Economics 442 Menzie D. Chinn
Fall 2020 Social Sciences 7418
University of Wisconsin-Madison

Problem Set 1 Answers

Due on 5:30pm CDT Friday, September 25 by submission to Canvas. Be sure to put your name on your problem set. Put "boxes" around your answers to the algebraic questions.

1. Suppose the economy is described by the following equations (so we are looking at a closed economy):

• Real Sector

(1)	$Y = Z$	Output equals aggregate demand, an equilibrium condition
(2)	$Z = C + I + G$	Definition of aggregate demand
(3)	$C = c_o + c_1 Y_D$	Consumption fn, c_1 is the marginal propensity to consume
(4)	$Y_D \equiv Y - T$	Definition of disposable income
(5)	$T = t_0 + t_1 Y$	Tax function; t_1 is marginal tax rate.
(6)	$I = b_0 - b_2 i$	Investment function
(7)	$G = GO_0$	Government spending on goods and services, exogenous

• Asset Sector

(8)	$\frac{M^d}{P} = \frac{M^s}{P}$	Equilibrium condition
(9)	$\frac{M^s}{P} = \frac{M_0}{P}$	Real money supply
(10)	$\frac{M^d}{P} = \mu_0 + Y - hi$	Real money demand

1.1 Solve for the IS curve (Y as a function of i).

$Y = Z = C + I + G$ substitute in for *C, I, G*

$Y = c_0 + c_1 Y_D + b_0 - b_2 i + GO_0$ substitute in for Y_d

$Y = a_0 + c_1 (Y - T + Tr) + b_0 - b_2 i + GO_0$ substitute in for tax, transfers functions

$Y = a_0 + c_1 (Y - t_1 Y + TR_0) + b_0 - b_2 i + GO_0$ bring the "Y" terms to left hand side.

$Y - c_1(Y - t_1 Y) = Y(1-c_1(1-t_1)) = c_0 + c_1 TR_0 + b_0 + GO_0 - b_2 i$

divide both sides by $(1- c_1 (1-t_1))$ and let $\Lambda_0 \equiv c_0 + c_1 TR_0 + b_0 + GO_0$

$\boxed{Y_0 = \bar{\gamma}(\Lambda_0 - b_2 i)}$ let $\bar{\gamma} = \frac{1}{[1 - c_1(1 - t_1)]}$

For your information, one can re-arrange to obtain:

$$i = -\frac{1 - c_1(1 - t_1)}{b_2} Y + \frac{\Lambda_0}{b_2}$$

In which case the slope of the IS curve is: $-\frac{1-c_1(1-t_1)}{b_2}$

1.2 Solve for the LM curve (i as a function of Y). What is the channel by which monetary influences affect the real goods sector in this model?

$$\frac{M_0}{P_0} = \frac{M^s}{P} = \frac{M^d}{P} = \mu_0 + Y - hi$$

Solving for the interest rate, i, yields the LM curve:

$$\boxed{i = \frac{\mu_0}{h} - \left(\frac{1}{h}\right)\left(\frac{M_0}{P_0}\right) + \left(\frac{1}{h}\right)Y}$$

Monetary policy influences (in part) interest rates. Interest rates in turn affect investment, and via the simple Keynesian multiplier ($\bar{\gamma}$) affects the entire real sector.

1.3 Solve for the equilibrium values of *Y*.

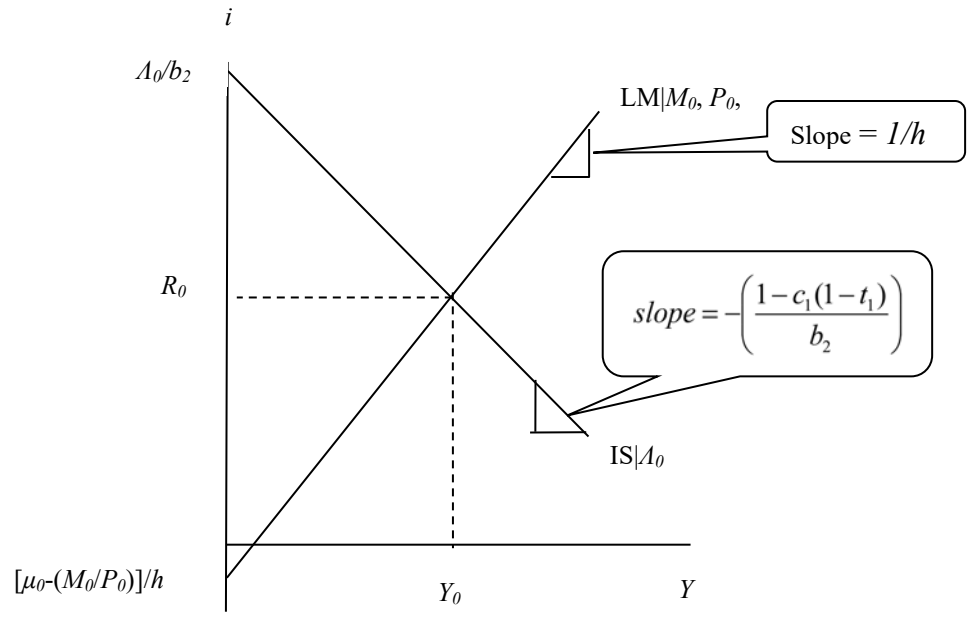
To solve for the equilibrium value of income, substitute the LM into the IS equation from 1.1:

$$Y = \left(\frac{1}{1 - c_1(1 - t_1)}\right) \times \left[\Lambda_0 - b_2\left\langle \frac{\mu_0}{h} - \frac{1}{h}\frac{M_0}{P_0} + \frac{1}{h}Y \right\rangle\right]$$

Move the term in parentheses (.) and the ***(b₂/h)Y*** term to the LHS; factoring and rearranging yields:

$$\boxed{Y_0 = \hat{\gamma}\left[\Lambda_0 - \frac{b_2\mu_0}{h} + \left(\frac{b_2}{h}\right)\left(\frac{M_0}{P_0}\right)\right]} \quad \text{where} \quad \hat{\gamma} \equiv \frac{1}{1 - c_1(1 - t_1) + b_2 / h}$$

1.4 Graph the IS and LM curves on one diagram. Clearly indicate the intercepts and the slopes.



2.1 Assume G increases by ΔGO, and is completely bond financed (and there are no portfolio effects here). Calculate the government spending multiplier.

Take the total differential of your answer to 1.3.

$$\Delta Y = \hat{\gamma}\left[\Delta\Lambda - \frac{b_2\mu_0}{h} + \left(\frac{b_2}{h}\right)\Delta\left(\frac{M}{P}\right)\right]$$

To find the government spending multiplier, set the changes in real money to zero and the money constant (μ_0), and divide both sides by ΔGO:

$$\boxed{\Delta Y = \hat{\gamma}\Delta GO \Rightarrow \frac{\Delta Y}{\Delta GO} = \hat{\gamma} \equiv \frac{1}{1 - c_1(1 - t_1) + b_2/h}}$$

2.2 Suppose instead t_0 decreases by Δt_0. Calculate the lump sum tax multiplier.

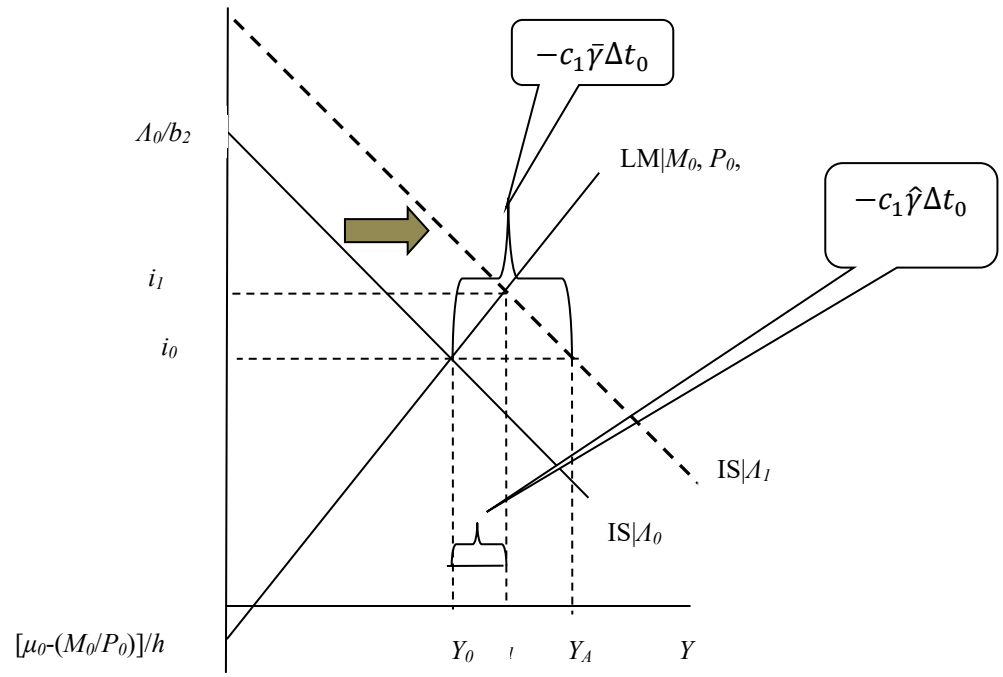
Take the total differential again:

$$\Delta Y = \hat{\gamma}\left[\Delta\Lambda - \frac{b_2\mu_0}{h} + \left(\frac{b_2}{h}\right)\Delta\left(\frac{M}{P}\right)\right]$$

To find the lump sum taxes multiplier, set the changes in real money to zero and the money constant, set the change in $\Delta\Lambda$ to equal $-c_1\,\Delta t_0$, and divide both sides by Δt_0:

$$\Delta Y = -\hat{\gamma}c_1\Delta t_0 \rightarrow {}^{\Delta Y}/_{\Delta t_0} = -\hat{\gamma}c_1 \equiv \frac{-c_1}{1 - c_1(1 - t_1) + b_2/h}$$

2.3 Redraw your answer to 1.4. Then in the same graph, show what happens to the equilibrium income and interest rate if lump sum taxes are cut by Δt_0. Include in your graph the level of income that would be achieved if somehow the interest rate stayed constant (label this point Y_A).



2.4 At the new equilibrium, do we know if investment is higher or lower than the level it started out at? Do we know if it is higher or lower than at Y_A?

Recall the investment function is given by:

$$I = b_0 - b_2 i$$

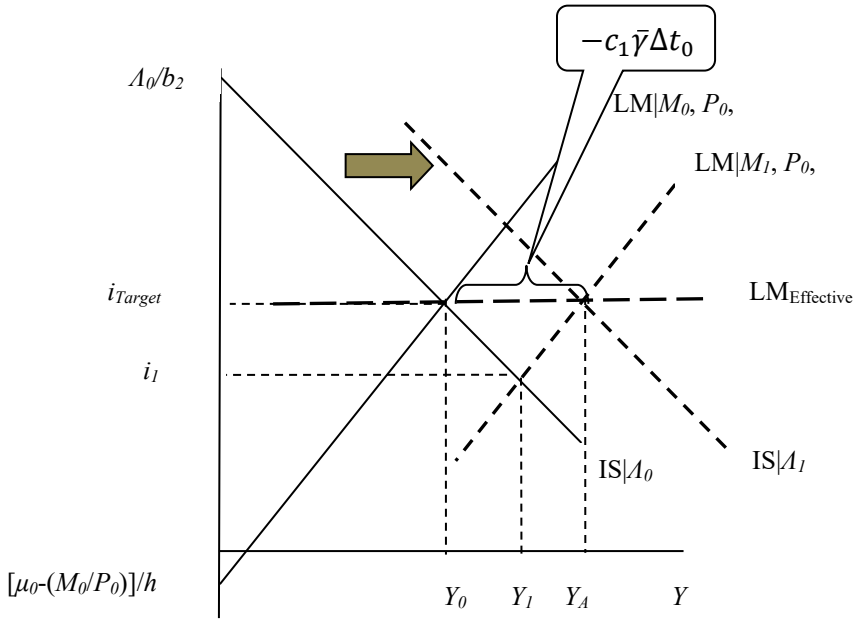
So the change in investment is given by:

$$\Delta I = \Delta b_0 - b_2 \Delta i$$

Notice that at the new equilibrium, the interest rate is higher (i_1). Hence, investment is unambiguously lower.

Regarding the second question, if one were at Y_A, *and* interest rate i_0, then investment would be unchanged.

2.5 Suppose the Fed targets the interest rate at i_0 (call this i_{target}). Returning to 2.3, show graphically what happens if lump sum taxes is decreased. What happens to the level of investment?



When the Fed targets the interest rate, and the target interest rate remains constant, then the LM is now the Effective LM. A decrease in lump sum taxes causing an increase in consumption in this case causes no change in the interest rate, so there is no crowding out of investment, and so investment is unchanged (monetary policy is accommodative).

Note that the answer is the same if we are in a liquidity trap.

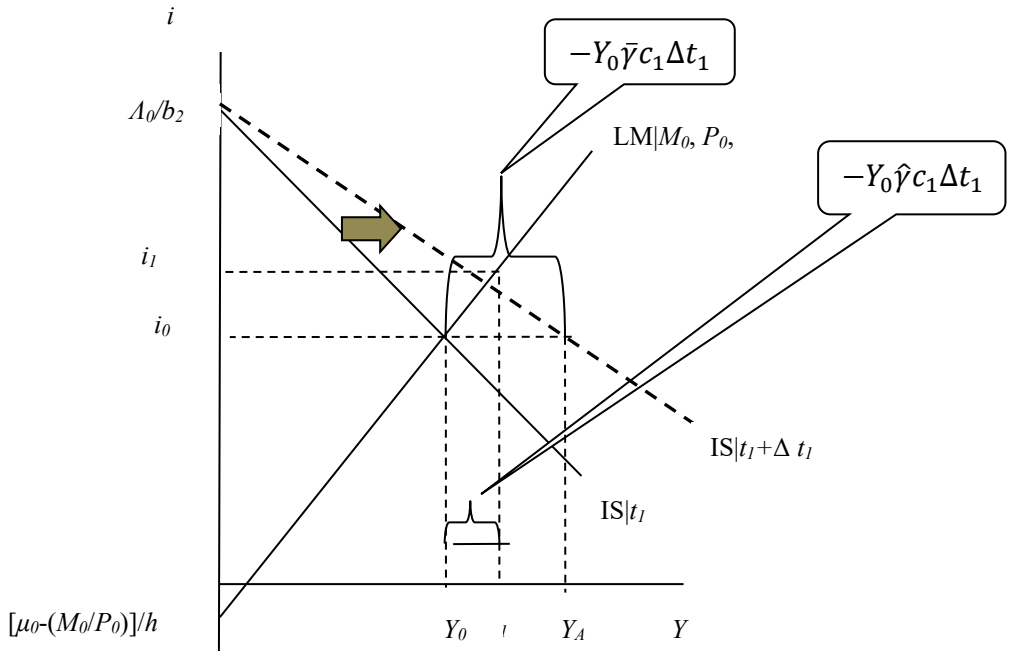
2.6. Return to 2.3. Show graphically what happens if the marginal tax rate, t_1, is reduced.

Recall the slope of the IS curve in this model is:

$$i = -\frac{1 - c_1(1 - t_1)}{b_2} Y + \frac{\Lambda_0}{b_2}$$

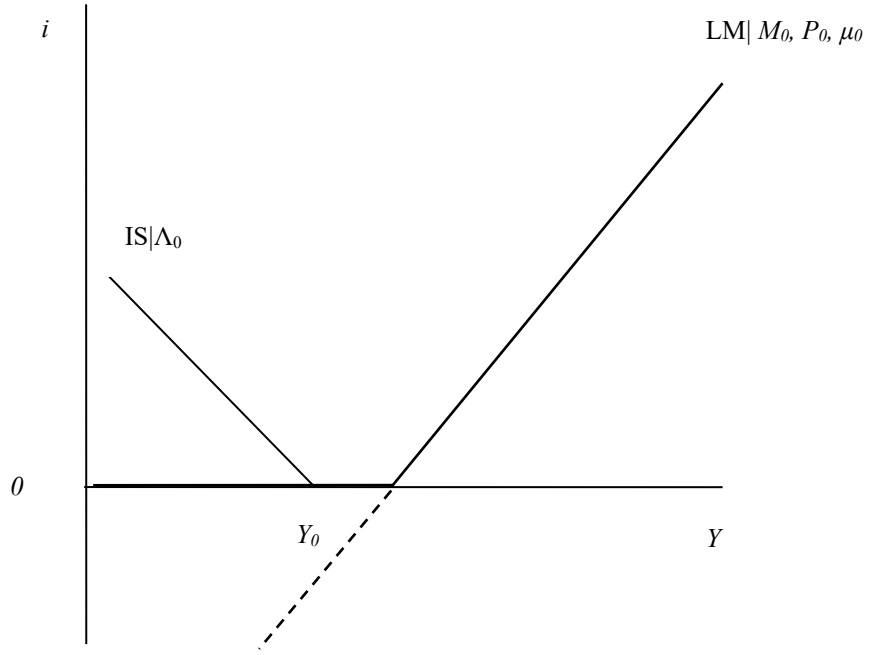
A decrease in the tax rate thus makes the slope less negative; this is shown in the figure below as a rotation outward in the IS curve. As a consequence, output and interest rates rise (just as they would in the case of a decrease in lump sum taxes).

The size of the increase in income requires taking a total differential of the solution for equilibrium income, keeping in mind the tax rate is in the $\hat{\gamma}$ term.

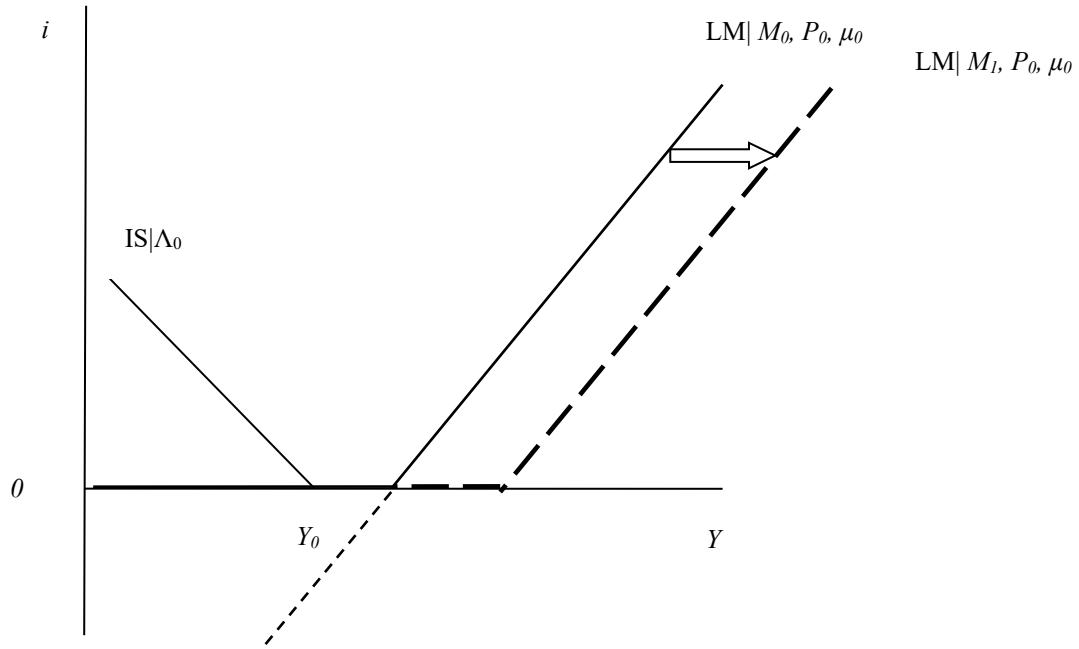


3. Consider a situation where the economy is in a liquidity trap.

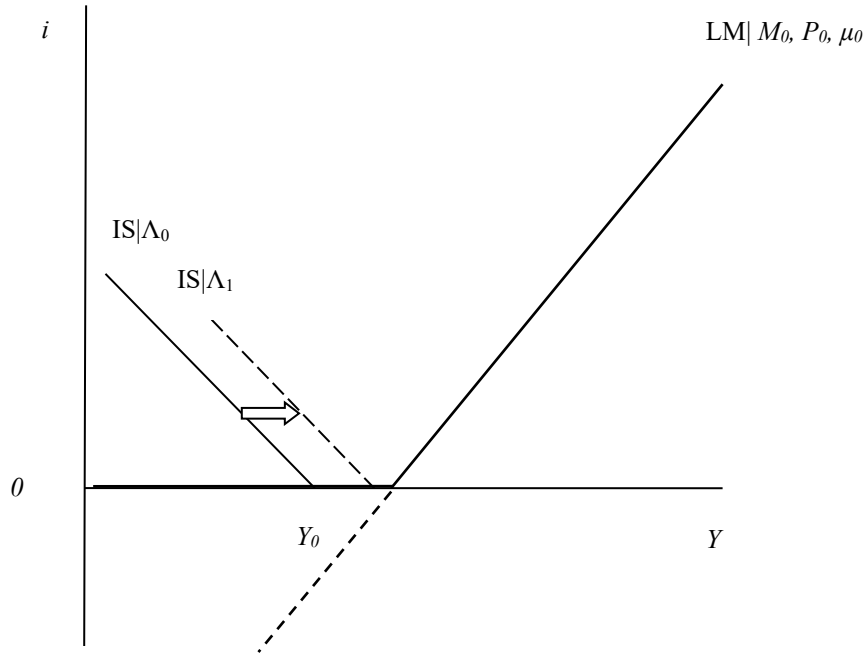
3.1 Draw a diagram illustrating the situation where interest rates are at the zero lower bound.



3.2 Show what happens to the equilibrium income and interest rate if the money supply is increased.



3.3 Show what happens if lump sum taxes are decreased. Does investment rise or fall?



The decrease in lump sum taxes shifts out the IS curve by . Since interest rates are unchanged, investment is unchanged using the investment equation given in Problem 1.

4. Consider an economy where the money demand function takes the following form:

$$\frac{M^d}{P} = \mu_0 + Y + j\left(\frac{MB}{P} + \frac{B}{P}\right) - hi$$

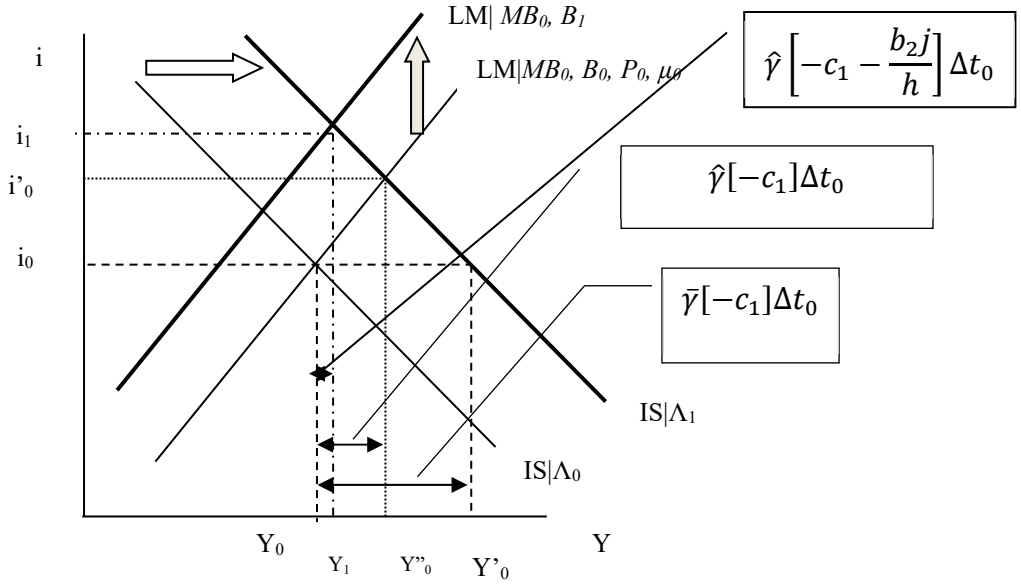
Assume investment depends on income and the interest rate, and the marginal tax rate is zero.

4.1 Further assume the budget is balanced to begin with. When taxes are cut by Δt_0, the government must borrow $\Delta(M/P)$. What happens to the LM curve when lump sum taxes decrease? Why?

The LM curve in this case is given by:

$$i = \frac{\mu_0}{h} - \left(\frac{1}{h}\right)\left(\frac{mMB_0}{P_0}\right) + \frac{j}{h}\left(\frac{MB_0}{P_0} + \frac{B_0}{P_0}\right) + \left(\frac{1}{h}\right)Y$$

If bonds are increased (equal to the budget deficit equal to the tax cut under given assumptions), then it's apparent the LM curve shifts up by (j/h)Δ(B/P).



4.2 Can you tell what happens to equilibrium income and interest rates when lump sum taxes decrease? Show, using and IS-LM diagram.

As shown in the above figure, interest rates unambiguously increase. Output might or might not increase, depending on the shifts and slopes of the curves. That's because the financing of the budget deficit might increase interest rates sufficiently that investment falls so much as to offset the spending increase associated with lower taxes.

4.3 What happens to investment? Explain, using an equation.

The investment equation is:

$$I = b_0 + b_1 Y - b_2 i$$

Then the total differential is then:

$$\Delta I = \Delta b_0 + b_1 \Delta Y - b_2 \Delta i$$

Since interest rates are higher, but the change in income is ambiguous, then the net impact is ambiguous.

4.4 Assume the Fed targets the interest rate at the level it was before the decrease in lump sum taxes. What is the impact on equilibrium income and interest rates of the decrease in lump sum taxes?

If the Fed targets the interest rate, then output will increase when taxes are cut (or government spending increased). That's because the interest rate is constant (the mathematical solution can be obtained by setting h=∞. Only the Keynesian effect remains.

$$\Delta Y = -\bar{\gamma} c_1 \Delta t_0$$