Problem Set 3 Answers

Due in Lecture on Wednesday, November 1st. Box in your answers to the algebraic questions.

1. Fiscal policy in an IS-LM model

Suppose the real side of the economy is given by:

\begin{align}
(11.1) \quad & Y = AD \\
(11.2) \quad & AD = C + I + G + X - IM \\
(11.3) \quad & C = \overline{C} + c(Y - T) \\
& T = \overline{T} \\
(12.1) \quad & I = \overline{I} - bi \quad \text{Investment depends on interest rate} \\
(11.4') \quad & IM = IM + mY - nq \quad \text{Real exchange rate fixed} \\
(11.11) \quad & X = \overline{X} + \nu q \\
& G = \overline{G}
\end{align}

and the monetary sector is given by:

\begin{align}
(12.3) \quad & \frac{M^d}{P} = \frac{M^s}{P} \quad \text{Equilibrium condition} \\
(12.4) \quad & \frac{M^s}{P} = \frac{M}{P} \quad \text{Money supply} \\
(12.5) \quad & \frac{M^d}{P} = kY - hi \quad \text{Money demand}
\end{align}

For now, we ignore the external balance condition.

1.1 Solve for the IS curve, with \( Y \) on the left hand side. Show your work.

Substitute in all the equations into equation (11.1)
\( Y = AD = \overline{C} + c(Y - \overline{T}) + \overline{I} - bi + \overline{G} + (\overline{X} + \nu \overline{q}) - (IM - nq + mY) \)

Solve for \( Y \):
\( Y = \overline{\alpha} [\overline{A} + \overline{X} - \overline{IM} + (n + \nu)\overline{q} - bi] \)

where \( \overline{\alpha} = \left( \frac{1}{1 - c + m} \right), \quad \overline{A} \equiv \overline{C} - c\overline{T} + \overline{I} + \overline{G} \)
1.2 Solve for the LM curve, with $i$ on the left hand side. Show your work.

Substitute (12.4) and (12.5) into (12.3):

$$kY - hi = \left( \frac{M}{P} \right)$$

$$- hi = \left( \frac{M}{P} \right) - kY$$

$$i = \left( \frac{1}{h} \right) \left( \frac{M}{P} \right) + \left( \frac{k}{h} \right) Y$$

1.3 Graph the IS and LM curves on a single graph. Show the vertical intercepts, the slopes, and the intersection of the two curves.

1.4 Solve for equilibrium income. Show your work.

$$Y = \bar{a}[\bar{A} + \bar{X} - \bar{M} + (n + v)\bar{q} - b \left( - \left( \frac{1}{h} \right) \left( \frac{M}{P} \right) \right) + \left( \frac{k}{h} \right) Y]$$
\[ Y(1 - c + m) = [\bar{\bar{A}} + \bar{X} - \bar{\bar{M}} + (n + v)\bar{q} - b \left( -\frac{1}{h} \left( \frac{M}{P} \right) + \left( \frac{k}{h} \right) Y \right)] \]

\[ Y(1 - c + m + bk/h) = [\bar{\bar{A}} + \bar{X} - \bar{\bar{M}} + (n + v)\bar{q} + \left( \frac{b}{h} \right) \left( \frac{M}{P} \right)] \]

\[ Y_0 = \tilde{\alpha} [\bar{\bar{A}} + \bar{X} - \bar{\bar{M}} + (n + v)\bar{q} + \left( \frac{b}{h} \right) \left( \frac{M}{P} \right)] \] where \( \tilde{\alpha} \equiv \left( \frac{1}{1 - c + m + b h} \right) \)

1.5 Calculate the change in income resulting from a given change in government spending on goods and services, \( \Delta G \).

Take the total differential of your answer to question 1.3:

\[ \Delta Y = \tilde{\alpha} \left[ \Delta \bar{A} + \Delta X + \Delta IM + (n + v)\Delta q + \left( \frac{b}{h} \right) \Delta \left( \frac{M}{P} \right) \right] \]

Set all the other changes to zero, and set : \( \Delta A = \Delta G \)

\[ \Delta Y = \tilde{\alpha} \Delta G \]

1.6 Show graphically what happens when government spending is increased. Clearly indicate the distance of the curve shifts, and the amount of the income change.

\[ (\bar{\bar{A}} + \Delta G + \bar{X} - \bar{\bar{M}} + (n + v)q) / b \]
1.7 Is the effect of government spending on income greater or less in this model, as compared to the simple Keynesian model? Explain why the difference occurs, in words.

The effect is less, because of crowding out. As income rises, demand for money rises. Since the money supply is fixed, the equilibrating interest rate in the money market must rise. As a consequence, investment falls, and this decreases aggregate demand and output relative to what it would have been if interest rates remain unchanged. As a consequence, output rises by only $\Delta Y = \hat{\alpha} \Delta G$, rather than $\Delta Y = \overline{\alpha} \Delta G$.

1.8 Answer 1.7 again, if the interest sensitivity of money demand were zero. Explain why this is true.

When the interest sensitivity of money demand is zero, increases in income lead to increases in money demand that cannot be reduced by higher interest rates (hence, the LM curve is vertical). Only when investment declines sufficiently to offset the increase in income is money demand equal to the original money supply, so that there is complete crowding out of investment, and crowding out of income.

1.9 Answer 1.7 again, if the interest sensitivity of investment were zero. Explain why this is true.
When the interest sensitivity of investment is zero, then the IS slope is infinity (the IS curve is vertical). A zero $b$ implies that the link between interest rates and the real side of the economy is cut, so there is zero crowding out of investment due to the higher interest rate resulting from higher transactions demand for money, that is in turn associated with higher income. This is shown in the figure below.

2. Monetary policy in an IS-LM model

Using the model laid out in Question 1,

Take the total differential of your answer to question 1.3:

$$\Delta Y = \alpha \left[ \Delta A + \Delta X + \Delta IM + (n + v)\Delta q + \left( \frac{b}{h} \right) \Delta \left( \frac{M}{P} \right) \right]$$

Set all the changes to zero with the exception of the real money supply:

$$\Delta Y = \alpha \left( \frac{b}{h} \right) \Delta \left( \frac{M}{P} \right)$$

2.2 Show graphically what happens when the real money stock is increased. Clearly indicate the distance of the curve shifts and the amount of the income change.
2.3 Suppose instead that the interest sensitivity of investment were very low. Show graphically the effect upon output and interest rates that results from an increase of the real money stock. Clearly indicate the distance of the curve shifts and the amount of the income change.
2.4 Suppose the interest sensitivity of money demand were infinite. Show graphically the effect upon output and interest rates that results from an increase of the real money stock. Clearly indicate the distance of the curve shifts and the amount of the income change.

The LM curve shifts sideways.

2.5 Assume non-zero values for the interest sensitivity of investment and money demand. Show graphically how the Fed could keep the interest rate constant as the government pursued an expansionary fiscal policy. Clearly indicate the distance of the curve shifts and the amount of the income change. What is the effect on output?

The monetary authorities could shift around the LM curve always keeping the interest rate constant. This is accommodative monetary policy. In this case, fiscal policy would be as effective as it would be in the simple Keynesian model. In the graph below, a shift out in the IS curve is matched by an increase in the LM curve sufficient to keep the interest rate constant.
3. Policy under Fixed Exchange Rates in the IS-LM-BP=0 model

Suppose the economy is given by the following set of equations.

\begin{align}
 Y &= \overline{\alpha} [\overline{A} + \overline{X} - \overline{IM} + (n + v)\overline{q} - bi] \quad \text{<IS curve>}
 \end{align}

where \( \overline{\alpha} = \left( \frac{1}{1 - c + m} \right) \), \( \overline{A} = \overline{C} - c\overline{T} + \overline{I} + \overline{G} \)

\begin{align}
 i &= -\left( \frac{1}{h} \right) \left( \frac{\overline{M}}{\overline{P}} \right) + \left( \frac{k}{h} \right) Y \quad \text{<LM curve>}
 \end{align}

\begin{align}
 i &= -\left( \frac{1}{\kappa} \right) [(\overline{X} - \overline{IM} + \overline{FA}) + (n + v)\overline{q} + \overline{I}^* + \left( \frac{m}{\kappa} \right) Y] \quad \text{<BP=0 curve>}
 \end{align}

3.1 Draw a graph of initial equilibrium, where the goods and money markets are in equilibrium, as is the balance of payments. Assume that \( m/\kappa > k/h \).
3.2 Show what happens if the government decreases government spending by $\Delta G$, both immediately, and over time in the absence of central bank sterilization. Note, the money base changes by an amount equal to the change in foreign exchange reserves,

$$\Delta MB = \Delta FXRes = BP = -ORT$$

And the change in the money stock is a multiple of the change in the money base,

$$\Delta M = \text{constant} \times \Delta MB$$
\[ \Delta MB = \Delta Res = BP = -ORT \]

The decrease in government spending shifts the IS curve in by \( \alpha \Delta G \) (this is indicated by arrow [1]). At the new level of income and interest rates, there is a balance of payments surplus, so that foreign exchange reserves increase, and – in the absence of sterilization – the money base increases, thereby increasing the money supply. Hence the LM shifts out (as indicated by arrow [2]), yielding \( Y_1, i_1 \).

3.3 Answer 3.2, assuming the central bank sterilizes changes in official reserves. At the new equilibrium, what is true about (i) the level of output; (ii) the level of investment; (iii) the real exchange rate; and (iv) the trade balance?

The decrease in government spending shifts the IS curve in by \( \alpha \Delta G \) (this is indicated by arrow [1]). At the new level of income and interest rates, there is a balance of payments surplus, so that foreign exchange reserves increase. Since the central bank is sterilizing, the money base is unchanged, so the money supply is also unchanged. Hence the LM stays in place, yielding \( Y_2, i_2 \). Income is lower, investment is higher, real exchange rate is constant, and the trade balance is larger.

3.4 Redraw 3.1, and show the impact of a monetary contraction, both immediately and over time. Assume over time, capital flows are sterilized.
3.5 Explain why the process you lay out in 3.4 occurs.

A decrease in \(M\) to \(M'\) decreases the real money stock, shifting the LM to the left (arrow [1]). Interest rates rise above that consistent with external equilibrium; there is a balance of payments surplus so that foreign exchange reserves increase. Since inflows are sterilized, the money supply stays fixed at \(M'\), so output and interest rates stabilize at \(Y_3\) and \(i_3\).

3.6 Answer 3.4 if capital flows are not sterilized.

In this case, the LM shifts back to where it started, so that the original income level and interest rate are restored at \(Y_0\) and \(i_0\). As capital flows in, foreign exchange reserves increase, increasing the money base, in the absence of sterilization. Consequently, the money supply increases, and the LM shifts back out (arrow [2]).
4. Policy under Floating Exchange Rates in the IS-LM-BP=0 model

Suppose the exchange rate is floating. I assume \( m/k > k/h \) (\( m/k < k/h \) is covered in the textbook; just reverse the example given in the textbook).

4.1 Assume the economy described above is under a floating exchange rate regime. Show graphically what happens if the government decreases government spending.

4.2 Explain your answer to 4.1.

The IS curve initially shifts in by \( \Delta G \) (arrow [1]). The interest rate \((i_1)\) is above that required to obtain balance of payments equilibrium \((i_{BP=0}|Y_1)\) leads to a appreciation of the real exchange rate to \( q' \). This in turn shifts inward the IS curve, and the BP=0 curve upward (as denoted by arrow shifts [2]). Final income and interest rate is \( Y_2 \) and \( i_2 \).

4.3 Now examine a monetary contraction from initial equilibrium, carefully distinguishing between initial impact, and the effect over time.

The LM shifts in first (arrow [1]). In this initial equilibrium with lower income \( Y_1 \), the resulting interest rate \((i_1)\) is greater than required for external equilibrium \((i_{BP=0}|Y_1)\). As a consequence, there is an incipient balance of payments surplus and the exchange rate appreciates. The resulting decrease in net exports means that the required interest rate for external equilibrium rises (the BP=0 curve shifts upward – arrow [2]). The decrease in net exports means that domestic aggregate demand falls, and the IS curve shifts in (arrow [2]). The equilibrium settles at income level \( Y_2 \) and interest rate \( i_2 \).
4.4 Explain why monetary policy has a larger effect in this open economy as opposed to that in a closed economy.

With floating exchange rates, monetary policy can affect aggregate demand through two channels (investment, and the trade balance via the exchange rate), rather than only investment, which is the only channel in closed economies.

4.5 Answer 4.1, assuming $\kappa = \infty$.

The decrease in government spending shifts the IS curve (arrow 1). The resulting interest rate $i_1$ is below the interest rate consistent with external balance ($i_0$). The balance of payments deficit of infinite magnitude induces an immediate currency depreciation that shifts out the IS curve (arrow 2). Only when the IS curve returns to its previous position will equilibrium be restored.