Public Affairs 854 | Menzie D. Chinn
Spring 2021 | Social Sciences 7418
University of Wisconsin-Madison

Problem Set 2 Answers

Due in Lecture on Thursday, March 4. $\boxed{\text{"}Box - in\text{"}}$ your answers to the algebraic questions.

1. Expenditure switching versus expenditure reduction

Eq.No.	Equation	Description
(1)	$Y = AD$	Output equals aggregate demand – an equilibrium condition
(2)	$AD \equiv C + I + G + EX - IM$	Definition of aggregate demand
(3)	$C = \overline{CO} + c(Y - T)$	Consumption function, c is the marginal propensity to consume
(4)	$T = \overline{TA} + tY$	Tax function; $\overline{TA}$ is lump sum taxes, t is tax rate.
(5)	$I = \overline{IN}$	Investment function
(6)	$G = \overline{GO}$	Government spending on goods and services
(7)	$EX = \overline{EXP} + vq$	Export spending
(8)	$IM = \overline{IMP} + mY - nq$	Import spending

Equilibrium income, Y_0, is given by:

(9) $$Y_0 = \left(\frac{1}{1 - c(1-t) + m}\right)[\overline{A} + \overline{EXP} - \overline{IMP} + (n+v)q] \quad \text{let} \quad \overline{\alpha} \equiv \left(\frac{1}{1 - c(1-t) + m}\right)$$

1.1 Solve for the total differential (break the change in Y into its constituent parts).

$$\boxed{\Delta Y = \overline{\alpha}[\Delta A + \Delta EXP - \Delta IMP + (n+v)\Delta q]}$$

1.2 Calculate the change in the trade balance given a \$1 (billion) increase in government expenditures.

Using the expression for the change in the trade balance,

$$\Delta TB = \Delta\overline{EXP} - \Delta\overline{IMP} - m\Delta Y + (n+v)\Delta q$$

set $\Delta EXP = \Delta IMP = \Delta q = 0$ and set $\Delta A = \Delta GO$ obtain:

$\boxed{\Delta TB = -m\overline{\alpha}\Delta GO}$ so for a one (billion) dollar *decrease* in government spending $\Delta GO = -1$,

$\Delta TB = m\overline{\alpha}$

1.3 Calculate the change in the trade balance given a one unit change in the real exchange rate (a one unit depreciation of the real value of the dollar). Remember: GDP responds to a change in the real exchange rate, q.

Note that the change in the trade balance is given by:

$$\Delta TB = \Delta EXP - \Delta IMP - m\Delta Y + (n+v)\Delta q$$

Recall that from your answer to 1.1 above,

$$\Delta Y = \overline{\alpha}[\Delta A + \Delta EXP - \Delta IMP + (n+v)\Delta q]$$

And holding government spending constant:

$$\Delta Y = \overline{\alpha}(n+v)\Delta q$$

So substituting into the expression for the change in TB, holding the autonomous components of exports and imports constant yields:

$$\Delta TB = (1 - m\overline{\alpha})(n+v)\Delta q$$

1.4 If the marginal propensity to import rises, then what is true about the relative effectiveness of expenditure switching versus expenditure reducing as a means of reducing a trade deficit?

As m rises, $\left|\frac{\Delta TB}{\Delta GO}\right|$ increases. On the other hand, $\frac{\Delta TB}{\Delta q} = (1 - m\overline{\alpha})(n+v)$, so as m rises, the effectiveness of exchange rate depreciation declines. Hence, the relative effectiveness of fiscal retrenchment increases.

2. Fiscal policy in an IS-LM model

Suppose the real side of the economy is given by:

(1)	$Y = AD$	Output equals aggregate demand – an equilibrium condition
(2)	$AD \equiv C + I + G + EX - IM$	Definition of aggregate demand
(3)	$C = \overline{CO} + c(Y - T)$	Consumption function, c is the marginal propensity to consume
(4)	$T = \overline{TA} + tY$	Tax function; $\overline{TA}$ is lump sum taxes, t is tax rate.
(5')	$I = \overline{IN} - bi$	Investment function
(6)	$G = \overline{GO}$	Government spending on goods and services
(7)	$EX = \overline{EXP} + vq$	Export spending
(8)	$IM = \overline{IMP} + mY - nq$	Import spending

and the monetary sector is given by:

Eq.No.	Equation	Description
(10)	$\frac{M^d}{P} = \frac{M^s}{P}$	Equilibrium condition
(11)	$\frac{M^s}{P} = \frac{\overline{M}}{P}$	Money supply
(12)	$\frac{M^d}{P} = kY - hi$	Money demand

For now, we ignore the external balance condition.

2.1 Solve for the IS curve, with Y on the left hand side. Show your work.

Substitute in (1)-(8) equations:

$$Y = AD = \overline{CO} + c(Y - \overline{TA} - tY) + \overline{IN} - bi + \overline{GO} + \overline{EXP} - \overline{IMP} - mY + (n+v)q$$

Collect up terms:

$$Y = \overline{A} + \overline{EXP} - \overline{IMP} + (cY - ctY - mY) + (n+v)q - bi \text{ where } \overline{A} \equiv \overline{CO} - c\overline{TA} + \overline{IN} + \overline{GO}$$

Shift "*Y*" terms to the left hand side:

$$Y - (cY - ctY - mY) = \overline{A} + \overline{EXP} - \overline{IMP} + (n+v)q - bi \rightarrow$$

$$Y[1 - c(1-t) + m] = \overline{A} + \overline{EXP} - \overline{IMP} + (n+v)q - bi$$

Divide both sides by the term in the square bracket to obtain income, *Y*:

$$\boxed{Y = \left(\frac{1}{1 - c(1-t) + m}\right)[\overline{A} + \overline{EXP} - \overline{IMP} + (n+v)q - bi]} \text{ let } \overline{\alpha} \equiv \left(\frac{1}{1 - c(1-t) + m}\right) \qquad \text{<IS>}$$

2.2 Solve for the LM curve, with *i* on the left hand side. Show your work.

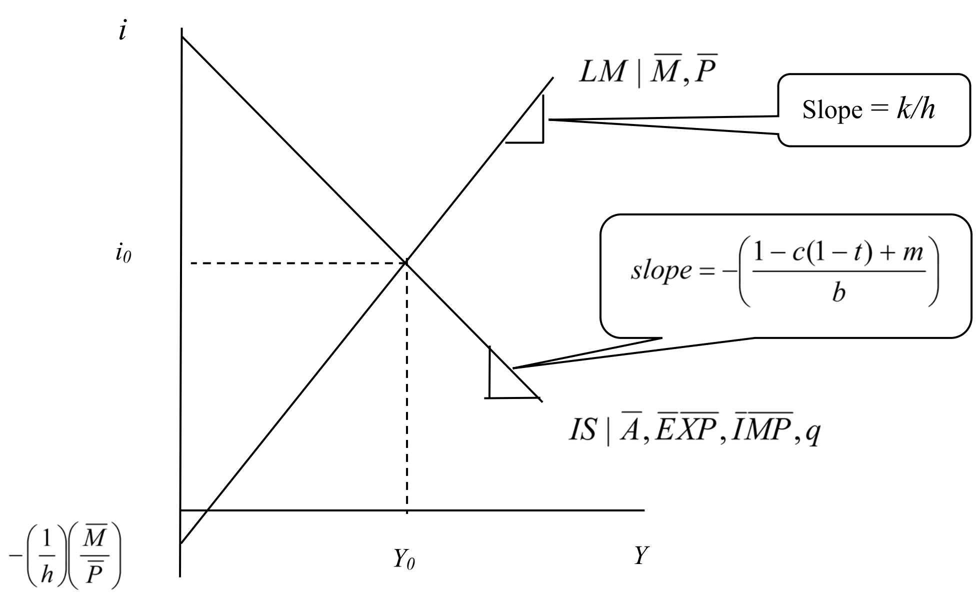
Substitute (10) and (11) into (9), to obtain:

$$\left(\frac{\overline{M}}{P}\right) = kY - hi$$

Solving for the interest rate yields:

$$\boxed{i = -\left(\frac{1}{h}\right)\left(\frac{\overline{M}}{P}\right) + \left(\frac{k}{h}\right)Y} \qquad \text{<LM>}$$

2.3 Graph the IS and LM curves on a single graph. Show the vertical intercepts, the slopes, and the intersection.



2.4 Solve for equilibrium income. Show your work.

Substitute the LM curve into the IS:

$$Y = \overline{\alpha}\left[\overline{A} + \overline{EXP} - \overline{IMP} + (n+v)q - b\left\langle -\left(\frac{1}{h}\right)\left(\frac{\overline{M}}{P}\right) + \left(\frac{k}{h}\right)Y\right\rangle\right]$$

Bring the multiplier and the *Y* term to the left hand side.

$$Y\left(1 - c(1-t) + m + \frac{bk}{h}\right) = \overline{A} + \overline{EXP} - \overline{IMP} + (n+v)q + \left(\frac{b}{h}\right)\left(\frac{\overline{M}}{P}\right)$$

Divide both sides by the term in parentheses to obtain equilibrium income:

$$\boxed{Y_0 = \hat{\alpha}\left[\overline{A} + \overline{EXP} - \overline{IMP} + (n+v)q + \left(\frac{b}{h}\right)\left(\frac{\overline{M}}{P}\right)\right]} \text{ where } \hat{\alpha} \equiv \left(\frac{1}{1 - c(1-t) + m + \frac{bk}{h}}\right)$$

2.5 Calculate the change in income resulting from a given increase in lump sum taxes, ΔTA.

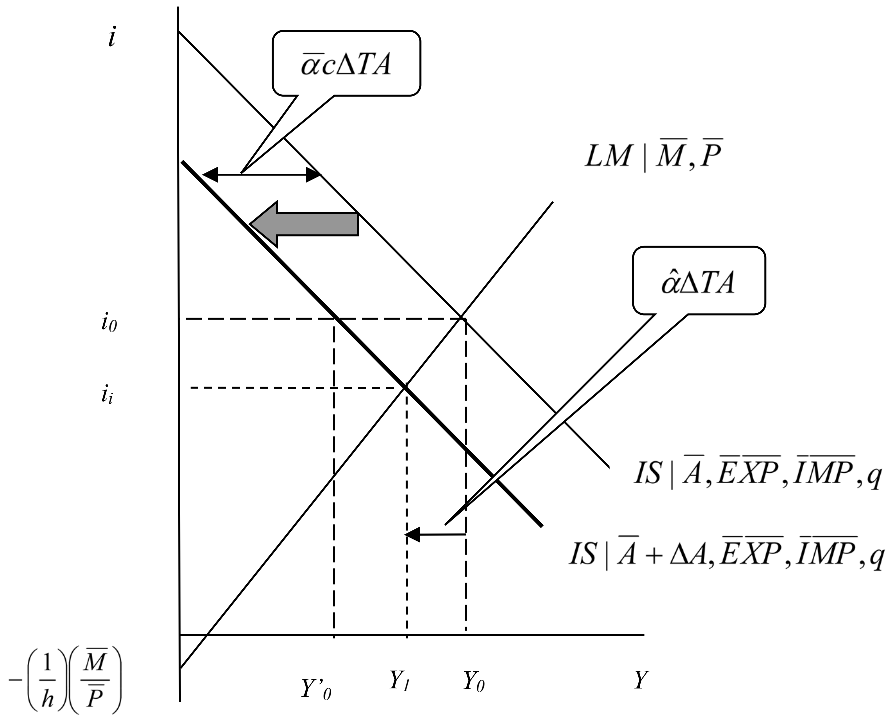
Take the total differential:

$$\Delta Y = \hat{\alpha}\left[\Delta A + \Delta EXP + \Delta IMP + (n+v)\Delta q + \left(\frac{b}{h}\right)\Delta\left(\frac{M}{P}\right)\right]$$

Set all the other changes to zero, and set $\Delta A = -c\Delta TA$:

$$\boxed{\Delta Y = -\hat{\alpha}c\Delta TA}$$

2.6 Show graphically what happens when lump sum taxes are increased. Clearly indicate the distance of the curve shifts, and the amount of the income change.

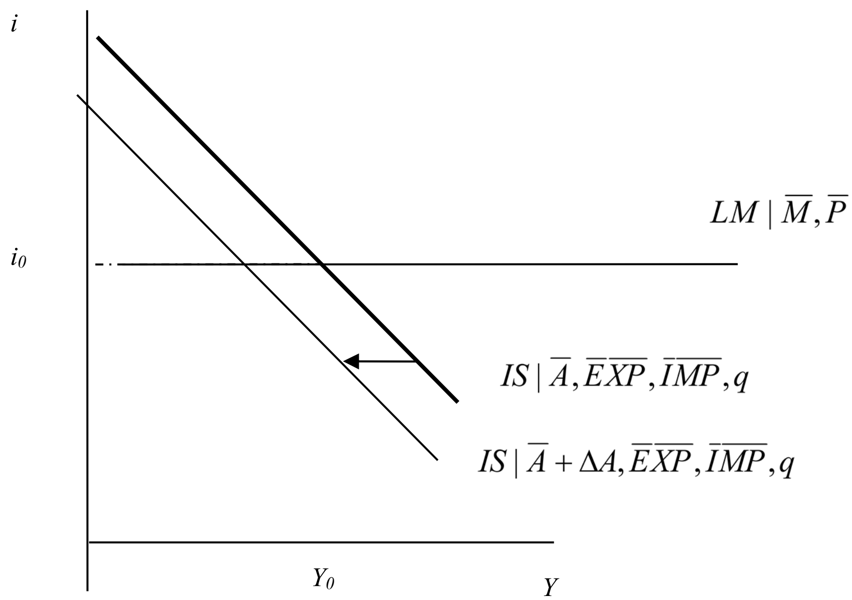


2.7 Is the effect of lump sum taxes on income greater or less in this model, as compared to the simple Keynesian model? Explain why the difference occurs, in words.

The effect is less in absolute value, because of crowding in of investment. As income fallss, demand for money falls. Since the money supply is fixed, the equilibrating interest rate in the money market must fall. As a consequence, investment rises, and this increases aggregate demand and output relative to what it would have been if interest rates remain unchanged. As a consequence, output falls by only, $\Delta Y = -\hat{\alpha}c\Delta TA$ rather than $\Delta Y = -\overline{\alpha}c\Delta TA$.

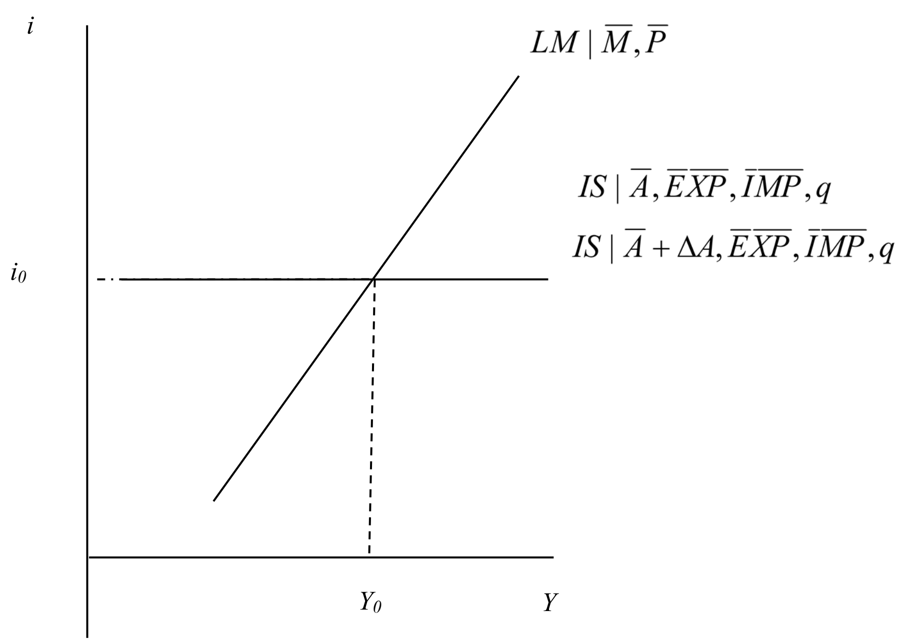
2.8 Answer 2.7 again, if the interest sensitivity of money demand were infinite. Explain why this is true.

If the interest sensitivity of money demand were infinite, then the LM curve would be perfectly flat. Then the shift inward is the same as the reduction in income. Notice that when h=∞, $\hat{\alpha} = \overline{\alpha}$.



2.9 Answer 2.7 again, if the interest sensitivity of investment were infinite. Explain why this is true.

If the interest sensitivity of investment were infinite, then the IS curve is horizontal, and does not shift down as lump sum taxes are reduced.



3. Monetary policy in an IS-LM model

Using the model laid out in Question 2,

3.1 Calculate the change in income for a given change in money supply, $\Delta(M/P)$ (you can assume that the price level P is fixed at 1).

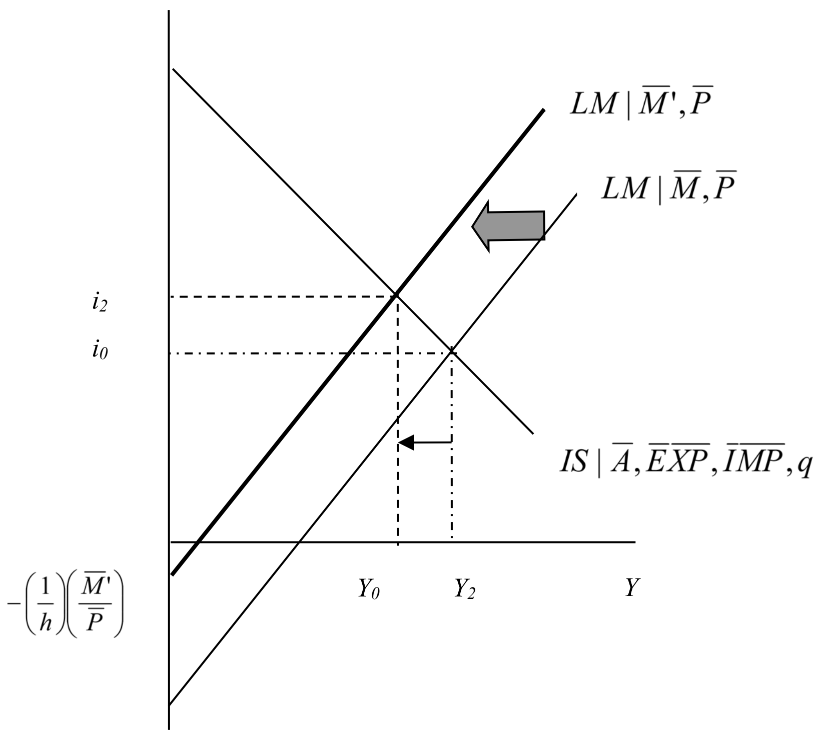
Take the total differential:

$$\Delta Y = \hat{\alpha}\left[\Delta A + \Delta EXP + \Delta IMP + (n+v)\Delta q + \left(\frac{b}{h}\right)\Delta\left(\frac{M}{P}\right)\right]$$

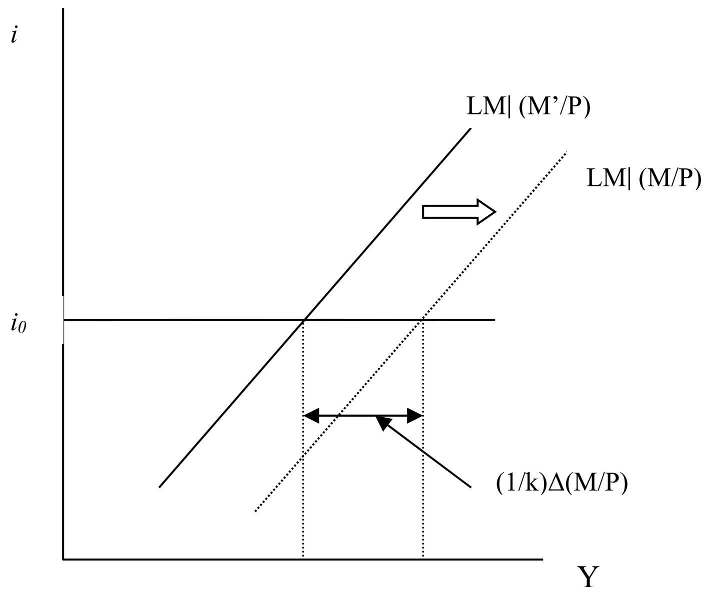
Set all the changes to zero with the exception of the real money supply:

$$\boxed{\Delta Y = \hat{\alpha}\left(\frac{b}{h}\right)\Delta\left(\frac{M}{P}\right)}$$

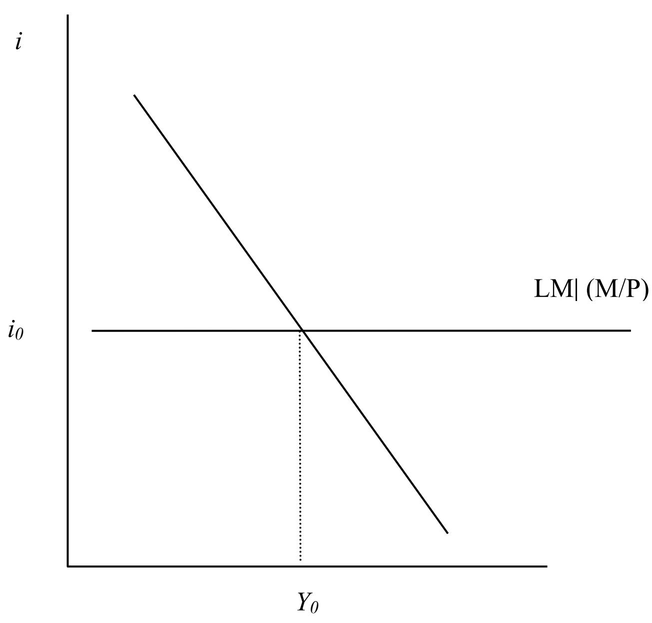
3.2 Show graphically what happens when the real money stock is decreased. Clearly indicate the distance of the curve shifts and the amount of the income change.



3.3 Suppose instead that the interest sensitivity of investment were very high. Show graphically the effect upon output and interest rates that result from an increase of the real money stock. Clearly indicate the distance of the curve shifts and the amount of the income change.



3.4 Suppose the interest sensitivity of money demand was infinite. Show graphically the effect upon output and interest rates that result from an increase of the real money stock. Clearly indicate the distance of the curve shifts and the amount of the income change.



An infinite sensitivity of money demand means that when real money is increased, interest rates drop an infinitely small amount, so investment rises an infinitesimally small amount, so output is essentially unchanged.