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Two Country Model and the Keynesian Multiplier

Suppose the export function in "The Keynesian Model of Equilibrium and the Trade Balance" was given by:

(7) $EX = \overline{EXP} + m^*Y^*$ Export function

Then the resulting equation would be:

(12) $$\boxed{Y = \left(\frac{1}{1-c(1-t)+m}\right)[\overline{A} + \overline{EXP} - \overline{IMP} + m^*Y^*]}$$

Letting t = 0 and 1-c = s:

(12') $$\boxed{Y = \left(\frac{1}{s+m}\right)[\overline{A} + \overline{EXP} - \overline{IMP} + m^*Y^*]}$$

For the foreign country:

(i) $Y^* = AD^* = \overline{CO}^* + c^*(Y^* - \overline{TA}^*) + \overline{IN}^* + \overline{GO}^* + \overline{EXP}^* + mY - \overline{IMP}^* - m^*Y^*$

But for $\overline{A}^* \equiv \overline{CO}^* - c^*\overline{TA}^* + \overline{IN}^* + \overline{GO}^*$

(ii) $Y^* = \overline{A}^* + c^*Y^* + \overline{EXP}^* + mY - \overline{IMP}^* - m^*Y^*$

(iii) $Y^* - c^*Y^* + m^*Y^* = \overline{A}^* + \overline{EXP}^* + mY - \overline{IMP}^*$

In this two country set-up where foreign exports are home imports and vice versa,

(iv) $Y^*(1 - c^* + m^*) = \overline{A}^* + \overline{IMP} + mY - \overline{EXP}$

(v) $$Y^* = \left(\frac{1}{s^* + m^*}\right)[\overline{A}^* + \overline{IMP} + mY - \overline{EXP}]$$

Equilibrium income is given by substituting (v) into (12'):

(vi) $$Y = \left(\frac{1}{s+m}\right)[\overline{A} + \overline{EXP} - \overline{IMP} + m^*\left(\frac{1}{s^*+m^*}\right)\{\overline{A}^* + \overline{IMP} + mY - \overline{EXP}\}]$$

Rearranging:

(vii) $Y(s+m) = [\breve{A} + \left(\frac{m^* m}{s^*+m^*}\right) Y]$ where $\breve{A} \equiv [\overline{A} + \overline{EXP} - \overline{IMP} + \left(\frac{m^*}{s^*+m^*}\right)\{\overline{A}^* + \overline{IMP} - \overline{EXP}\}]$

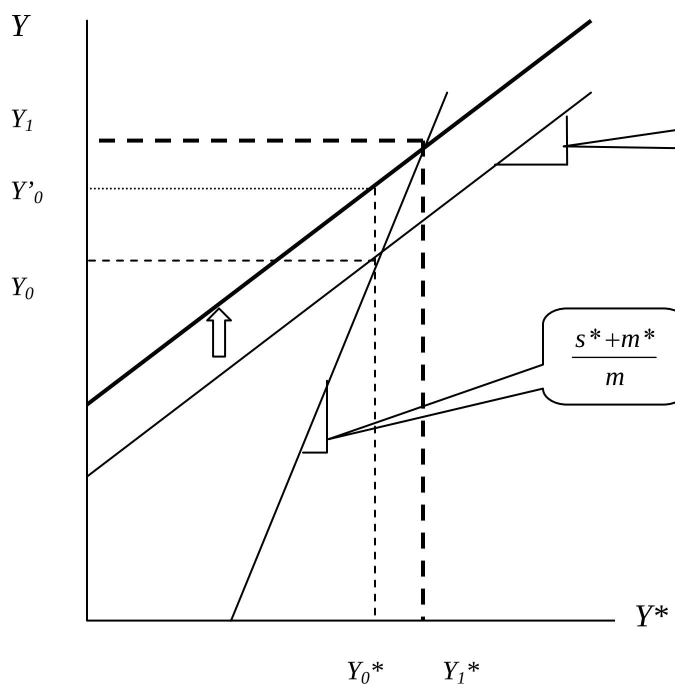
Solving

(viii) $Y(s+m-\left(\frac{m^* m}{s^*+m^*}\right)) = [\breve{A}]$

(ix) $Y_0 = \breve{\alpha}\breve{A}$ where $\breve{\alpha} \equiv \left(\frac{1}{s+m-\left(\frac{m^* m}{s^*+m^*}\right)}\right)$ and hence $\frac{\Delta Y}{\Delta GO} = \breve{\alpha} \geq \frac{1}{s+m}$

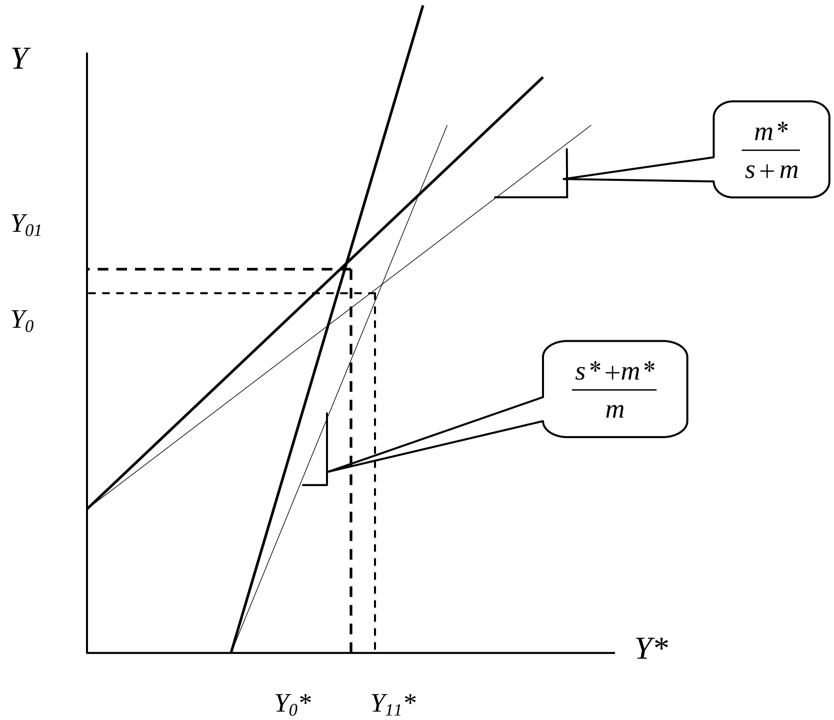
Notice for m^*=0, $\breve{\alpha} = \overline{\alpha}$. And that for the strange case of m^*=1, $\breve{\alpha} \equiv \left(\frac{1}{s+m-\left(\frac{m}{s^*+1}\right)}\right)$

The model can be graphically depicted as:



Consider a change in government spending, ΔGO. The distance Y'_0-Y_0 is equal to $\overline{\alpha}\Delta GO$. However, because the greater imports results in higher income in the foreign country, then exports rise, giving an extra boost to home income, and the change in income is $\breve{\alpha}\Delta GO \geq \overline{\alpha}\Delta GO$.

What happens when the foreign marginal propensity to import rises? Then both curves should get steeper.



The way this is drawn, the equilibrium income at home rises when *m** rises. The multiplier should also rise with *m**, by analogy. But it is actually difficult to pin this down definitely since this conclusion depends on how the rotations are drawn. To figure out whether this result holds mathematically for all parameter values, we need to obtain $\frac{\partial \breve{\alpha}}{\partial m^*}$. Using the expression for the multiplier in (ix), re-express the multiplier:

(x) $$\breve{\alpha} \equiv \left(s + m - \left(\frac{m^* m}{s^* + m^*} \right) \right)^{-1}$$

take the derivative with respect to *m**, using the chain rule and the product rule.

(xi) $$\frac{\partial \breve{\alpha}}{\partial m^*} \equiv (-1) \times \left(s + m - \left(\frac{m^* m}{s^* + m^*} \right) \right)^{-2} \left((-1) \frac{\partial \left(\frac{m^* m}{s^* + m^*} \right)}{\partial m^*} \right)$$

Substitute out for $\breve{\alpha} \equiv \left(s + m - \left(\frac{m^* m}{s^* + m^*} \right) \right)^{-1}$ to obtain:

(xii) $$\frac{\partial\breve{\alpha}}{\partial m^*} \equiv \breve{\alpha}^2 \left(\frac{\partial\left(\frac{m^*m}{s^*+m^*}\right)}{\partial m^*} \right) \text{ so } \frac{\partial\breve{\alpha}}{\partial m^*} > 0 \text{ iff } \left(\frac{\partial\left(\frac{m^*m}{s^*+m^*}\right)}{\partial m^*} \right) > 0$$

(xiii) $$\left(\frac{\partial\left(\frac{m^*m}{s^*+m^*}\right)}{\partial m^*} \right) = \left(\frac{m}{s^*+m^*} \right) + (-1)(m^*m)(s^*+m^*)^{-2}$$

(xiv) $$\left(\frac{\partial\left(\frac{m^*m}{s^*+m^*}\right)}{\partial m^*} \right) = \frac{m(s^*+m^*)}{(s^*+m^*)^2} - \frac{(m^*m)}{(s^*+m^*)^2}$$

so the required condition holds if $s^* > 0$, which is always true. Hence the multiplier increases with m^*.

[**Note**: You would not be required to replicate this derivation in an exam. This handout is for your information only]