidity) $w_0 \geq 0$ and some start-up assets. By placing a mortgage on these assets, the entrant borrows **initial loan** from a lender. Denote by B the total liquidity holding at the end of period 0 (the **precautionary liquidity**), i.e. the start-up money w_0 plus the initial loan $B - w_0$. As the usual financial contract, the initial loan contract C describes the following.

- $B - w_0$: the amount of the initial loan
- M: the set of available messages that will be sent in the beginning of period 2, which can be anything, e.g. unverified information (the incumbent's capacity) q_2, the anticipated gross profit $R^1(\mathbf{q})$, or their combination.
- $D(\cdot|\cdot) : M \times Q_1 \to \mathbb{R}$: the (monetary) repayment in period 4, given the entrant's capacity $q_1 \in Q_1$ and the message m. Since both variables q_1, q_2 and the contract are verifiable, the court can enforce the repayment of this D in period 4.

[4]Although we call q_i the capacity of firm i, it indeed should be seen as a summary variable of all committed strategies, e.g. capacity, quantity of product that takes so long time to produce, the volume of advertisement (or more adequately the size of target consumers reached by the advertisement).

[5]We presume that the entrant's equilibrium profit without threat of predation is positive. It is natural since otherwise the entrant would not enter the market.

- $\beta(\cdot|\cdot) : M \times Q_1 \to [0, 1]$: the liquidation policy, i.e. the probability that the lender takes over the mortgaged assets in the end of period 4. We allow a stochastic liquidation policy in the initial contract. That is, the probability of liquidation β can take any value in $[0, 1]$, not only $\{0, 1\}$.

We assume the contract C is made public and thus common knowledge for everyone in the economy, as well as it is verifiable in the court.

At the beginning of period 2, the entrant announces the message $m \in M$, after the capacity $\mathbf{q}$ is committed in period 1. Now he faces the cash-in-advance constraint. As the precautionary liquidity B may not suffice the capacity cost $C^1(\mathbf{q})$, the entrant can ask for the **additional loan** in period 2. The additional lender may or may not be the same as the initial lender. We assume that the additional lender also shares the same message m with the initial lender, which will reduce the information problem on borrowing the additional loan to the revelation by the initial loan contract. As the initial lender takes the start-up asset as collateral, we assume that the additional lender has priority to be repaid.[6] So, in this period 2, the entrant pay the capacity cost $C^1(\mathbf{q})$ from the precautionary liquidity B plus the additional loan. We assume that the financial market for the additional loan is competitive to simplify the additional lender's decision.

In period 3, the additional loan should be repaid right after the entrant gets the sales and the gross profit $R^1(\mathbf{q})$. If the additional lender is not sure about the whole repayment from the verifiable information available in period 2, she will not agree on lending. The verifiable case will clarify the condition to borrow the additional loan (or to continue the production without it). Notice that after the additional lender gets the whole repayment, the entrant should have liquidity as much as $\pi(\mathbf{q}) + B$ in the end of period 3.

In period 4, the initial loan is repaid according to the repayment schedule D. Besides, with probability $\beta(m|q_1)$, the initial lender liquidates the mortgaged asset and gains the liquidation value $\underline{V} > 0$. Otherwise, the entrant retains the whole control on the assets and gains the private benefit $\overline{V} > \underline{V}$. In contrast to the monetary repayment D, define the **total repayment** δ as the monetary repayment D plus the expected loss by liquidation of the mortgaged assets:

$$\delta(m|q_1) := D(m|q_1) + \beta(m|q_1)\bar{V},$$

which the entrant would minimize if he had the freedom to choose the message m.

These are the events about the entrant's financing. Now let us focus on the initial loan contract C. This contract does not matter when the incumbent's capacity q_2 is verifiable, because the amount of the net profit π^1 becomes verifiable: the court can enforce the entrant to repay the whole amount of the initial loan as long as $\pi(\mathbf{q})$ is nonnegative and the entrant has sufficient liquidity in the end of period 3. When q_2 is unverifiable, the repayment depends on the entrant's self report m. Therefore the initial lender needs the collateral to force him to repay as much as possible and not to go into false bankruptcy. We will discuss the detail of the assumed unverifiability later.

We impose two standard constraints on the initial loan contract:

- *the entrant's limited liability constraint*: the entrant's liquidity holding cannot be negative after the repayment of all the loans, i.e. in the end of period 4; and,
- *the lender's participation condition*: the equilibrium repayment (plus the liquidation value if the lender gets the control of the business) must cover the loan $B - w_0$.

[6] Although we just assume this financial structure, this is realistic as we imagine the following: the entrant puts up the physical assets to start the business as collateral for an initial loan, and the inventories and the accounts receivable for an additional loan (Hart, 1995, p.111).

When the entrant reports $\tilde{q}_2$ as the incumbent's capacity in period 2 and continues the production, the limited-liability constraint requires that the repayment $D(\tilde{q}_2|q_1)$ should be within the liquidity holding $\pi(\tilde{q}_2|q_1) + B$ in the beginning of period 4 inferred from this report $\tilde{q}_2$. If he gives up the production and exits from the market, the limited liability does not matter, because it is verifiable that the entrant exits in period 2 and nothing is spent from B and thus the court can enforce the entrant to repay the whole amount of the initial loan $B - w_0$.

So we define the **limited-liability constraint** as

$$D(\tilde{q}_2|q_1) \leq \pi^1(q_1, \tilde{q}_2) + B \qquad \text{whenever } \tilde{q}_2 \in Q_2^S(q_1), \tag{1}$$

where $Q_2^S(q_1) \subset Q_2$ is the set of the incumbent's capacities that allow the entrant to stay in the market given q_1. In Section 5.1 we will see that the limited liability constraint implies a nontrivial condition for the entrant to avoid predation, presuming a truth-telling (quasi-)direct mechanism $M = Q_2$.

In general, the message space M may be different from Q_2. But in Section 5.3, we see that an outcome from any contract (mechanism) C is also obtained from a "quasi-direct" mechanism $\hat{C}$, where given q_1 the entrant announces the rival's capacity $q_2 \in Q_2^S(q_1)$ if he wants to stay or otherwise a message $m \in M_0(q_1)$ that prevents him from obtaining the additional loan: $\hat{M}(q_1) = Q_2^S(q_1) \cup M_0(q_1)$. Then we impose the limited liability constraint (1) on this reduced quasi-direct mechanism $\hat{C}$. We also discuss the underlying assumption on this technique in Section 6.

3 Benchmark: No Cash-in-advance Constraint

As a benchmark, here we glance the 'regular' case where the entrant has no cash-in-advance constraint as well as the incumbent. Then, as is usual, the entrant proceeds the production and stays in the market, without need to raise precautionary liquidity, as he can pay all the cost after he gets the revenue in period 3.

So as long as the entrant will get positive profit from $\mathbf{q}$ determined in period 1 and decides to stay, the incumbent cannot exclude him anyhow from the market in this model. Hence, given the entrant q_1, the incumbent has no better choice than maximizing $\pi^2(q_1, \cdot)$. Thus we can solve the game as a usual 'Cournot' competition (taking q_i as the 'output' level): the benchmark capacity $\mathbf{q}^\dagger$ is determined by

$$q_i^\dagger = \arg\max_{q_i \in Q^i} \pi^i(q_i, q_j^\dagger) \quad \text{for each } i = 1, 2, \; j \neq i,$$

or[7]

$$\pi_1^1(\mathbf{q}^\dagger) = 0, \quad \pi_2^2(\mathbf{q}^\dagger) = 0. \tag{2}$$

In sum, without the CIA constraint (if the entrant also has "long purse"), there is no threat of predation and no need to raise precautionary liquidity on the entry. This is our benchmark.[8]

[7] Here we assume that the solution for this lies in $Q_1 \times Q_2$.

[8] Notice that we can determine the capacity levels, namely the outcome in the product market, without clarifying whether the rival's q_2 is verifiable or not. (Though, we might need it so as to justify the absence of the CIA constraint.) This comes directly from the absence of the CIA constraint: the entrant do not need any loan before getting sales.

4 Verifiable Case: Excess Liquidity against Predation

Here we look at the benchmark case where the rival's capacity is verifiable, and we specify the notions of "threat of predation" and of "excess liquidity against predation" in our model. Moreover, we find that the threat of predation does not affect the equilibrium outcome in this benchmark case.

By backward induction, we first consider the entrant's financing of additional loan in period 2. The entrant needs an additional loan if his precautionary liquidity B is below his capacity cost $c^1(\mathbf{q})$. Since both firms' capacities $\mathbf{q} = (q_1, q_2)$ are assumed here to be verifiable and thus it is verifiable that the entrant gains the gross profit $R^1(\mathbf{q})$ in period 3, the additional loan is available in period 2 if and only if the anticipated gross profit $R^1(\mathbf{q})$ and the precautionary liquidity B cover the capacity cost $c^1(\mathbf{q})$:

$$R^1(\mathbf{q}) + B \geq c^1(\mathbf{q}), \quad \text{i.e. } \pi^1(\mathbf{q}) + B \geq 0. \tag{3}$$

This inequality works as **the liquidity constraint** under the verifiability of the incumbent's capacity. If this inequality is satisfied, the additional lender is sure and can verify in the court that the additional loan is unspent and the entrant can repay it, and thereby the lender agrees on the loan; otherwise the lender is sure that the additional loan is spent to cover the operating loss and the entrant cannot repay it, and the lender refuses the loan.[9] So the inequality (3) is the sufficient and equivalent condition for the entrant to finance the capacity cost with the additional loan and continue the production in period 2.

The incumbent may produce predatory excess capacity because of this liquidity constraint. He can break this condition by increasing his capacity q_2, which lowers the entrant's anticipated net profit π^1. When he succeeds in such predation, the entrant is forced to exit from the market and the incumbent enjoys the predatory profit by monopolizing the market. So the liquidity constraint brings *the threat of predation* to the entrant.

But, the threat of predation is limited as we think of a rational incumbent. As we see in Fig. 2, there is a threshold level of the incumbent's capacity $\bar{q}_2^P(q_1)$ where the predatory profit begins to fall below the optimal profit without predation, given the entrant's capacity q_1:

$$\pi^2(0, \bar{q}_2^P(q_1)) = \max_{q_2 \in Q_2} \pi^2(q_1, q_2). \tag{4}$$

Larger predatory capacity brings to the entrant larger operating loss, but predatory capacity over the threshold $\bar{q}_2^P$ is *implausible* because it makes the incumbent's profit worse than that without predation (as $R^2_{22} < 0$) and so the incumbent himself never conducts it. So we call the threshold capacity level $\bar{q}_2^P(q_1)$ **the maximum predatory capacity** and the entrant's loss due to this maximum predatory capacity $-\pi^1(q_1, \bar{q}_2^P(q_1))$ **the maximum predatory loss** $\bar{L}^P(q_1)$:

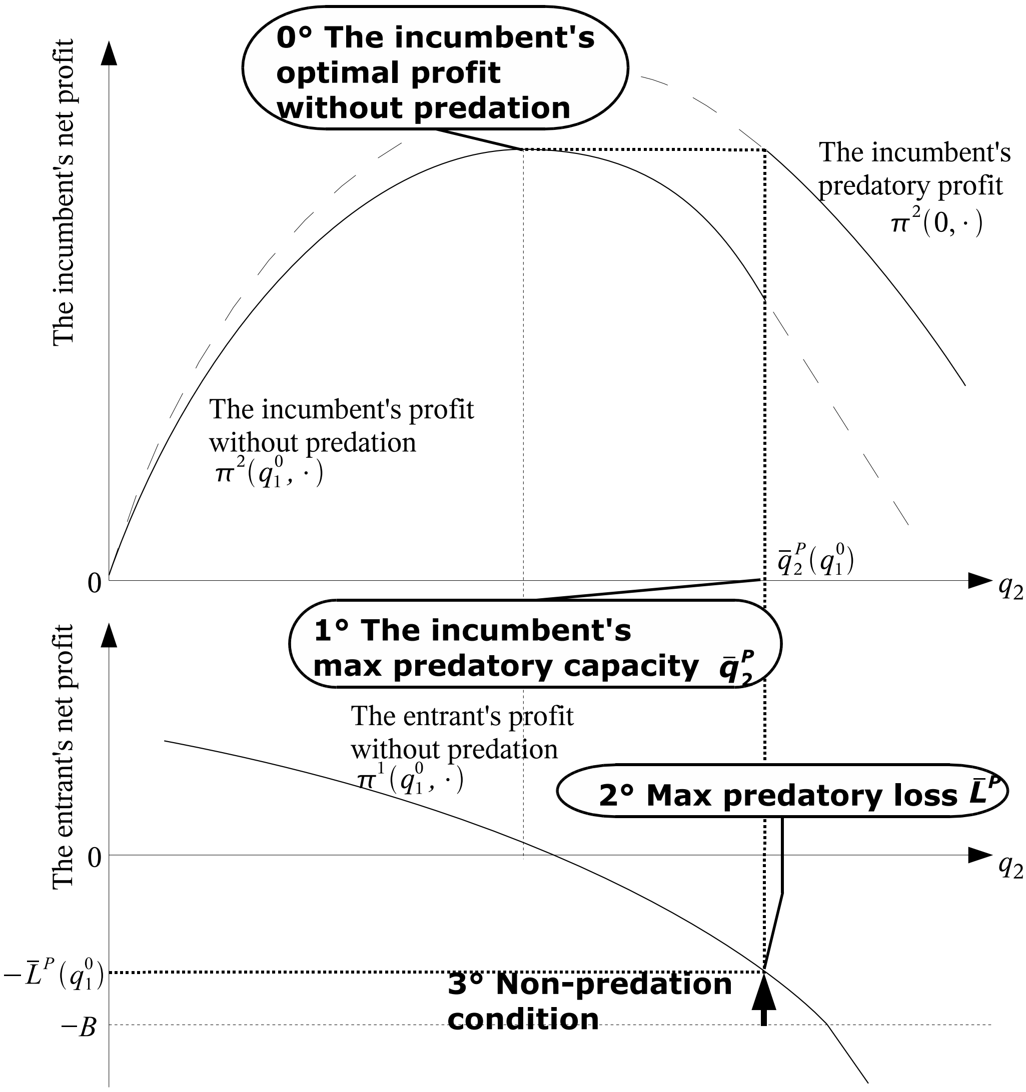
$$\bar{L}^P(q_1) = -\pi^1(q_1, \bar{q}_2^P(q_1)). \tag{5}$$

The liquidity constraint (3) suggests that the entrant can survive any plausible predation if the entrant has the precautionary liquidity enough to cover the maximum predatory loss. So we have **the non-predation condition under the verifiability of the rival's capacity (strategy)**:

$$B \geq \bar{L}^P(q_1). \tag{6}$$

If the entrant has precautionary liquidity larger than the maximum predatory loss, he avoids predation and the net profit in period 3 is $\pi^1(\mathbf{q})$ as he planed in period 1. Then

[9] If (3) is not satisfied, the entrant cannot pay the capacity cost sorely from his precautionary liquidity since $R^1(\mathbf{q}) > 0$.



0° We want to see the equilibrium where the entrant prevents predation; the incumbent's equilibrium capacity should be the maximizer of the duopoly profit given q_1^0.

1° The incumbent could benefit from predation if and only if the incumbent could push the entrant to exit by predatory excess capacity less than $\bar{q}_2^P(q_1^0)$.

2° So $\bar{L}^P(q_1^0)$ is the maximum possible loss of the incumbent in case of predation.

3° As long as the entrant can stay in the market even if he suffers the loss of $\bar{L}^P(q_1^0)$, the incumbent will not prey on him by profitable predation; thus, the incumbent gives up predation. To guarantee the entrant's stay, the liquidity constraint requires him to possess the precautionary liquidity B more than the loss $\bar{L}^P(q_1^0)$.

Figure 2: The maximum predatory loss $\bar{L}^P$ and the non-predation condition given q_1^0.

the precautionary liquidity is much larger than the required liquidity to suffice the liquidity constraint at the actual capacity $\mathbf{q}$. Accordingly the threat of predation creates the entrant's need for *excess liquidity against predation*.

In period 1, each firm determines his own capacity so as to maximize his net profit, though the entrant faces the non-predation condition (6). Given the entrant's precautionary liquidity $B^{\ddagger}$, the equilibrium capacity profile $\mathbf{q}^{\ddagger} = (q_1^{\ddagger}, q_2^{\ddagger})$ is the solution of

$$q_1^{\ddagger} = \arg\max_{q_1 \in Q_1} \left\{ \pi^1(q_1, q_2^{\ddagger}) \mid B^{\ddagger} \geq \bar{L}^P(q_1) \right\}, \tag{7a}$$

$$q_2^{\ddagger} = \arg\max_{q_2 \in Q_2} \pi^2(q_1^{\ddagger}, q_2). \tag{7b}$$

Although the non-predation condition seems to restrict the entrant's equilibrium capacity, the verifiability of the incumbent's capacity gets rids of this restriction. If the non-predation condition is satisfied and the actual predation is totally eliminated, the entrant will surely earn the equilibrium net profit $\pi^1(\mathbf{q}^{\ddagger})$ in period 3 and the initial loan will not be used for the production. Since this future net profit is verifiable under the verifiability of the rival's capacity, the court can enforce the entrant to repay the whole amount of the initial loan. So the initial lender agrees to lend any amount of loan. The entrant can thus obtain sufficient initial loan, i.e. raise B large enough to make the non-predation condition (6) slack.

As a result, when the rival's capacity is verifiable, the equilibrium capacity levels $\mathbf{q}^{\ddagger}$ are the same as the benchmark equilibrium $\mathbf{q}^{\dagger}$ where the entrant does not have the CIA constraint and thus he is free from threat of predation. The entrant raise enough precautionary liquidity $B^{\ddagger} \geq \bar{L}^P(q_1^{\ddagger})$.

Proposition 1. *Consider the case where the entrant faces the cash-in-advance constraint and the rival's capacity q_2 is verifiable.*

1) There is threat of predation: without enough precautionary liquidity on the entrant, the incumbent would exclude the entrant by setting the excess capacity. So the entrant needs to raise excess precautionary liquidity on the entry so as to avoid the predation, even if he has a chance to borrow an additional loan after the entry.

2) But, the verifiability enables the entrant to obtain sufficient precautionary liquidity to avoid the incumbent's predation. Consequently, though the threat of predation exists, it does not affect the equilibrium outcome.

5 Unverifiable case: distortion in the product market

5.1 Non-predation condition given a truth-telling quasi-direct mechanism

Here we simply presume that a contract C lets the entrant directly tell the unverified information q_2 truthfully when he stays in the market, so as to focus on the effect of threat of predation on the product market. We call such a contract a truth-telling quasi-direct mechanism. We add the prefix 'quasi' to distinguish it from a direct mechanism where the entrant (or an agent in general) always tell the unverified (or privately observed) information directly. Here the entrant need not tell it when he chooses to exit, because it does not affect the ability to repay the loan. Formally we define a truth-telling quasi-direct mechanism as follows.

Definition 1 (truth-telling quasi-direct mechanism)**.** Given a strategy profile, the contract is a **quasi-direct mechanism** under this strategy profile if

1) for each $q_1 \in Q_1$, the message space $M(q_1)$ contains a subset of Q_2: so it is written as $M(q_1) = Q_2^S(q_1) \cup M_0(q_1)$ with $Q_2^S(q_1) \subset Q_2$ and $Q_2^S(q_1) \cap M_0(q_1) = \emptyset$; and,

2) the additional lender offers any amount of additional loan if the entrant tells any $\tilde{q}_2 \in Q_2^S(q_1)$, and rejects any amount of loan if he tells any $m \in M_0(q_1)$.

Furthermore, it is **truth-telling** if

3) when the incumbent's actual capacity q_2 is in $Q_2^S(q_1)$, then the entrant announces it truthfully; and,

4) when the entrant announces any $\tilde{q}_2 \in Q_2^S(q_1)$, the players in the economy (especially both initial and additional lenders) believes it as truth.

We consider a valid contract in the sense we argued in Section 2. A quasi-direct mechanism should satisfy the limited liability constraint (1). Besides, given firm i's equilibrium capacity q_i^* $(i = 1, 2)$, the initial lender should earn non-negative profit as her participation condition, which is reduced in a truth-telling quasi-direct mechanism to

$$D(q_2^*|q_1^*) + \beta(q_2^*|q_1^*)\underline{V} \geq B - w_0.$$

Furthermore, given the quasi-direct mechanism, let $\underline{\delta}(q_2)$ be the minimum total repayment $\delta = D + \beta\bar{V}$ to stay in the market for each $q_1 \in Q_1$:

$$\underline{\delta}(q_2) := \min_{\tilde{q}_2 \in Q_2^S(q_1)} \delta(\tilde{q}_2|q_1).$$

Provided that the entrant wants to stay, he could choose the message $\tilde{q}_2$ that yields this minimum total repayment. So the incentive compatibility for a truth-telling mechanism is

$$\delta(q_2|q_1) = \underline{\delta}(q_2) \quad \text{for any } q_2 \in Q_2^S(q_1). \tag{8}$$

We investigate a (pure-strategy) non-predatory equilibrium where the entrant stays on the equilibrium path and the two firms share the product market with capacity $\mathbf{q}^* = (q_1^*, q_2^*)$. As we argued in the last section, this requires that the entrant can stay in the market for any plausible excess capacity, i.e.

$$q_2 \in Q_2^S(q_1^*) \quad \text{as long as } \pi^2(0, q_2) > \pi^2(\mathbf{q}^*).$$

(With the continuity) this is equivalent to

$$q_2^*, \bar{q}_2^P(q_1^*) \in Q_2^S(q_1^*).$$

That is, he commits himself to stay against the maximum predatory capacity, not only against the equilibrium capacity.

We see the constraints at the equilibrium capacity q_2^* and at the maximum predatory capacity $\bar{q}_2^P(q_1^*)$ together yield a non-trivial condition for a non-predatory equilibrium. First, at the maximum predatory capacity $\bar{q}_2^P(q_1^*) \in Q_2^S(q_1^*)$, the limited liability constraint need hold:

$$D(\bar{q}_2^P(q_1^*)|q_1^*) \leq B + \pi^1(q_1^*, \bar{q}_2^P(q_1^*)) = B - \bar{L}^P(q_1^*).$$

If this is violated, then it is clear (for all players in the economy as well as us) that the entrant will not be able to repay D as written in the contract. So the actual monetary repayment must be reduced. As we see D as the final monetary repayment, this limited liability constraint must hold. The constraint is most restrictive at $\bar{q}_2^P(q_1^*)$ (given q_1^*), because it implies the largest plausible predatory loss and thus the smallest liquidity holding of the entrant in the beginning of period 4. Since $\beta \in [0,1]$ and $\bar{V} > 0$, this constraint implies an upper bound on the total repayment in the case of predation:

$$\delta(\bar{q}_2^P(q_1^*)|q_1^*) = D(\bar{q}_2^P(q_1^*)|q_1^*) + \beta(\bar{q}_2^P(q_1^*)|q_1^*)\bar{V} \leq B - \bar{L}^P(q_1^*) + \bar{V}. \tag{9}$$

Second, since $\mathbf{q}^*$ is the equilibrium outcome in the product market, the participation condition matters at $\mathbf{q}_2^*$: since $B - w_0$ is the amount of the initial loan, it requires

$$D(q_2^*|q_1^*) + \beta(q_2^*|q_1^*)\underline{V} \geq B - w_0.$$

Since $\bar{V} > \underline{V}$, this yields the lower bound of the total repayment.

$$\delta(q_2^*|q_1^*) = D(q_2^*|q_1^*) + \beta(q_2^*|q_1^*)\bar{V} \geq B - w_0 \quad (\because \bar{V} > \underline{V}, \beta \geq 0). \tag{10}$$

Finally, we combine these two bounds by the incentive compatibility. Because the entrant (given q_1^*) is free to choose any $\tilde{q}_2$ from $Q_2^S(q_1^*)$ when he wants to stay in the market, he should minimize the total repayment. So the total repayment should be constant among any possible $q_2 \in Q_2^S(q_1^*)$. Therefore, we have

$$B - \bar{L}^P(q_1^*) + \bar{V} \geq \delta(\bar{q}_2^P(q_1^*)|q_1^*) = \delta(q_2^*|q_1^*) \geq B - w_0.$$

$$\therefore \ \bar{V} + w_0 \geq \bar{L}^P(q_1^*). \tag{11}$$

This is **the non-predation condition under the unverifiability of the rival's capacity**:[10] In Section 5.3, we see the generality of this condition starting from an arbitrary contract.

The entrant faces the non-predation condition (11) in deciding his capacity in period 1. The equilibrium capacitys $\mathbf{q}^* = (q_1^*, q_2^*)$ are determined as

$$q_1^* = \arg\max_{q_1 \in Q_1} \left\{ \pi^1(q_1, q_2^*) \;\middle|\; \bar{V} + w_0 \geq \bar{L}^P(q_1) \right\}, \tag{12a}$$

$$q_2^* = \arg\max_{q_2 \in Q_2} \pi^2(q_1^*, q_2). \tag{12b}$$

While under the verifiability of the incumbent's capacity the non-predation condition $B \geq \bar{L}^P(q_1)$ is not restrictive because of the freedom to raise the precautionary liquidity B by the initial loan, under the unverifiability the non-predation condition $\bar{V} + w_0 \geq \bar{L}^P(q_1)$ is restrictive for the entrant with small internal capital w_0. A low-capitalized entrant thus reduces his own capacity from that in a usual Cournot competition.

We can summarize the result in this section as follows. It is clear from the argument above that we can generalize the result from the truth-telling quasi-direct mechanism to an arbitrary (not truth-telling) quasi-direct mechanism, as long as it requires the same incentive compatibility (8) for q_2^* and $\bar{q}_2^P(q_1^*)$. In the next section, we see that the non-predatory condition actually distorts the equilibrium outcome in the product market.

Theorem 1. *Consider a valid quasi-direct mechanism that satisfy the incentive compatibility* (8) *under the unverifiability of the rival's capacity. The entrant's equilibrium capacity q_1^* is determined in* (12a)*, restricted by the non-predatory condition* (11).

In summary, the threat of predation requires the lender to commit the continuation of the production even if the entrant announces the predatory loss that is never realized in equilibrium, and this commitment is also the source of the borrower's opportunism aiming at the remission of the loan under the limited liability. So, under the unverifiability of the rival's predation, the truth-telling incentive must be reduced to avoid predation; thus, the low-capitalized entrant cannot make a valid contract.

[10] This should be written in a contract to restrict the entrant's capacity q_1. Otherwise, the entrant could set larger q_1^* and then eventually the contract would become invalid.

5.2 Distortion in the Product Market

Because our model specifies the maximum predatory loss as (5), we can evaluate from the reduced form (12) how much the equilibrium capacitys are distorted under the threat of unverifiable predation.

In this section we want to determine q_1^*, q_2^* and $\bar{q}_2^P(q_1^*)$ by calculus (first-order condition etc.). To justify it, we assume the following property on the strategy space:

Assumption 1. Let $\mathbf{q}^*$ be the solution of (12) and $\bar{q}_2^P(q_1^*)$ be the solution of (4) when $\mathbf{q}$ may take any value in $\mathbb{R}^2$. The capacity spaces Q_1, Q_2 are assumed to contain these capacity levels:

$$q_1^* \in Q_1, \qquad q_2^*, \bar{q}_2^P(q_1^*) \in Q_2.$$

We focus on the entrant who has so small internal capital that violates the non-predation condition at the benchmark capacity level $q_1^\dagger$, i.e. $w_0 < \bar{L}^P(q_1^\dagger) - \bar{V}$; otherwise there is no distortion of both firms' capacity levels.

First, in general the low-capitalized entrant *reduces* his capacity (and the incumbent *increases* his in response) from the benchmark $\mathbf{q}^\dagger$.

Corollary 1. *Suppose Assumption 1. If the entrant has internal capital w_0 below $\bar{L}^P(q_1^\dagger) - \bar{V}$, the unverifiability of the rival's capacity does not allow the entrant to borrow sufficient precautionary liquidity to avoid predation. Consequently, the entrant's capacity shrinks while the incumbent's expands, compared to the benchmark equilibrium $\mathbf{q}^\dagger$.*

In contrary, if the entrant has an internal capital $w_0 > \bar{L}^P(q_1^\dagger) - \bar{V}$, the entrant has no financial difficulty in borrowing precautionary liquidity to avoid predation. Then the equilibrium capacities are not distorted, as is the verifiable case (cf. Prop. 1).

Proof. See Appendix B. □

This is because the maximum predatory loss $\bar{L}^P(q_1)$ increases with q_1 around the benchmark $q_1^\dagger$; when the non-predation condition (11) is violated at $q_1^\dagger$, the entrant must reduce the maximum predatory loss $\bar{L}^P$ by setting smaller q_1. Recall the definition of $\bar{L}^P$, and take its derivative with regard to q_1:

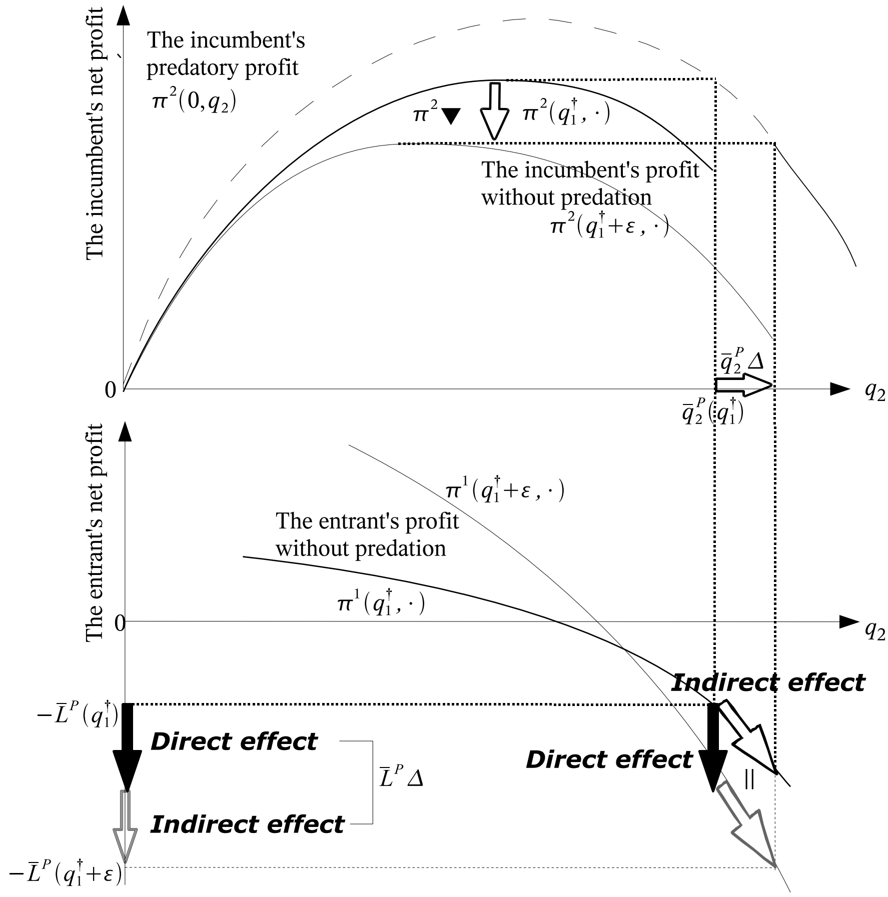
$$\frac{d\bar{L}^P}{dq_1}(q_1) = \underbrace{[\text{Marginal operating loss in predation } - \pi_1^1(q_1, \bar{q}_2^P)]}_{\text{Direct effect}} \underbrace{-\pi_2^1(q_1, \bar{q}_2^P) \times \frac{d\bar{q}_2^P}{dq_1}(q_1)}_{\text{Indirect effect}},$$

$$\text{where } \frac{d\bar{q}_2^P}{dq_1}(q_1) = \frac{\pi_1^2(q_1, q_2^{BR}(q_1))}{\pi_2^2(0, \bar{q}_2^P(q_1))} > 0.$$

We refer to the former term of the RHS as the **direct effect** of the marginal increase in the entrant's capacity q_1 on the maximum predatory loss $\bar{L}^P$, and to the latter as the **indirect effect**. The direct effect represents the increase of the entrant's maximum predatory loss, with the incumbent's capacity fixed, caused by the increase in the entrant's capacity itself; the indirect one represents the increase caused by the change in the incumbent's maximum predatory capacity $\bar{q}_1$.

The direct effect is positive at the entrant's benchmark capacity level $q_1^\dagger$.[11]
Since the entrant's marginal profit $\pi_1^1(\cdot)$ is assumed to be non-increasing to the incumbent's capacity q_2 and $\bar{q}_2^P(q_1^\dagger)$ is larger than the incumbent's equilibrium capacity $q_2^\dagger$ (by $\pi_{22}^2(\cdot) < 0$),

[11] Since the direct effect may be non-positive at q_1^* in equilibrium, the proof in Appendix B is modified from the explanation below. But the procedure of the proof is consistent.



Direct effect Provided $\bar{q}_2^P$ was unchanged, the entrant's profit changes by the marginal profit due to the increase of q_1. As the marginal profit should be negative at $(q_1^\dagger, \bar{q}_2^P(q_1^\dagger))$, this effect increases $\bar{L}^P$ at $q_1^\dagger$.

Indirect effect As q_1 increase, the incumbent's profit without predation shrinks at any q_2; his net profit of predation gets larger. This allows more predatory capacity, i.e. $\bar{q}_2^P$ increases. This also increases $\bar{L}^P$.

Since both the direct and the indirect effects are positive at $q_1 = q_1^\dagger$, the maximum predatory loss $\bar{L}^P$ increases with the entrant's capacity; thus the low-capitalized entrant should reduce his capacity from the benchmark one $q_1^\dagger$ to satisfy the non-predation condition (11). Hence, unless the entrant satisfies this condition at the benchmark equilibrium, his response curve must shift downward at least around the benchmark equilibrium, while the incumbent's remains the same as the benchmark.

Figure 3: Effect of the increase in the entrant's capacity q_1 on the maximum predatory loss $\bar{L}^P$, around the benchmark equilibrium $\mathbf{q}^\dagger$.

the entrant's marginal revenue is smaller in predation than in the benchmark equilibrium. Hence we have

$$\begin{aligned}[\text{Direct effect at } q_1^\dagger] &= [\text{Marginal operating loss in predation } -\pi_1^1(q_1^\dagger, \bar{q}_2^P)] \\ &\geq -[\text{Marginal operating profit in the benchmark } \pi_1^1(\mathbf{q}^\dagger)] = 0.\end{aligned}$$

The indirect effect is also positive if $d\bar{q}_2^P/dq_1$ is positive, as $R_2^1(\cdot) < 0$. Positive $d\bar{q}_2^P/dq_1$ means that the increase of the entrant's capacity allows the incumbent to be still better off by predation with further excess supply. This is the case. Increase of the entrant's capacity level q_1 decreases the incumbent's profit without predation $\pi^2(q_1, q_2^{BR}(q_1))$, while his (would-be) predatory profit $\pi^2(0, \tilde{q}_2)$ remains the same for any predatory capacity $\tilde{q}_2$. Accordingly, as the entrant increases his capacity q_1, the incumbent's net benefit of the predation becomes larger, and thus the maximum predatory capacity $\bar{q}_2^P$ gets higher, i.e. $d\bar{q}_2^P/dq_1 > 0$. In short, because increase of the entrant's capacity level intensifies predation, the indirect effect on the maximum predatory loss $\bar{L}^P$ is positive.

It is worth mentioning that this shift does not occur for the highly-capitalized entrant with $w_0 > \bar{L}^P(q_1^\dagger) - \bar{V}$; otherwise, the financial difficulty due to the unverifiable predation shrinks the entrant's capacity, and then the incumbent's expands as the best response.

Socially Inefficiency under Threat of Predation

To justify the political/legal intervention on predation, we need to show that the threat of unverifiable predation reduces social welfare, in addition to distorting the firms' capacity levels. Since we work on a very general demand structure R, our model may have both positive and negative results: in general the social welfare may and may not decrease under the threat of predation.

Yet we have one concrete case where the social welfare decreases.

Corollary 2. *Suppose Assumption 1. Furthermore, assume that the capacity cost function is linear in the own capital, i.e.* $C^i(\mathbf{q}) = c_i q_i$ *and the gross profit function is linear respectively in the total capacity (for price) and in the own capacity (for quantity supplied), i.e.* $R^i(\mathbf{q}) = \{a - (q_1 + q_2)\}q_i$ *with sufficiently high demand level* a *compared to the entrant's unit cost* c_1*:* $a/c_1 > -7 + \sqrt{66} \approx 1.12$*. If the two firms have little difference in their productivity, namely* c_1 *and* c_2 *are close enough, then the maximum predatory loss at the benchmark capacity* $\bar{L}^P(q_1^\dagger)$ *increases with the incumbent's unit cost* c_2*. That is, as the incumbent has less efficient technology, the low-capitalized entrant is more likely to shrink his capacity under the threat of unverifiable predation.*

Proof. Calculate $\bar{L}^P(q_1^\dagger)$ in this case with $c_1 = c_2$, and differentiate it with regard to c_2. □

In this case, given the total supply, it is socially inefficient to have the entrant supply less than the incumbent, because both two firms have linear production technology and the entrant has better one. Hence, the equilibrium capacity under the threat of unverifiable predation is more socially inefficient than the benchmark.

Product Market Environment to Motivate Threat of Predation

The low-capitalized entrant reduces his capacity when the maximum predatory loss is so huge that the non-predation condition is violated at the benchmark. Here we make a list of the situations where the maximum predatory loss becomes large.

As seen above, the maximum predatory loss increases with the entrant's capacity. The entrant's benchmark capacity gets larger if he has *better productivity* in capacity building (less c_1) or demand for the entrant's product has *lower price elasticity* against increasing capacity(higher R^1_1).

Next, consider the case where the incumbent can reduce the entrant's gross profit so much from the benchmark $R^1(\cdot, q_2^\dagger)$ by a small predatory capacity. This happens if the entrant's product is little differentiated from the incumbent's and has *high substitutability* in the two firms' capacity (large R^1_2).[12] Then, even if the entrant could earn positive profit at the benchmark, he suffers predatory loss and thus has to reduce his capacity if he has little internal capital.

5.3 The Revelation Principle under Strategic Uncertainty

In the last sections, we see that the non-predation condition distorts the equilibrium outcome in the product market, presuming a truth-telling quasi-direct mechanism as the initial loan contract. Yet the entrant and the initial lender might try to write a better contract so as to prevent the entrant from the opportunism: so we need to think a broader range of possible contracts to check whether our non-predation condition is robust when we allow any form of the loan contract.

Therefore we prove the revelation principle so as to reduce any outcome under an arbitrary contract to an outcome under a quasi-direct mechanism, though several versions of the principles are already proved and widely used: for a correlated equilibrium under perfect information (Osborne and Rubinstein, 1994, Proposition 47.1), for a Bayesian Nash equilibrium under incomplete information (Fudenberg and Tirole, 1991, Section 7.2), and recently for a perfect Bayesian equilibrium under incomplete information without the principle's perfect commitment (Bester and Strautz, 2001). Unlike the incomplete information game, the unverified information q_2 is chosen by the outsider of the contract — the incumbent, not by the nature, and most of alternatives (which could be 'type' in a Bayesian game) do not realize on a (pure-strategy) equilibrium path. But in our setting, non-equilibrium path should play a crucial role, because we want to see the effect of *threat* of predation on the product market in a *non-predatory* equilibrium.

Thus we adopt *sequential equilibrium* to select a reasonable off-path believes and actions after the contract is written in period 0. That means in essence that we select the pair of strategy and belief if 1) the strategy is rational under the belief and 2) the belief is stable ('consistent') when the strategy is perturbed to be completely mixed (take every action with positive probability). This perturbation pins down the belief, especially the off-path one, so that the (perturbed) belief is consistent with the (perturbed) strategy by Bayes rule.[13]

Stability under perturbation in strategy means robustness under strategic uncertainty. In the perturbation, every action are chosen with some probability. Although the players anticipate the opponent's action correctly on the equilibrium path, the perturbation requires an off-path belief to be robust in the emergence of uncertainty in the opponents' strategy (caused by the perturbation).

If we stick to a general — possibly continuous — strategy space, it is hard (even only) to define a sequential equilibrium and to prove its existence, because we need to think about convergence of non-finite probability measures (as mixed strategies). To our knowledge,

[12]See $\mathbf{q}$ as the output levels of the two firms, rather than just capacities. Suppose the representative customer has quasi-linear utility $u(x_1, x_2) - p_1 x_1 - p_2 x_2$ on the products and money. Then, the Walrasian demand $\mathbf{d}(\mathbf{p})$ should satisfy $u_i(\mathbf{d}(\mathbf{p})) = p_i$, which determines the inverse demand function $\mathbf{p}(\mathbf{q})$ s.t. $p^i(\mathbf{q}) = u_i(\mathbf{q})$ and the revenue function $R^i(\mathbf{q}) = p^i(\mathbf{q})q_i$. When the substitutability u_{12} is high, $R^1_2 = p^1_2 q_2 = u_{12} q_1$ is large.

[13]We do not require the rationality of the perturbed strategy, which selects the equilibrium more strictly. That is a trembling-hand perfect equilibrium, stronger than a sequential equilibrium.

there is literature about a trembling-hand perfect equilibrium in a normal-form continuous game (Méndez-Naya, García-Jurabo, and Cesco, 1995), but not about a sequential equilibrium in an extensive-form one. Instead, we assume a finite strategy space. This is enough for our analysis, because as we see in Section 5.1, only $\bar{q}_2^P(q_1^*)$ and q_2^* are crucial to induce the non-predatory condition and the distortion in the product market.

The following properties of a (Bayesian Nash) equilibrium (not necessarily sequential) are established as a lemma (Lemma 1 in Appendix A) for the revelation principle. First, the additional lender never lends money if the message m (given q_1) imply a chance (any positive probability in the posterior belief) of default in the repayment of the additional loan; otherwise, she lends any amount of money. This is because of the competitive financial market: the additional lender cannot gain positive profit and thus cannot take any risk.

Second, given $\mathbf{q}$, the entrant chooses a message that induces the rejection of the additional loan if he wants to exit from the market. Otherwise, he chooses a message so as to minimize the total repayment $\delta(m|q_1)$ among all messages that enable him to borrow the additional loan. This is the incentive compatibility.

Based on these two properties, we can categorize the equilibrium outcome (for each q_1) into three cases as below. For each q_1, denote by $M_0(q_1) \subset M$ the set of messages that are sent under some $q_2 \in Q_2$ and let the entrant exit, and by $M_1(q_1) \subset M \setminus M_0(q_1)$ the set of messages that are sent under some $q_2 \in Q_2$ and let the entrant stay. Recall that $Q_2^S(q_1)$ is the set of the incumbent's capacity level $q_2 \in Q_2$ that lets the entrant stay.

1) a pooling-stay case $M_0(q_1) = \emptyset$: the entrant stays for any q_2, i.e. $Q_2^S(q_1) = Q_2$;

2) a pooling-exit case $M_1(q_1) = \emptyset$: the entrant exits for any q_2, i.e. $Q_2^S(q_1) = \emptyset$;

3) a separating case $M_0(q_1), M_1(q_1) \neq \emptyset$: stay or exit depends on q_2, i.e. $\emptyset \neq Q_2^S(q_1) \subsetneq Q_2$.

Focusing on sequential equilibria, we can convert any message space M to a quasi-direct mechanism (not necessarily being truth-telling) $\hat{M} = Q_1^S \cup M_0$, so that we keep the equilibrium outcome in the subgame after the contract is written (the equilibrium capacity $\mathbf{q}^*$ and the probability that entrant stays in the market after each possible $\mathbf{q} \in Q_1 \times Q_2$). This is our revelation principle (Theorem 2 in Appendix A), which is summarized as follows.

Theorem 2 (the Revelation Principle)**.** *Suppose the strategy space is (arbitrarily) finite and satisfies Assumption 1. Consider a sequential equilibrium and a contract that is valid through the perturbation in the sequential equilibrium. Based on the category above, we can convert the message space for each q_1 to the quasi-direct mechanism as below, keeping the same equilibrium outcome after the contract is written:*

1) *A pooling-stay case is reduced to a* **pooling-stay mechanism***, where the message space $\hat{M}(q_1)$ is the whole Q_2 and the entrant sends all $\tilde{q}_2 \in Q_2$ with the equal probability, regardless of the actual q_2. The posterior belief is the same as the prior, i.e. the incumbent's (mixed) strategy of q_2, regardless of the message $\tilde{q}_2$. The net repayment $\delta(\tilde{q}_2|q_1)$ must be constant among all $\tilde{q}_2 \in Q_2$, given $q_1 \in Q_1$.*

2) *A pooling-exit case is reduced to a* **pooling-exit mechanism***, where the message space $\hat{M}(q_1)$ is $M_0(q_1)$ in the original mechanism. The entrant's strategy of m and the posterior belief are the same as the original mechanism.*

3) *A separating case is reduced to a truth-telling quasi-direct mechanism with $\hat{M}(q_1) = Q_1^S(q_1) \cup M_0(q_1)$, using $Q_1^S(q_1)$ and $M_0(q_1)$ in the original equilibrium.*

Proof. See Appendix A for the proof and the detail of this theorem, as well as the detail of Lemma 1 that gives the above categorization. □

Now we obtain the same non-predation condition for a non-predatory equilibrium (so throwing pooling-exit cases) by reducing an arbitrary contract to a quasi-direct mechanism and using Theorem 1.

Theorem 3. *Suppose the strategy space is (arbitrarily) finite and satisfies Assumption 1. Consider a sequential equilibrium and a contract that is valid through the perturbation in the sequential equilibrium. Given the entrant's equilibrium capacity q_1^* determined in* (12a)*, under the unverifiability of the rival's capacity, the entrant can finance precautionary liquidity B not less than the maximum predatory loss $\bar{L}^P(q_1)$ in period 0 and avoid the actual predation, only if he has sufficient internal capital w_0 so as to satisfy* (11)*.*

Proof. Combine the revelation principle (Theorem 2) with Theorem 1. □

Still we might need a few explanation for the pooling-stay case, because the limited liability condition must hold for all $q_2 \in Q_2^S(q_1^*) = Q_2$ in the pooling-stay mechanism. Although the pooling-stay mechanism seems to let the lender anticipate the equilibrium q_2^* correctly, this is done by not giving any information about actual q_2, not by telling the true q_2. So, in the perturbation (namely when the incumbent tries other capacity levels), the monetary repayment D as well as the total one δ cannot be adjusted to different q_2. Since we want a contract that is robust to strategic uncertainty, the contract should be still valid under the perturbation. Therefore, we require the limited liability constraint for any $\tilde{q}_2 \in Q_2^S(q_1^*) = Q_2$.

It would relax the constraint if the entrant and the additional lender could modify the contract to the perturbation. But such a possibility is excluded from our setting and contradicts with the pooling itself. First, in our model, perturbation takes place after the contract is written in period 0. Second, under the pooled-stay mechanism, the lender would not know the perturbed q_2. So there is no chance to adjust the contract to the perturbation.

Hence we have the same non-predatory condition for a pooled-stay mechanism as well as for a truth-telling quasi-direct mechanism. Moreover, the stronger limited liability constraint restricts the monetary repayment more than the separating case. Even if the incumbent sets $q_2 > q_2^P(q_1^*)$, the entrant stays and thus the limited liability holds for this q_2 in a pooling-stay equilibrium, while in a truth-telling mechanism, he can choose to exit.

In summary, a non-predation equilibrium needs the non-predation condition even if we allow arbitrary mechanism for a loan contract, as long as we require the robustness of the contract to strategic uncertainty.

Proposition 2. *Consider the case where the entrant faces the cash-in-advance constraint and the rival's capacity q_2 is not verifiable. Assume that the strategy space is (arbitrarily) finite.*

1) There is threat of predation, and thus the entrant needs to raise excess precautionary liquidity to avoid the predation.

2) Thanks to the unverifiability, the entrant with small start-up capital and less valuable asset cannot finance sufficient precautionary liquidity to keep the benchmark capacity, as long as the entrant and the initial lender need to write a contract robust against strategic uncertainty. With Assumption 1, this implies the entrant shrinks his capacity compared to the benchmark, so as to stay in the market.

6 Discussion on the Underlying Assumptions

In this section, we discuss the structural assumptions underlying in our model and in our revelation principle for the unverified case. Besides, we see an empirical research that supports out results. This discussion clarifies applicability of each proposition and guides us to policy implication that we shall make in the next section.

Unverifiability

First, let us consider unverifiablity of the incumbent's capacity (or precommitted strategy) q_2 and the entrant's profit π^1. We should distinguish unverifiablity from unobservability. Even if q_2 is unverifiable, the entrant may directly observe q_2 or predict it with high accuracy, by good marketing research. He could present the marketing data about the rival's strategy and its impact on his own business to the lenders so as to convince them of profitability of his business plan to enter the market; actually we assume the lenders also have the correct prediction on the equilibrium outcome.

What we mean by unverifiability is that nobody (especially the lenders) cannot verify that the observation or the prediction coincides with the actual q_2 (or π^1). Actually, like us, the entrant and the lenders can predict the market outcome by assuming the rationality of the incumbent (and some epistemologic assumptions to guarantee a Nash equilibrium), but this is only the prediction.[14] Furthermore, because the incumbent is a rival in the product market, it is hard to expect that he would assure to provide the information of the actual q_2 for an evidence in the entrant's financial lawsuit, which would eventually help the entrant's financing to enter the market and compete with the incumbent himself.

Moreover, we should notice that in the unverifiable case, the repayment does not rely on the court. Our unverifiability prevents the court from enforcing the whole repayment of the loan. So the lender herself has to encourage the entrant to repayment by using liquidation of collateral as threat.

We might feel that our verifiable and unverifiable cases are too extreme. In between, we could think of stochastically verifiable case where the lender gets verifiable information about q_2 or π^1 with some probability. On the other hand, so-called "costly state verification", usually meaning that a principal (the lenders) *surely* obtains the verifiable information at some cost, should fall into our verifiable case.

Although we emphasis plausibility of unverifiable case, we do not insist that unverifiable q_2 is always the case. Actually the entrant should try to make things verifiable to get good finance. For example, in a "main bank system", a borrower (the entrant) can have his business activity monitored by the "main bank" (the lender) through keeping all transactions in the bank's account and inviting a banker as an accounting director, which guarantees verifiability of the borrower's liquidity holding and enable the lender to enforce the whole repayment of the loan (possibly without help of the court).

The loan structure and the CIA constraint

Second, in our model, there are two types of loans — the initial and the additional loans. Additional loan is allowed just to give the entrant a second chance to raise liquidity. Without it, the model would seem too restrictive. Besides, having two types of loans, we separate loan to protect the entrant against predation from loan just to pay the cost; one of our

[14] This could be enough for the antitrust lawsuit, but here we argue the financial lawsuit to enforce the loan repayment. So the court needs to know whether the entrant actually has enough money to repay the loan.

propositions (part 1 of Propositions 1 and 2) is the existence of excess liquidity holding due to threat of predation.

But additional loan does not affect the result: the firm would obtain enough initial loan if and only if the non-predation condition (in each case) holds. If the additional loan is not allowed and the initial lender can withdraw the loan before the entrant proceeds production in period 2, then the case falls into a separating case. If the initial lender commits himself not to withdraw, then it falls into pooling-stay case.

On the other hand, we can think of wide range of financing as additional loan. For example, if the entrant is allowed to defer the payment of the capacity cost C^1 from period 2 to period 3 or he gets the advance payment for his product, it is considered as the additional loan in our model. Our "cash-in-advance constraint" means that the entrant should cover all the capacity cost by his precautionary liquidity, the bank loan (or the additional investment), and such deferred payment of costs and advance draw of sales. So the constraint exits when the entrant do not have committed line to cover all capacity costs by any these means. The key in our loan structure is *commitment* of the initial financing on entry and uncommitment of the additional financing just before finishing production.[15]

This point leads us to reconsider the meaning of the 'entrant' in our model. Although we can think of an entrant to have this constraint because he is new to the industry and thus has no credit to get deferred payment or advance draw, theoretically our 'entrant' can be an incumbent. For example, we should think the entrant as our 'incumbent' if the 'entrant' is a big company and can use profit from its other businesses or he has government's support to enter the market.

Strategic Uncertainty

We select equilibria by sequential equilibrium, namely by robustness to strategic uncertainty, for the unverifiable case to induce Proposition 2. For the other two cases and Proposition 1, we did not need strategic uncertainty to pin down the equilibrium.

In particular, we argued the limited liability constraint should hold for $q_2 = \bar{q}_2^P(q_1^*)$ because the incumbent indeed sets this capacity in the perturbation and the valid contract must be still valid. That is, when the entrant and the lenders are not sure about whether the incumbent actually sets the capacity to what the party predicts from economic analysis, i.e. the equilibrium q_2^*, they should write the contract so as to prepare for the perturbed case and keep the contract consistent in the perturbation. Actually, when threat of predation is a matter for them, they should be worry about uncertainty in the rival's strategy.

On the other hand, if they were so sure somehow about their prediction of $q_2 = q_2^*$, they would not have to think other possibility. This is like the verifiable case, where they *ex post* exclude the other possibility after q_2 is set. Here they *ex ante* exclude it.

But we should notice that the behavioral condition to reduce strategic uncertainty (to expect the rival setting the equilibrium strategy) is quite strong. It needs not only the rival's rationality, but also the rival's correct prediction about others' strategies etc. For example, even when the entrant has enough precautionary liquidity to avoid predation $B > \bar{L}^P(q_1^*)$ by getting additional loan approved, still the incumbent could prey on him if the incumbent would expect that additional lenders would not offer the loan for unprofitable business, i.e. $\pi^1 < 0$, though they are sure about the whole repayment.

[15] Recall that in the epigraph in the beginning of this paper, Telser considers "liquid asset together with *the credit lines*" as a precautionary liquidity.

Empirical support

Lerner (1995)'s empirical study on the disk drive industry in 1980–88 is consistent with the predictions of our model. After hedonic regressions of products' price to their attributes,[16] he tests whether price wars were triggered by entries of financially weak rivals.

Lerner identifies a financially weak firm in two aspects. First, the firm specializes in disk drive manufacturing, which means the absence of internal financing from other business. Second, the firm's equity capital is below the median of all the samples. In 1980–83, a venture company was able to easily raise the internal capital with equity finance. In this era of "capital market myopia," prices were wholly determined by the products' attributes: there was no predatory pricing by "long-purse" incumbents. On the contrary, in 1984–88, when entrepreneurs suddenly faced difficulty in equity financing, prices were significantly low in the presence of the financially weak rivals after the product prices are controlled by their attributes: predatory pricing was executed against financially weak firms.

We can compare this empirical analysis with Proposition 2 for the unverifiable case. In early 1980s, "capital market myopia" enabled the entrants to raise the internal capital w_0. So they sufficed the non-predatory condition and could borrow the outside loan to raise enough precautionary liquidity and to let the incumbent give up predation. In contrast, the hard time of equity financing in the late 80s forced the entrants to enter the industry with short internal capital. So they did not obtain sufficient precautionary liquidity and had to allow the incumbent to be more aggressive.

7 Implication on Competition Policy

First of all, we cannot overpass the assumptions on our propositions when we apply them to practice. There is no distortion in the product market if the assumptions (unverifiability, strategic uncertainty etc.) do not hold; the classic negative view like Telser (1966) is right in this case. This is why we discuss the underlying assumptions in so much detail. Besides, in equilibrium, predation is anyway avoided by the entrant's two counter-strategies — raising precautionary liquidity and shrinking his own business. So it would not be simple to identify predation in practice.

Hence we emphasize ex-ante prevention to reduce threat of predation, rather than ex-post punishment on the actual predation. So we need much broader perspective for competition policy than antitrust legal judgment. For example, it would help entrepreneurs to avoid threat of predation and reduce the predatory distortion if they have stronger relation with banks to let them monitor the business and easier access to equity market to raise start-up internal capital. The discussion we made in detail in Section 6 would help policy makers and entrepreneurs to create the product and financial market environment with less threat of predation and less predatory distortion.

However we still need to keep the legal punishment on predatory conduct. First, though no predation should occur in equilibrium, it depends on so much epistemologic and behavioral assumptions to have the equilibrium result in reality. Especially we argued in the last section that if the incumbent did not expect that excess precautious liquidity would help the entrant to raise additional loan, he would conduct predation. Besides, if the antitrust authority punishes predation severely, the entrant and the lenders can eliminate its possibility; then strategic uncertainty gets reduced and adequate initial loan becomes easier to obtain.

[16] Mainly, diameters, densities, access time, products' ages, and years of observation. His observations are prices while ours are quantities (capacity levels) in a Cournot market.

Anyway, because our model predicts only the distortion on the product market by threat of predation and not the actual predation (elimination of the entrant from the market), we do not suggest the legal criterion to identify the predation from our equilibrium analysis. Rather our model would suggest to use the incumbent's 'intent'[17] as the evidence of predation more than economic analysis.

Finally, we should notice that there must be excess precautionary liquidity even if there is no distortion in product market. The excess liquidity is just kept to show the entrant's finance health and commitment to stay in the market. So it does not contribute any production. When the liquidity supply is limited, such demand for precautionary liquidity crowds out real liquidity demand to pay for production and investment (Holmström and Tirole, 1998). Hence policy that reduces threat of predation contributes macroeconomic efficiency through releasing excess liquidity holding.

8 Conclusion

We see the effect of threat of predation on the product market and the entrant's finance. In our model, the outcome in the product market is evaluated by the firms's precommitted strategies like capacity investment and threat of predation comes from the entrant's cash-in-advance constraint to pay the precommitment costs. We see threat of predation causes the demand for excess precautionary liquidity that is never spent. Furthermore, we proved that if the incumbent's strategy and thus the entrant's final profit are unverifiable and the loan contract needs to prepare for perturbation from the equilibrium outcome, the entrant faces short supply of excess liquidity and has to shrink his business.

From this result, we first suggest macroeconomic impact of anti-predation policy, which will release excess precautionary liquidity and reduce the crowd out in financial market. Besides, our model suggests ex-ante prevention of predation rather than ex-post punishment, because we focus on the equilibrium where the entrant eventually avoids the predation by raising excess liquidity and shrinking his business and stays in the market. We discussed about structural assumptions on our propositions, which would help the competition policy maker (and the entrepreneurs themselves) to construct the anti-predation market environment. Still we insist to punish predation if it is identified by the incumbent's 'intention' (rather than economic equilibrium analysis), because it will reduce the uncertainty in the incumbent's strategy and help the entrant to obtain the adequate precautionary liquidity to avoid predation.

A Proof of the Revelation Principle (Theorem 2)

In this Appendix, we show the revelation principle on sequential equilibrium in the finite version of the subgame after the contract is accepted: we prove Lemma 1 and Theorem 2 in Section 5.3.

We first define the finitely approximation of the subgame after the initial contract is accepted, and its sequential equilibrium. Then, we obtain the revelation principle for these sequential equilibria.

Henceforth, let π^i be the operating profit and $\delta(m|q_1)$ be the total repayment to the initial lender:

$$\pi^i(q_i, q_{-i}) := R^i(q_i, q_{-i}) - c_i q_i (i = 1, 2),$$
$$\delta(m|q_1) := D(m|q_1) + \beta(m|q_1)\bar{V}.$$

[17] For the use of 'intent' in the antitrust court, see Comanor and Frech III (1993).

Since we assume $D \geq 0$ and $\bar{V} > 0$, the total repayment δ is non-negative.

Formal Definition of a Finite Subgame

We restrict the sets of both firms' capacity levels (the capacity spaces) and the message space to (arbitrary) finite ones, Q_1, Q_2, M ($\sharp Q_1, \sharp Q_2, \sharp M \in [2, \infty)$.)[18]

Under the contract C with the message space M, the strategy space is given as follws:

$$\begin{array}{rr} \text{the entrant's capacity} & \sigma_1 \in \Delta Q_1, \\ \text{the incumbent's capacity} & \sigma_2 \in \Delta Q_2, \\ \text{the entrant's message conditional on } (q_1, q_2) \in Q_1 \times Q_2 & \sigma_m(\cdot|q_1, q_2) \in \Delta M, \\ \text{the additional lender's decision conditional on } (m, q_1) \in M \times Q_1 & \sigma_a(\cdot|q_1, m) \in \Delta A. \end{array}$$

Here the set A consists of 0 (rejecting the loan) and 1 (accepting the loan), and ΔX denotes the set of probability measures on the set X. In particular, with a finite set X, ΔX is an $\sharp X$-dimentional simplex, i.e. $\Delta X := \{\sigma \in \mathbf{R}_+^{\sharp X} \mid \sum_{x \in X} \sigma(x) = 1\}$. The posterior belief $\mu(\cdot|q_1, m)$ is a probability measure on Q_2, conditional on $q_1 \in Q_1$ and $m \in M$.

Sequential Equilibrium

Here we investigate sequential equilibrium (σ^*, μ^*) of a finite subgame.

Consistent belief μ^* must be a limit of beliefs μ^k that is wholly determined from some completely mixed strategy $\sigma^k \in \mathring{\Sigma}$ by Bayes rule: we must have the sequence of $\sigma^k \in \mathring{\Sigma}$ that holds $\sigma^k \to \sigma^*$ and

$$\mu^k(q_2|q_1, m) := \frac{\sigma_m^k(m|q_1, q_2)\sigma_2^k(q_2)}{\sum_{q_2' \in Q_2^n} \sigma_m^k(m|q_1, q_2')\sigma_2^k(q_2')} \longrightarrow \mu^*(q_2|q_1, m)$$

as $k \to \infty$, for each $q_2 \in Q_2, q_1 \in Q_1, m \in M$.

The additional lender employs the strategy $\sigma_a^*(\cdot|q_1, m)$ on additional lending decision so as to maximize

$$\begin{aligned} & E_{a,q_2}\left[a(\min\{0, \pi^1(q_1, q_2) + B\})|q_1, m\right] \\ = \; & \sigma_a(1) \sum_{q_2 \in Q_2} (\min\{0, \pi^1(q_1, q_2) + B\})\mu^*(q_2|q_1, m), \end{aligned}$$

with regard to $\sigma_a \in \Delta A$, at each information set $(q_1, m) \in Q_1 \times M$. Hence, the additional lender strictly prefers the pure strategy $a = 0$ to $a = 1$ and takes $\sigma_a^*(1|q_1, m) = 0$, when $\pi^1(q_1, q_2) + B < 0$ for some q_2 in the support of $\mu^*(q_2|q_1, m)$. Otherwise, $a = 0$ and $a = 1$ are indifferent. We focus on equilibrium where he chooses $\sigma_a^*(1|q_1, m) = 1$ in this indifferent case; this is justified by giving an infinitesimal utility to $a = 1$.

The entrant follows the strategy $\sigma_m^*(\cdot|q_1, q_2)$ on message so as to maximize

$$\begin{aligned} & E_{a,m}\left[a\{\pi^1(q_1, q_2) + B - D(m|q_1) + (1 - \beta(m|q_1))\bar{V}\} + (1 - a)(w_0 + \bar{V})|q_1, q_2\right] \\ = \; & w_0 + \bar{V} + \sum_{m \in M^n} \sigma_a^*(1|q_1, m)\left[\pi^1(q_1, q_2) - \delta(m|q_1) + B - w_0\right]\sigma_m(m|q_1, q_2), \end{aligned}$$

[18] The boundedness of the capacity space Q_i is easily justified if there exists $\bar{q}_i < \infty$ such that $\pi^i(\bar{q}_i, 0) = 0$ and $\partial \pi^i / \partial q_i(\bar{q}_i, 0) < 0$; since larger capacity than $\bar{q}_i$ results loss (negative profit) no matter to the rival's capacity, the firm i won't choose such huge capacity level. Anyway all we need for this finite subgame is only Assumption 1; so the restriction would be so small.

with regard to $\sigma_m(\cdot|q_1, q_2) \in \Delta M$, at each information set $(q_1, q_2) \in Q_1 \times Q_2$. Let $\underline{\delta}(q_2)$ be the minimal total repayment when the entrant stays in the market:

$$\underline{\delta}(q_1) := \min\{\delta(m|q_1)|\sigma_a^*(1|q_1, m) = 1\}.$$

Given the additional lender's strategy specified above, the entrant sends only messages in the set $\{m|\delta(m|q_1) = \underline{\delta}(q_1), \sigma_a^*(1|q_1, m) = 1\}$ ($\{m|\sigma_a^*(1|q_1, m) = 0\}$, resp.) with possitive probability, if $\pi^1(q_1, q_2) - \underline{\delta}(q_1) - F + B - w_0$ is positive (negative, resp). If it is just zero, messages in both sets can be sent. Given q_1 (and C), let $q_2^S(q_1)$ be the threshold incumbent capacity:

$$\pi^1(q_1, q_2^S(q_1)) - \underline{\delta}(q_1) + B - w_0 = 0. \tag{13}$$

Given q_1, denote by $M_1(q_1)$ ($M_0(q_1)$, resp) the set of messages that is actually sent with possitive probability by some $q_2 \leq q_2^S(q_1)$ and satisfies $\sigma_a^*(1|q_1, m) = 1$ (by some $q_2 \geq q_2^S(q_1)$ and satisfies $\sigma_a^*(1|q_1, m) = 0$, resp). Given q_1, q_2, let $P^*(q_1, q_2)$ be the probability of sending message in $M_1(q_1)$, namely the equilibrium probability of staying in the market:

$$P^*(q_1, q_2) := \sum_{m' \in M_1(q_1)} \sigma_m^*(m'|q_1, q_2).$$

The incumbent sets the strategy σ_2^* on capacity level so as to maximize

$$\begin{aligned} & E_{a,m,q_1,q_2}\left[a\pi^2(q_1, q_2) + (1-a)\pi^2(0, q_2)\right] \\ = & \sum_{q_1 \in Q_1} \sum_{q_2 \in Q_2} \sum_{m \in M} \left\{\sigma_a^*(1|q_1, m)\pi^2(q_1, q_2) + \sigma_a^*(0|q_1, m)\pi^2(0, q_2)\right\} \sigma_m^*(m|q_1, q_2)\sigma_2(q_2)\sigma_1^*(q_1), \end{aligned}$$

with regard to $\sigma_2 \in \mathring{\Delta} Q_2$. Given σ_a^*, σ_m^* clarified above as well as σ_1^*, the incumbent's expected profit under strategy σ_2 is

$$\begin{aligned} \sum_{q_1 \in Q_1} \Big[& \sum_{q_2 < q_2^S(q_1)} \pi^2(q_1, q_2)\sigma_2(q_2) \\ & + \left\{P^*(q_1, q_2^S(q_1))\pi^2(q_1, q_2^S(q_1)) + (1 - P^*(q_1, q_2^S(q_1)))\pi^2(0, q_2^S(q_1))\right\} \sigma_2(q_2^S(q_1)) \\ & + \sum_{q_2 > q_2^S(q_1)} \pi^2(0, q_2)\sigma_2(q_2)\Big]\sigma_1^*(q_1). \end{aligned} \tag{14}$$

The entrant sets the strategy σ_1^* so as to maximize

$$\begin{aligned} & E_{a,m,q_1,q_2}\left[a(\pi^1(q_1, q_2) - \delta(m|q_1) + B) + (1-a)w_0\right] \\ = & \; w_0 + \sum_{q_1 \in Q_1^n} \sum_{q_2 \in Q_2^n} \sum_{m \in M^n} \sigma_a^*(1|q_1, m)\left[\pi^1(q_1, q_2) - \delta(m|q_1) + B - w_0\right] \\ & \times \sigma_m^*(m|q_1, q_2)\sigma_2^*(q_2)\sigma_1(q_1), \end{aligned}$$

with regard to $\sigma_1 \in \mathring{\Delta} Q_1$. Given σ_a^*, σ_m^* clarified above as well as σ_2^*, the entrant's expected profit under strategy σ_1 is equal to

$$w_0 + \sum_{q_1 \in Q_1^n} \sum_{q_2 < q_2^S(q_1)} \left[\pi^1(q_1, q_2) - \underline{\delta}(q_1) + B - w_0\right]\sigma_2^*(q_2)\sigma_1(q_1). \tag{15}$$

Noticing that we did not argue consistency of the belief to determine the optimal strategies, we find that the following properties should hold foe all Bayesian Nash equilibria, not only for sequential equilibria.

Lemma 1. *Suppose Assumption 1. The following properties characterize all (Bayesian Nash, not necessarily sequential) equilibria under* (C, M)*.*
(a) The additional lender's optimal strategy $\sigma_a^*(\cdot|q_1, m)$ *is*

$$\sigma_a^*(1|q_1, m) = 1 \qquad \textit{iff } \pi^1(q_1, q_2) + B \geq 0 \quad \textit{for all } q_2 \in \textit{support}(\mu^*(q_2|q_1, m)),$$
$$= 0 \qquad \textit{iff } \pi^1(q_1, q_2) + B < 0 \quad \textit{for some } q_2 \in \textit{support}(\mu^*(q_2|q_1, m)).$$

(b) i) Given q_1*, suppose that both sets of* $\{m|\sigma_a^*(1|q_1, m) = 1\}$ *and* $\{m|\sigma_a^*(1|q_1, m) = 0\}$ *are non-empty (a separating case). Then,* $M_0(q_1)$ *and* $M_1(q_1)$ *are nonempty. The entrant's optimal message strategy* $\sigma_m^*(\cdot|q_1, q_2)$ *satisfies*

$$\sigma_m^*(\cdot|q_1, q_2) > 0 \quad \textit{only on the set}$$
$$\begin{cases} \{m|\sigma_a^*(1|q_1, m) = 1, \delta(m|q_1) = \underline{\delta}(q_1)\} & \textit{if } q_2 < q_2^S(q_1), \\ \{m|\sigma_a^*(1|q_1, m) = 0\} & \textit{if } q_2 > q_2^S(q_1), \\ \textit{both of these two sets above} & \textit{if } q_2 = q_2^S(q_1), \end{cases}$$

where $q_2^S(q_1)$ *is the threshold capacity given by* (13)*. Given* q_1 *and* q_2*, the entrant stays in the market at period 3, with the probability of*

$$P^*(q_1, q_2) \begin{cases} = 1 & \textit{if } q_2 < q_2^S(q_1), \\ \in [0, 1] & \textit{if } q_2 = q_2^S(q_1), \\ = 0 & \textit{if } q_2 > q_2^S(q_1). \end{cases}$$

Otherwise, either $M_0(q_1)$ *or* $M_1(q_1)$ *(not both) is empty.*
ii) If $M_0(q_1) = \emptyset$ *(a pooling-stay case), then* $P^*(q_1, q_2) = 1$ *for any* $q_2 \in Q_2$*. For any* $m \in M_1(q_1)$*, we have* $\delta(m|q_1) = \underline{\delta}(q_1)$ *and any* q_2 *in the support of* $\mu^*(q_2|q_1, m))$ *satisfies* $\pi^1(q_1, q_2) + B \geq 0$*.*
iii) If $M_1(q_1) = \emptyset$ *(a pooling-exit case), then* $P^*(q_1, q_2) = 0$ *for any* $q_2 \in Q_2$*. For any* $m \in M_0(q_1)$*, there exists some* q_2 *in the support of* $\mu^*(q_2|q_1, m))$ *such that* $\pi^1(q_1, q_2) + B < 0$*.*
(c) The incumbent's capacity strategy σ_2^* *maximizes (14), given* σ_1^* *and* $P^*(q_1, q_2)$*. As well, the entrant's capacity strategy* σ_1^* *maximizes (15), given* σ_2^**.*

The Revelation Principle

So far we formalized the finite subgame after period 1 and its sequential equilibrium, under an arbitrary mechanism C with an arbitrary message space M. Here we obtain the Revelation Principle: a sequential equilibria in (C, M) is reduced to a sequential equilibrium in the *quasi*-direct mechanism $\hat{C}$ with the message space $\hat{M}(q_1) = M_0(q_1) \cup Q_2^S(q_1)$ for each q_1. Here, given q_1, $Q_2^S(q_1)$ is the set of capacity levels q_2 at which the entrant stays in the market with positive probability in the original equilibrium:

$$Q_2^S(q_1) := \{q_2 \in Q_2 | P^*(q_1, q_2) > 0\}.$$

In the quasi-direct mechanism, the entrant announces the true q_2 if he wants to stay in the market; otherwise he still sends any message in $M_0(q_1)$ so that the additional lender rejects the loan. A (conventional) direct mechanism with message space Q_2 (only the unverified information) may not possess the same equilibrium outcome; as announce of $\tilde{q}_2 \in Q_2/Q_2^S(q_1)$ implies $\mu^*(\tilde{q}_2|q_1, \tilde{q}_2) = 1$, the belief allows the acceptance of additional lending despite what the entrant wanted, as long as $\pi^1(q_1, \tilde{q}_2) + B \geq 0$. This is why we keep all the messages of $M_0(q_1)$ in our message space, so as to leave the belief of these messages that induce the exit.

Theorem 2. *Suppose Assumption 1. Suppose that a mixed strategy profile* $\sigma^* = \{\sigma_1^*, \sigma_2^*, \sigma_m^*, \sigma_a^*\}$ *is a sequential equilibrium under a message space* M *and a mechanism* $C = \{B, D, \beta\}, D(\cdot|q_1) : M \to \mathbb{R}_+, \beta(\cdot|q_1) : M \to [0,1]$ *for each* $q_1 \in Q_1$*.*

Then, there exists a sequential equilibrium $(\hat{\sigma}^*, \hat{\mu}^*)$ *that results the same capacity strategies* (σ_1^*, σ_2^*) *and the same probability* P^* *that the entrant stays in the market, under the message space* $\hat{M}(q_1) = M_0(q_1) \cup Q_2^S(q_1)$ *and the quasi-direct mechanism* $\hat{C} = \{B, \hat{D}, \hat{\beta}\}$*,* $\hat{D}(\cdot|q_1) : \hat{M}(q_1) \to \mathbb{R}_+, \hat{\beta}(\cdot|q_1) : \hat{M}(q_1) \to [0,1]$ *for each* $q_1 \in Q_1$*.*

Here, $\hat{C}$ *must satisfy*

$$\hat{\delta}(\tilde{q}_2|q_1) = \underline{\delta}(q_1) \quad \textit{for all } q_1 \in Q_1, \tilde{q}_2 \in Q_2^S(q_1), \tag{16}$$

and the same D, β *for messages in* $M_0(q_1)$*:* $\hat{D}(m|q_1) := D(m|q_1), \hat{\beta}(m|q_1) := \beta(m|q_1)$ *for each* $m \in M_0(q_1)$*.*

The profile $(\hat{\sigma}^*, \hat{\mu}^*)$ *is specified as follows:*

$$(\hat{\sigma}^*) \qquad \hat{\sigma}_i^*(q_i) := \sigma_i^*(q_i) \quad \textit{for each } q_i \in Q_i, i \in \{1,2\};$$

(Separating case) If both $M_0(q_1)$ *and* $M_1(q_1)$ *are nonempty, then*

$$(\hat{\sigma}^*) \begin{cases} \hat{\sigma}_m^*(\tilde{q}_2|q_1,q_2) := 0 & \textit{for each } q_2 \in Q_2, \tilde{q}_2 \in Q_2^S(q_1) \setminus \{q_2\}, \\ \hat{\sigma}_m^*(q_2|q_1,q_2) := P^*(q_1,q_2) & \textit{for each } q_2 \in Q_2^S(q_1), \\ \hat{\sigma}_m^*(m|q_1,q_2) := \sigma_m^*(m|q_1,q_2); & \textit{for each } q_1 \in Q_1, q_2 \in Q_2, m \in M_0(q_1), \\ \hat{\sigma}_a^*(1|q_1,\tilde{q}_2) := 1, & \textit{for each } \tilde{q}_2 \in Q_2^S(q_1), \\ \hat{\sigma}_a^*(1|q_1,m) := 0 & \textit{for each } m \in M_0(q_1); \end{cases}$$

$$(\hat{\mu}^*) \begin{cases} \hat{\mu}^*(q_2|q_1,\tilde{q}_2) := \mathbf{1}_{\{q_2=\tilde{q}_2\}}, & \textit{for each } q_2 \in Q_2, \tilde{q}_2 \in Q_2^S(q_1), \\ \hat{\mu}^*(q_2|q_1,m) := \mu^*(q_2|q_1,m) & \textit{for each } q_2 \in Q_2, m \in M_0(q_1). \end{cases}$$

(Pooling-stay case) If $M_0(q_1) = \emptyset$ *and* $M_1(q_1) \neq \emptyset$*, then* $\hat{M}(q_1) = Q_2^S(q_1) = Q_2$ *and*

$$(\hat{\sigma}^*) \begin{cases} \hat{\sigma}_m^*(\tilde{q}_2|q_1,q_2) := 1/\sharp Q_2 & \textit{for each } q_2 \in Q_2, \tilde{q}_2 \in Q_2, \\ \hat{\sigma}_a^*(1|q_1,\tilde{q}_2) := 1, & \textit{for each } \tilde{q}_2 \in Q_2^S(q_1), \end{cases}$$

$$(\hat{\mu}^*) \quad \hat{\mu}^*(q_2|q_1,\tilde{q}_2) := \sigma_2^*(q_2), \quad \textit{for each } q_2 \in Q_2, \tilde{q}_2 \in Q_2.$$

(Pooling-exit case) If $M_0(q_1) \neq \emptyset$ *and* $M_1(q_1) = \emptyset$*, then* $\hat{M}(q_1) = M_0(q_1)$ *and*

$$(\hat{\sigma}^*) \begin{cases} \hat{\sigma}_m^*(m|q_1,q_2) := \sigma_m^*(m|q_1,q_2); & \textit{for each } q_2 \in Q_2, m \in M_0(q_1), \\ \hat{\sigma}_a^*(1|q_1,m) := 0 & \textit{for each } m \in M_0(q_1); \end{cases}$$

$$(\hat{\mu}^*) \quad \hat{\mu}^*(q_2|q_1,m) := \mu^*(q_2|q_1,m) \quad \textit{for each } q_2 \in Q_2, m \in M_0(q_1).$$

Proof. Define the repayment $\hat{D}$ and the liquidation policy $\hat{\beta}$ for each $q_1 \in Q_1$ and $\tilde{q}_2 \in Q_2^S(q_1)$ as

$$\begin{aligned} \hat{D}(\tilde{q}_2|q_1) &:= \sum_{m \in M_1} D(m|q_1) p(m|q_1,\tilde{q}_2), \\ \hat{\beta}(\tilde{q}_2|q_1) &:= \sum_{m \in M_1} \beta(m|q_1) p(m|q_1,\tilde{q}_2). \end{aligned}$$

Here $p(m|q_1, q_2)$ is the probability to send the message $m \in M_1(q_1)$ in equilibrium given the actual capacity (q_1, q_2), conditional on stay in the market:[19]

$$p(m|q_1, q_2) := \sigma_m^*(m|q_1, q_2)/P^*(q_1, q_2).$$

Then, Lemma 1 implies (16).

We show that the strategy profile $\hat{\sigma}^* = \{\sigma_1^*, \sigma_2^*, \hat{\sigma}_m^*, \hat{\sigma}_a^*\}$ specified in the theorem is a sequential equilibrium under the belief $\hat{\mu}^*$.

Consistency of belief. The belief $\hat{\mu}^*$ must possess consistency with a sequence of complete-mixed strategy. Based on the sequence of completely mixed strategy profiles $\{\sigma^k\}$ converging to σ^* in the original sequential equilibrium, we set the sequence $\{\hat{\sigma}^k\}$ and then prove the consistency of the belief.

For each $k \in \mathbf{N}$, define $\{\hat{\sigma}_1^k, \hat{\sigma}_2^k\}$ as

$$\begin{aligned}
\hat{\sigma}_1^k(q_1) &:= \sigma_1^k(q_1) && \in (0,1),\\
\hat{\sigma}_2^k(q_2) &:= \frac{1}{\sqrt{k}\#Q_2} + \left(1 - \frac{1}{\sqrt{k}}\right)\sigma_2^k(q_2) && \in (0,1).
\end{aligned}$$

Since $\sigma_i^k \to \sigma_i^*$, we have $\hat{\sigma}_i^k \to \sigma_i^* = \hat{\sigma}_i^*$ for each $i \in \{1, 2\}$.

Separating case Fix $q_1 \in Q_1$ such that $M_0(q_1) = \emptyset$ and $M_1(q_1) \neq \emptyset$. For each $k \in \mathbf{N}$, define $\{\hat{\sigma}_m^k, \hat{\sigma}_a^k\}$ as

$$\begin{aligned}
\hat{\sigma}_m^k(\tilde{q}_2|q_1, q_2) &:= \frac{1}{k\#Q_2^S(q_1)} + \left(1 - \frac{1}{k}\right) I(q_2, \tilde{q}_2)P^k(q_1, q_2) && \in (0,1),\\
\hat{\sigma}_m^k(m|q_1, q_2) &:= \left(1 - \frac{1}{k}\right)\left\{\sigma_m^k(m|q_1, q_2) + \frac{\left(1 - I^S(q_2, q_1)\right)P^k(q_1, q_2)}{\#M_0(q_1)}\right\} && \in (0,1),\\
\hat{\sigma}_a^k(1|q_1, \tilde{q}_2) &:= 1 - 1/k && \in (0,1),\\
\hat{\sigma}_a^k(1|q_1, m) &:= 1/k && \in (0,1)
\end{aligned}$$

for each $q_2 \in Q_2, \tilde{q}_2 \in Q_2^S(q_1), m \in M_0(q_1)$. Here let P^k, I^S and I be

$$\begin{aligned}
P^k(q_1, q_2) &:= \sum_{m' \in M/M_0(q_1)} \sigma_m^k(m'|q_1, q_2) \ \in (0,1),\\
I^S(q_2, q_1) &:= \mathbf{1}_{\{q_2 \in Q_2^S(q_1)\}}, \qquad I(q_1, \tilde{q}_2) := \mathbf{1}_{\{q_2 = \tilde{q}_2\}}.
\end{aligned}$$

Notice that $\sum_{\tilde{q}_2 \in Q_2^S(q_1)} I(q_2, \tilde{q}_2) = I^S(q_2, q_1)$ and $P^k(q_1, q_2) + \sum_{m \in M_0(q_1)} \sigma_m^k(m|q_1, q_2) = 1$. This guarantees

$$\begin{aligned}
&\sum_{\tilde{q}_2 \in Q_2^S(q_1)} \hat{\sigma}_m^k(\tilde{q}_2|q_1, q_2) + \sum_{m' \in M_0(q_1)} \hat{\sigma}_m^k(m'|q_1, q_2)\\
=&\frac{1}{k} + \left(1 - \frac{1}{k}\right)\left\{\left(\sum_{\tilde{q}_2 \in Q_2^S(q_1)} I(q_2, \tilde{q}_2) + 1 - I^S(q_2, q_1)\right)P^k(q_1, q_2) + \sum_{m \in M_0(q_1)} \sigma_m^k(m|q_1, q_2)\right\} = 1,\\
&\therefore \quad \hat{\sigma}_m^k(\cdot|q_1, q_2) \in \mathring{\Delta}\hat{M}(q_1)
\end{aligned}$$

[19]Here P^* is based on the original equilibrium σ_m^* and $P^*(q_1, q_2)$ is positive iff $q_2 \in Q_2^S(q_1)$.

for all $q_1 \in Q_1, q_2 \in Q_2$.

As specified in $(\hat{\sigma}^*)$, $\hat{\sigma}_a^*$ is the limit of $\hat{\sigma}_a^k$ as $k \to \infty$. According to Lemma 1 (b), we obtain $P^k \to P^*$ and thus $\hat{\sigma}_m^k \to \hat{\sigma}_m^*$ as $k \to \infty$.

We see that the Bayesian belief $\hat{\mu}^k$ determined from $(\hat{\sigma}_2^k, \hat{\sigma}_m^k)$ actually converges to $\hat{\mu}^*$. For a while, omit q_1 from arguments in functions. For each $q_2 \in Q_2, m \in M_0$, the belief is

$$\begin{aligned}
&\hat{\mu}^k(q_2|m) \\
&:= \frac{\hat{\sigma}_m^k(m|q_2)\hat{\sigma}_2^k(q_2)}{\sum_{q_2' \in Q_2} \hat{\sigma}_m^k(m|q_2')\hat{\sigma}_2^k(q_2')} \\
&= \frac{\left\{\frac{1}{\sqrt{k}\#Q_2} + \left(1 - \frac{1}{\sqrt{k}}\right)\sigma_2^k(q_2)\right\}\left\{\sigma_m^k(m|q_2) + \frac{(1-I^S(q_2))P^k(q_2)}{\#M_0}\right\}}{\sum_{q_2' \in Q_2}\left\{\frac{1}{\sqrt{k}\#Q_2} + \left(1 - \frac{1}{\sqrt{k}}\right)\sigma_2^k(q_2')\right\}\left\{\sigma_m^k(m|q_2') + \frac{(1-I^S(q_2'))P^k(q_2')}{\#M_0}\right\}} \\
&= \frac{\frac{1}{\sqrt{k}\#Q_2}\left\{\sigma_m^k(m|q_2) + \frac{(1-I^S(q_2))P^k(q_2)}{\#M_0}\right\} + \left(1 - \frac{1}{\sqrt{k}}\right)\left\{\mu^k(q_2|m)S^k(m) + \frac{\sigma_2^k(q_2)(1-I^S(q_2))P^k(q_2)}{\#M_0}\right\}}{\frac{1}{\sqrt{k}\#Q_2}\left\{s^k(m) + \sum_{q_2' \notin Q_2^S}\frac{P^k(q_2')}{\#M_0}\right\} + \left(1 - \frac{1}{\sqrt{k}}\right)\left\{S^k(m) + \sum_{q_2' \notin Q_2^S}\frac{\sigma_2^k(q_2')P^k(q_2')}{\#M_0}\right\}},
\end{aligned}$$

where $s^k(m) := \sum_{q_2' \in Q_2} \sigma_m^k(m|q_2')$ converges to $s^*(m) := \sum_{q_2' \in Q_2} \sigma_m^*(m|q_2') < \infty$, and $S^k(m) := \sum_{q_2' \in Q_2} \sigma_2^k(q_2')\sigma_m^k(m|q_2')$ to $S^*(m) := \sum_{q_2' \in Q_2} \sigma_2^*(q_2')\sigma_m^*(m|q_2') < \infty$. By construction, $P^*(q_2) = 0$ for any $q_2 \notin Q_2^S$ and thus $(1 - I^S(q_2))P^*(q_2) = 0$ for all q_2. This implies

$$\begin{aligned}
&\lim_{k\to\infty} \hat{\mu}^k(q_2|m) \\
&= \frac{0 \times \left\{\sigma_m^*(m|q_2) + \frac{0}{\#M_0}\right\} + 1 \times \left\{\mu^*(q_2|m)S^*(m) + \frac{\sigma_2^*(q_2)\cdot 0}{\#M_0}\right\}}{0 \times \left\{s^*(m) + \sum_{q_2' \notin Q_2^S}\frac{0}{\#M_0}\right\} + 1 \times \left\{S^*(m) + \sum_{q_2' \notin Q_2^S}\frac{\sigma_2^*(q_2')\cdot 0}{\#M_0}\right\}} \\
&= \mu^*(q_2|m).
\end{aligned}$$

For $\tilde{q}_2 \in Q_2^S(q_1), q_2 \in Q_2/\{\tilde{q}_2\}$, the belief is

$$\begin{aligned}
\hat{\mu}^k(q_2|\tilde{q}_2) &:= \frac{\hat{\sigma}_m^k(\tilde{q}_2|q_2)\hat{\sigma}_2^k(q_2)}{\sum_{q_2' \in Q_2}\hat{\sigma}_m^k(\tilde{q}_2|q_2')\hat{\sigma}_2^k(q_2')} \\
&= \frac{\frac{1}{k\#Q_2^S}\hat{\sigma}_2^k(q_2)}{\frac{1}{k\#Q_2^S} + \left(1 - \frac{1}{k}\right)P^k(\tilde{q}_2)\hat{\sigma}_2^k(\tilde{q}_2)} \quad (\because I(q_2, \tilde{q}_2) = 0 \text{ by } q_2 \neq \tilde{q}_2) \\
&= \left[\frac{1}{\hat{\sigma}_2^k(q_2)} + (k-1)\#Q_2^S\frac{\hat{\sigma}_2^k(\tilde{q}_2)}{\hat{\sigma}_2^k(q_2)}P^k(\tilde{q}_2)\right]^{-1}
\end{aligned}$$

Here the inverse of RHS satisfies

$$[\cdots] \geq (k-1)\#Q_2^S\frac{\hat{\sigma}_2^k(\tilde{q}_2)}{\hat{\sigma}_2^k(q_2)}P^k(\tilde{q}_2),$$

and we have

$$
\begin{aligned}
(k-1)\#Q_2^S\frac{\hat{\sigma}_2^k(\tilde{q}_2)}{\hat{\sigma}_2^k(q_2)} &= (k-1)\#Q_2^S\frac{1+\left(\sqrt{k}-1\right)\#Q_2\sigma_2^k(\tilde{q}_2)}{1+\left(\sqrt{k}-1\right)\#Q_2\sigma_2^k(q_2)} \\
&> (k-1)\#Q_2^S\frac{1}{1+\left(\sqrt{k}-1\right)\#Q_2} \qquad (\because \sigma_2^k(\tilde{q}_2)>0, \sigma_2^k(q_2)<1) \\
&= \left(\frac{1}{(k-1)\#Q_2^S}+\frac{\#Q_2}{(\sqrt{k}+1)\#Q_2^S}\right)^{-1}, \\
\therefore 0 \le \hat{\mu}^k(q_2|\tilde{q}_2) &< \left(\frac{1}{(k-1)\#Q_2^S}+\frac{\#Q_2}{(\sqrt{k}+1)\#Q_2^S}\right)\frac{1}{P^k(\tilde{q}_2)}.
\end{aligned}
$$

By construction we have $P^*(\tilde{q}_2)>0$ for any $\tilde{q}_2\in Q_2^S$ and thus

$$
\begin{aligned}
0 \le \lim_{k\to\infty}\hat{\mu}^k(q_2|\tilde{q}_2) &\le 0/P^*(\tilde{q}_2)=0, \\
\therefore \quad \lim_{k\to\infty}\hat{\mu}^k(q_2|\tilde{q}_2) &= 0.
\end{aligned}
$$

Because this holds for all $q_2\in Q_2/\{\tilde{q}_2\}$, we have

$$
\lim_{k\to\infty}\hat{\mu}^k(\tilde{q}_2|\tilde{q}_2)=1.
$$

Therefore the belief $\hat{\mu}^*$ speicified in $(\hat{\sigma}^*)$ is actually consistent with $\hat{\sigma}^*$.

Pooling-stay case Fix $q_1\in Q_1$ such that $M_0(q_1)=\emptyset$ and $M_1(q_1)\neq\emptyset$. For each $k\in\mathbf{N}$, set $\{\hat{\sigma}_m^k,\hat{\sigma}_a^k\}$ as

$$
\begin{aligned}
\hat{\sigma}_m^k(\tilde{q}_2|q_1,q_2) &:= 1/(k\#Q_2) \quad \in (0,1), \\
\hat{\sigma}_a^k(1|q_1,\tilde{q}_2) &:= 1-1/k \quad \in (0,1),
\end{aligned}
$$

for each $q_2,\tilde{q}_2\in Q_2$. Then the strategies are completely mixed and converge to σ^* given such q_1.

For $\tilde{q}_2,q_2\in Q_2$, the Bayesian belief is determined as

$$
\begin{aligned}
\hat{\mu}^k(q_2|\tilde{q}_2) &:= \frac{\hat{\sigma}_m^k(\tilde{q}_2|q_2)\hat{\sigma}_2^k(q_2)}{\sum_{q_2'\in Q_2}\hat{\sigma}_m^k(\tilde{q}_2|q_2')\hat{\sigma}_2^k(q_2')} \\
&= \frac{\hat{\sigma}_2^k(q_2)/(k\#Q_2)}{\sum_{q_2'\in Q_2}\hat{\sigma}_2^k(q_2')/(k\#Q_2)} \qquad = \hat{\sigma}_2^k(q_2).
\end{aligned}
$$

Hence as $k\to\infty$ we have

$$
\lim_{k\to\infty}\hat{\mu}^k(\tilde{q}_2|q_2)=\hat{\sigma}_2^*(q_2).
$$

Therefore the belief $\hat{\mu}^*$ speicified in $(\hat{\sigma}^*)$ is actually consistent with $\hat{\sigma}^*$.

Pooling-exit case Fix $q_1 \in Q_1$ such that $M_0(q_1) \neq \emptyset$ and $M_1(q_1) = \emptyset$. For each $k \in \mathbf{N}$, set $\{\hat{\sigma}_m^k, \hat{\sigma}_a^k\}$ as

$$\hat{\sigma}_m^k(m|q_1, q_2) := \sigma_m^k(m|q_1, q_2) \quad \in (0,1);$$
$$\hat{\sigma}_a^k(1|q_1, m) := 1/k \quad \in (0,1)$$

for each $q_2 \in Q_2, m \in M_0(q_1)$.

For each $q_2 \in Q_2, m \in M_0$, the Bayesian belief is determined as

$$\begin{aligned}
\hat{\mu}^k(q_2|m) &:= \frac{\hat{\sigma}_m^k(m|q_2)\hat{\sigma}_2^k(q_2)}{\sum_{q_2' \in Q_2} \hat{\sigma}_m^k(m|q_2')\hat{\sigma}_2^k(q_2')} \\
&= \frac{\left\{\frac{1}{\sqrt{k}\#Q_2} + \left(1 - \frac{1}{\sqrt{k}}\right)\sigma_2^k(q_2)\right\}\sigma_m^k(m|q_2)}{\sum_{q_2' \in Q_2}\left\{\frac{1}{\sqrt{k}\#Q_2} + \left(1 - \frac{1}{\sqrt{k}}\right)\sigma_2^k(q_2')\right\}\sigma_m^k(m|q_2')} \\
&= \frac{\frac{1}{\sqrt{k}\#Q_2}\sigma_m^k(m|q_2) + \left(1 - \frac{1}{\sqrt{k}}\right)\mu^k(q_2|m)S^k(m)}{\frac{1}{\sqrt{k}\#Q_2}s^k(m) + \left(1 - \frac{1}{\sqrt{k}}\right)S^k(m)},
\end{aligned}$$

where $s^k(m) := \sum_{q_2' \in Q_2} \sigma_m^k(m|q_2')$ converges to $s^*(m) := \sum_{q_2' \in Q_2} \sigma_m^*(m|q_2') < \infty$, and $S^k(m) := \sum_{q_2' \in Q_2} \sigma_2^k(q_2')\sigma_m^k(m|q_2')$ to $S^*(m) := \sum_{q_2' \in Q_2} \sigma_2^*(q_2')\sigma_m^*(m|q_2') < \infty$. This implies

$$\lim_{k \to \infty} \hat{\mu}^k(q_2|m) = \frac{0 \times \sigma_m^*(m|q_2) + 1 \times \mu^*(q_2|m)S^*(m)}{0 \times s^*(m) + 1 \times S^*(m)} = \mu^*(q_2|m).$$

Therefore the belief $\hat{\mu}^*$ speicified in ($\hat{\sigma}^*$) is actually consistent with $\hat{\sigma}^*$.

Sequential rationality. We prove the optimality of the strategy profile $\hat{\sigma}^*$ given the belief $\hat{\mu}^*$. Given $(\hat{\sigma}_a^*, \hat{\sigma}_m^*)$, the probability for the entrant to stay is the same probability as $P^*(q_1, q_2)$ in the original equilibrium. Hence the incumbent's expected profit under any strategy σ_2 remains the same, given the entrant's capacity strategy σ_1^*. So does for the entrant's. Therefore σ_1^*, σ_2^* are still the optimal capacity strategies in the equilibrium $(\hat{\sigma}^*, \hat{\mu}^*)$. Next, we check the strategies of message and of the additional lender in each case.

Separating case Fix $q_1 \in Q_1$ such that $M_0(q_1) = \emptyset$ and $M_1(q_1) \neq \emptyset$. According to Lemma 2 (a), the additional lender's strategy $\hat{\sigma}_a^*$ under the belief $\hat{\mu}^*$ should be

$$\hat{\sigma}_a^*(1|q_1, \tilde{q}_2) = 1 \quad \text{for all } \tilde{q}_2 \in Q_2^S(q_1),$$
$$\hat{\sigma}_a^*(1|q_1, m) = 0 \quad \text{for all } m \in M_0(q_1).$$

Notice that $\tilde{q}_2 \in Q_2^S(q_1)$ implies $\tilde{q}_2 \leq q_2^S(q_1)$ and $\pi^1(q_1, \tilde{q}_2) + B \geq w_0 + \underline{\delta}(q_1) \geq 0$, and that for any $m \in M_0(q_1)$ we have $\sigma_a^*(1|q_1, m) = 0$, namely $\pi^1(q_1, q_2) + B < 0$ for some q_2 in the support of $\mu^*(q_2|q_1, m) = \hat{\mu}^*(q_2|q_1, m)$. So $\hat{\sigma}_a^*$ specified in ($\hat{\sigma}^*$) is actually an optimal strategy.

Applying Lemma 2 (b) to the entrant's message strategy $\hat{\sigma}_m^*$, we find

$$\hat{\sigma}_m^*(\cdot|q_1, q_2) > 0 \quad \text{only on the set}$$
$$\begin{cases} Q_2^S(q_1) & \text{if } q_2 < q_2^S(q_1), \\ M_0(q_1) & \text{if } q_2 > q_2^S(q_1), \\ \text{both of these two sets above} & \text{if } q_2 = q_2^S(q_1), \end{cases}$$

and the messages in each set are indifferent. So $\hat{\sigma}_m^*$ specified in ($\hat{\sigma}^*$) is actually an optimal strategy.

Pooling-stay case Fix $q_1 \in Q_1$ such that $M_0(q_1) = \emptyset$ and $M_1(q_1) \neq \emptyset$. In the original equilibrium, the Bayesian belief and the strategy profile in the perturbation satisfy for any $q_2 \in Q_2, m \in M_1(q_1)$

$$\sigma_2^k(q_2)\sigma_m^k(m|q_1, q_2) = \mu^k(q_2|m) \sum_{q_2' \in Q_2} \sigma_2^k(q_2')\sigma_m^k(m|q_1, q_2'),$$

$$\therefore \quad \sigma_2^k(q_2) = \sum_{m \in M_1(q_1)} \mu^k(q_2|q_1, m) \sum_{q_2' \in Q_2} \sigma_2^k(q_2')\sigma_m^k(m|q_1, q_2').$$

Fix $\tilde{q}_2 \in Q_2$. Because $\hat{\mu}^*(q_2|q_1, \tilde{q}_2) = \hat{\sigma}_2^*(q_2)$, we have at the limit

$$\hat{\mu}^*(q_2|q_1, \tilde{q}_2) = \hat{\sigma}_2^*(q_2) = \sum_{m \in M_1(q_1)} \mu^*(q_2|q_1, m) \sum_{q_2' \in Q_2} \sigma_2^*(q_2')\sigma_m^*(m|q_1, q_2').$$

If $\hat{\mu}^*(q_2|q_1, \tilde{q}_2) > 0$, then there exists some $m \in M_1(q_1)$ such that

$$\mu^*(q_2|q_1, m) \sum_{q_2' \in Q_2} \sigma_2^*(q_2')\sigma_m^*(m|q_1, q_2') > 0, \quad \therefore \mu^*(q_2|q_1, m) > 0.$$

Applying Lemma 1 a) to the original equilibrium, we find

$$\pi^1(q_1, q_2) + B \geq 0.$$

This holds for all q_2 with $\hat{\mu}^*(q_2|q_1, \tilde{q}_2) > 0$. Now applying it to the new equilibrium, we have $\sigma_a^1(1|q_1, \tilde{q}_2) = 1$. So $\hat{\sigma}_a^*$ specified in ($\hat{\sigma}^*$) is actually an optimal strategy.

Because any message $\tilde{q}_2 \in Q_2$ induces the same additional lender's strategy $\sigma_a^*(1|q_1, \tilde{q}_2) = 1$ and the same total payment $\underline{\delta}(q_1)$, all messages in $\hat{M}(q_1) = Q_2$ are indifferent for the entrant. So $\hat{\sigma}_m^*$ specified in ($\hat{\sigma}^*$) is an optimal strategy.

Pooling-exit case In this case the strategy and belief profile is exactly same as the original. So the optimality holds from it.

Therefore we have found the consistency of belief $\hat{\mu}^*$ with $\hat{\sigma}^*$ and the sequential rationality of the strategy profile $\hat{\sigma}^*$, and thus the profile $(\hat{\mu}^*, \hat{\sigma}^*)$ is a sequential equilibrium under the quasi-direct mechanism $(\hat{C}, \hat{M})$. □

B Proof of Corollary 1

Before the proof, let us rephrase Corollary 1 a bit more mathematically:

Corollary 1. *Suppose that the entrant's internal capital w_0 and private value $\bar{V}$ are smaller than the maximum predatory loss in the benchmark equilibrium $\bar{L}^P(q_1^\dagger)$:*

$$\bar{V} + w_0 < \bar{L}^P(q_1^\dagger) = -\pi^1(q_1^\dagger, \bar{q}_2^P(q_1^\dagger)) \tag{17}$$

Then, compared with the benchmark $q_i^\dagger$, the entrant's capacity level q_1^* shrinks while the incumbent's q_2^* expands:

$$q_1^* < q_1^\dagger, \quad q_2^* > q_2^\dagger.$$

Proof. Recall the definition of *the maximum predatory loss* $\bar{L}^P(q_1)$

$$\bar{L}^P(q_1) = -\pi^1(q_1, \bar{q}_2^P(q_1)),$$

and that of *the incumbent's maximum predatory capacity* $\bar{q}_2^P(q_1)$

$$\pi^2(0, \bar{q}_2^P(q_1)) = \pi^2(q_1, q_2^{BR}(q_1)), \text{ and } \pi_2^2(0, \bar{q}_2^P(q_1)) < 0,$$

where $q_2^{BR}(q_1)$ is the incumbent's optimal capacity without predation, namely

$$q_2^{BR}(q_1) = \arg\max_{q_2} \pi^2(q_1, q_2).$$

Differentiating the above two equations with regard to q_1, we have

$$\frac{d\bar{L}^P}{dq_1}(q_1) = -\pi_1^1(q_1, \bar{q}_2^P(q_1)) - \pi_2^1(q_1, \bar{q}_2^P(q_1))\frac{d\bar{q}_2^P}{dq_1}(q_1), \tag{18}$$

and

$$\pi_2^2(0, \bar{q}_2^P(q_1))\frac{d\bar{q}_2^P}{dq_1}(q_1) = \pi_1^2(q_1, q_2^{BR}(q_1)).$$

The latter yields

$$\frac{d\bar{q}_2^P}{dq_1}(q_1) = \frac{\pi_1^2(q_1, q_2^{BR}(q_1))}{\pi_2^2(0, \bar{q}_2^P(q_1))} > 0$$

by $\pi_1^2(\cdot) < 0$ and the definition of $\bar{q}_2^P$.

The equilibrium capacity $\mathbf{q}^*$ is a solution of (12). With the Lagrange multiplier λ of the non-predation condition (11), $\mathbf{q}^*$ should satisfy the first order conditions:[20]

$$\pi_1^1(\mathbf{q}^*) = \lambda\frac{d\bar{L}^P}{dq_1}(q_1^*), \quad \pi_2^2(\mathbf{q}^*) = 0.$$

If $\lambda = 0$ then the equilibrium capacity $\mathbf{q}^*$ was exactly the same as the benchmark. Because the non-predatory condition (11) is assumed to be violated at $\mathbf{q}^\dagger$ and thus $\mathbf{q}^\dagger$ cannot be a solution of (12), the Lagrange multiplier λ must be positive.

Substituting (18) into the entrant's FOC yields

$$\pi_1^1(\mathbf{q}^*) - c_1 = \lambda\left[-\pi_1^1(q_1^*, \bar{q}_2^P(q_1^*)) - \pi_2^1(q_1^*, \bar{q}_2^P(q_1^*))\frac{d\bar{q}_2^P}{dq_1}(q_1^*)\right].$$

Since $\pi_{12}^1(\cdot) \leq 0$ and $q_2^* = q_2^{BR}(q_1^*) < \bar{q}_2^P(q_1^*)$, we have

$$\pi_1^1(\mathbf{q}^*) \geq \pi_1^1(q_1^*, \bar{q}_2^P(q_1^*)).$$

[20]To determine the entrant's best response uniquely, it is sufficient that the marginal revenue minus the implicit marginal "cost of financing" decreases to the entrant's capacity level q_1, given the incumbent's q_2:

$$\begin{aligned}&\frac{\partial}{\partial q_1}\left[\pi_1^1(\mathbf{q}) - \lambda\left\{-\pi_1^1(q_1, \bar{q}_2^P) - \pi_2^1(q_1, \bar{q}_2^P)\frac{d\bar{q}_2^P}{dq_1}(q_1)\right\}\right]\\ &= \pi_{11}^1(\mathbf{q}) + \lambda\left\{\pi_{11}^1(q_1, \bar{q}_2^P) + 2\pi_{12}^1(q_1, \bar{q}_2^P)\frac{d\bar{q}_2^P}{dq_1}(q_1) + \pi_{22}^1(q_1, \bar{q}_2^P)\frac{d^2\bar{q}_2^P}{dq_1^2}(q_1)\right\} < 0\end{aligned}$$

This is guaranteed if π^1 is linear in q_2 ($\pi_{22}^1(\cdot) = 0$), for example $\pi^1(\mathbf{q}) = (a - q_1 - q_2)q_1 - c_1 q_1$.

Combining these two expressions, we obtain

$$(1+\lambda)\pi_1^1(\mathbf{q}^*) \geq -\lambda\pi_2^1(q_1^*, \bar{q}_2^P(q_1^*))\frac{d\bar{q}_2^P}{dq_1}(q_1^*).$$

As we have verified that $d\bar{q}_2^P/dq_1 > 0$, this implies

$$\pi_1^1(\mathbf{q}^*) > 0.$$

That is, the entrant's response curve shifts downward (at least near the equilibrium $\mathbf{q}^*$). On the contrary, that of q_2 obviously remains the same. We therefore conclude that the entrant's equilibrium capacity shrinks, while the incumbent's expands, compared with the benchmark.

□

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