Problem Set 2 Answers

Due in Lecture on Monday, October 9th. "Box-in" your answers to the algebraic questions.

1. Elasticities approach.

1.1 Suppose that each one percent depreciation in the US dollar induces a 0.70 increase in exports and a 0.25 decrease in imports. Starting from a position where exports equals imports, what will be the impact on the trade balance?

Since the Marshall-Lerner condition fails to hold, then the depreciation of the exchange rate from a position of initial balance (TB=0) will result in a deteriorated trade balance. Specifically, for a 10% depreciation, exports in domestic currency units rise by 7 percent and real imports fall by 2.5 percent. But since import prices rise by 10%, imports measured in domestic currency terms rise by 8 percent. 7+2.5-10 = -0.5 percent.

1.2 Suppose the US experiences the exchange rate depreciation while running a large trade surplus. What will happen to the trade balance?

The answer is indeterminate. If the surplus is sufficiently large, the trade balance will improve, despite the fact that the Marshall-Lerner conditions do not hold. Otherwise the surplus will shrink.

1.3 Suppose that instead of the elasticities being constant, they are smaller in the short run, and larger in the long run. What is the time path of the trade balance over time (starting from initial balance)?

These are the conditions in which the trade balance would worsen to begin with, before ultimately improving. This phenomenon is termed “the J-curve”.

2. Equilibrium income and multipliers. Consider the following model of the economy:

<table>
<thead>
<tr>
<th>Eq.No.</th>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>( Y = AD )</td>
<td>Output equals aggregate demand, an equilibrium condition</td>
</tr>
<tr>
<td>(2)</td>
<td>( AD = C + I + G + X - IM )</td>
<td>Definition of aggregate demand</td>
</tr>
<tr>
<td>(3)</td>
<td>( C = c(Y - T + TR) )</td>
<td>Consumption function, ( c ) is the MPC</td>
</tr>
<tr>
<td>(4)</td>
<td>( T = \tilde{T} + tY )</td>
<td>Tax function; ( \tilde{T} ) is lump sum taxes, ( t ) is tax rate</td>
</tr>
<tr>
<td>(5)</td>
<td>( TR = \tilde{TR} )</td>
<td>Transfers function</td>
</tr>
<tr>
<td>(6)</td>
<td>( I = I )</td>
<td>Investment function</td>
</tr>
<tr>
<td>(7)</td>
<td>( G = \tilde{G} )</td>
<td>Government spending on goods and services</td>
</tr>
<tr>
<td>(9)</td>
<td>( X = \tilde{X} )</td>
<td>Export spending</td>
</tr>
<tr>
<td>(10)</td>
<td>( IM = \tilde{IM} + mY )</td>
<td>Import spending</td>
</tr>
</tbody>
</table>
There is no real exchange rate effect now because the real exchange rate is assumed constant (and so its effect is subsumed into the constant in (9) and (10)). In your answers to the questions below, show your work, and “box in” your answers.

2.1 Solve for $Y$, setting $\bar{A} \equiv \bar{C} + \bar{I} + \bar{G} + c(\bar{TR} - \bar{T})$.

Substituting (3) - (10) into (2), and using (1):

$Y = AD = \bar{C} + cY - c\bar{T} - ctY + c\bar{TR} + \bar{I} + \bar{G} + \bar{X} - \bar{IM} - mY$

Collect up terms:

$Y = \bar{A} + \bar{X} - \bar{IM} + (cY - ctY - mY)$ where $\bar{A} \equiv \bar{C} + c(\bar{TR} - \bar{T}) + \bar{I} + \bar{G}$

Shift "$Y$" terms to the left hand side:

$Y - (cY - ctY - mY) = \bar{A} + \bar{X} - \bar{IM} \Rightarrow$

$Y[1 - c(1 - t) + m] = \bar{A} + \bar{X} - \bar{IM}$

Divide both sides by the term in the square bracket to obtain equilibrium income, $Y_0$:

$Y_0 = \bar{\alpha}[\bar{A} + \bar{X} - \bar{IM}]$ let $\bar{\alpha} \equiv \left(\frac{1}{1 - c(1 - t) + m}\right)$

2.2 Calculate the change in income for a given change in lump sum transfers. Show your work!

Take a total differential of the answer to 5.1:

$Y = \bar{\alpha}[DA + DX - DI M]$ 

Set $\Delta A = c\Delta \bar{TR}$

$\Delta Y = \bar{\alpha}c\Delta TR$

2.3 Show what the multiplier is for a change in lump sum transfers.

Divide both sides by $\Delta TR$

$\Delta Y \over \Delta TR = \bar{\alpha}c$

2.4 Calculate the change in income for a given change in government spending. Show your work!

Take a total differential of the answer to 5.1:

$Y = \bar{\alpha}[\Delta A + \Delta X - \Delta IM]$ 

Set $\Delta A = \Delta G$

$\Delta Y = \bar{\alpha}\Delta G$
2.5 Calculate the change in the trade balance for a given change in lump sum transfers. Hint: 
\[ TB = X - IM \], so 
\[ \Delta TB = \Delta X - \Delta IM - m\Delta Y \]. Show your work!

Substitute the answer to 4.3 into the expression for the change in the trade balance (and holding the 
real exchange rate constant):

\[ \Delta TB = \Delta X - \Delta IM - m\Delta Y = \Delta X - \Delta IM - m\alpha c\Delta TR \]

Since the autonomous components of imports and exports are unchanged,
\[ \Delta TB = -m\alpha c\Delta TR \]

2.6 Solve for a change in the budget surplus resulting from the change in lump sum transfers, 
recalling your answer to 4.3.

\[ BuS \equiv T - G - TR = \bar{T} + tY - \bar{G} - \bar{TR} \]
\[ \Delta BuS = \Delta T + t\Delta Y - \Delta G - \Delta TR \]

Setting \( \Delta T = 0 = \Delta GO \)

\[ \Delta BuS = tc\alpha \Delta TR - \Delta TR = -(tc\alpha)\Delta TR < 0 \]

Hence, both the budget balance and the trade balance will deteriorate in response to an increase in 
lump sum transfers.

2.7 Suppose autonomous exports increase. Show what are the implications for the trade balance 
and the budget balance. Do they move in the same or different directions?

\[ \Delta TB = \Delta X - \Delta IM - m\Delta Y = \Delta X - m\alpha \Delta X = (1 - m\alpha)\Delta X > 0 \]

\[ \Delta BuS = \Delta T + t\Delta Y - \Delta G - \Delta TR = \Delta T + t\alpha \Delta X - \Delta G - \Delta TR ; \text{ setting change in government spending} 
\text{and lump sum taxes and transfers to zero} \]
\[ \Delta BuS = t\alpha \Delta X > 0 \]

Both trade balance and budget balance respond positively to an autonomous export increase.

3. Expenditure Switching/Expenditure Reduction

Suppose equations (9) and (10) in the above model were altered to:

\[
\begin{align*}
X &= \bar{X} + vq & \text{Export spending} \\
IM &= \bar{IM} + mY - nq & \text{Import spending}
\end{align*}
\]

3.1 Solve for equilibrium income. What is the government spending multiplier in this model?

Substitute in the equations, but using the new export and import equations, to obtain:
\[
Y_0 = \alpha[A + \bar{X} - \bar{I}M + (n + v)q]
\]

Let \[\alpha = \frac{1}{1 - c(1 - t) + m}\]

Take a total differential of the equilibrium solution.

\[
Y = \alpha[A + \Delta X - \Delta IM + (n + v)q]
\]

Set \[\Delta A = \Delta G, \Delta X = \Delta IM = 0\]

\[\Delta Y = \bar{\alpha} \Delta G\]

3.2 Solve for the multiplier for a given unit exchange rate devaluation.

Take the total differential:

\[
Y = \alpha[A + \Delta X - \Delta IM + (n + v)q]
\]

Set \[\Delta A = \Delta X = \Delta IM = 0\]

\[\Delta Y = \bar{\alpha} (n + v) \Delta q\]

3.3 (optional) In order to improve the trade balance by $1 billion, what would have to happen to government spending on goods and services; alternatively, what would have to happen to the real exchange rate?

For government spending:

\[\Delta TB = \Delta X - \Delta IM - m \Delta Y + (n + v) \Delta q\]

\[1 = -m \bar{\alpha} \Delta G \Rightarrow \Delta G = \frac{1}{-m \bar{\alpha}}\]

i.e., government spending decreases.

For the real exchange rate:

\[\Delta TB = \Delta X - \Delta IM - m \Delta Y + (n + v) \Delta q\]

\[\Delta TB = -m[\bar{\alpha}(n + v) \Delta q] + (n + v) \Delta q\]

\[1 = (n + v)[1 - m \bar{\alpha}] \Delta q \Rightarrow \Delta q = \frac{1}{(n + v)[1 - m \bar{\alpha}]}\]

i.e., exchange rate depreciates.