Problem Set 4 Answers

Due in lecture on Wednesday, December 9. Be sure to put your name on your problem set. Put “boxes” around your answers to the algebraic questions.

1. Consider a Taylor rule of the following form:

\[ i_t^{\text{Fed Funds}} = \pi_t + 0.5 \times (y_t - y^*_t) + 0.5 \times (\pi_t - \pi^*_t) + r^*_t \]

Calculate the implied Fed funds rate for 2017Q3, assuming the equilibrium real rate is 2.0%, and target inflation rate is 2%. You will need to obtain information on the output gap and inflation rate (calculated using quarterly data). Show your work.

You can obtain information St. Louis Fed FRED system (https://fred.stlouisfed.org/) on potential GDP and actual GDP, to calculate the output gap. You can also obtain data for personal consumption expenditure deflator inflation from there as well.

<table>
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<tr>
<th>Year</th>
<th>GDPC1</th>
<th>GDP09_POT</th>
<th>LOG(GDPC1/GDP09_POT)</th>
<th>PCEPI</th>
<th>PCEPI/PCEPI(-4)-1</th>
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</thead>
<tbody>
<tr>
<td>2016Q3</td>
<td>16778.15</td>
<td>16864.3</td>
<td>-0.005122</td>
<td>111.0340</td>
<td>0.011598</td>
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<td>2016Q4</td>
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<td>0.016174</td>
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<tr>
<td>2017Q1</td>
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<td>16992.2</td>
<td>-0.005249</td>
<td>112.1980</td>
<td>0.020121</td>
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<tr>
<td>2017Q2</td>
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<td>17058.2</td>
<td>-0.001591</td>
<td>112.2730</td>
<td>0.015540</td>
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<td>2017Q3</td>
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<td>17125.5</td>
<td>0.002580</td>
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<td>0.015049</td>
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Answer:
The 2017Q3 output gap calculated in log terms is 0.26%; the PCE-all inflation rate (4 quarters) is 1.50%. Putting these into the Taylor equation yields:

\[ i_t^{\text{Fed Funds}} = 0.015 + 0.5 \times (0.003) + 0.5 \times (0.015 - 0.02) + 0.02 \]

\[ i_t^{\text{Fed Funds}} = 0.034 \]

I.e., the target Fed funds rate should be 3.4 percentage points.

Note the above calculations rely on year-on-year PCE inflation (as in the example discussed in lecture). Quarter-on-quarter annualized inflation is 1.55%. Then the resulting calculation is:

\[ i_t^{\text{Fed Funds}} = 0.0155 + 0.5 \times (0.003) + 0.5 \times (0.0155 - 0.02) + 0.02 \]

\[ i_t^{\text{Fed Funds}} = 0.035 \]

2. Consider a CC-LM model as laid out in the Bernanke-Blinder article.
2.1 Show what happens, graphically, if the Fed undertakes open market operations, by buying Treasury bonds.

2.2 Explain the economics behind why the curve(s) shift(s).

Because the supply of loans depends on deposits (money supply), and deposits depend on reserves, any increase in reserves will tend to increase the amount of loans (as well as the money supply). Both of these outcomes tend to raise GDP, due to both lower bond rate ($i$) and lower loan interest rate ($\rho$).

Optional derivations:

\[
L' = \lambda(\rho, i, Z)D(1 - \tau)
\]

\[D(i, y) = mR\]

\[dY = \frac{(Y_{\rho}i_{i} + Y_{i})di + Y_{i}dR}{(1 - Y_{i})\rho_{i}}\]
To figure out the horizontal shift of the CC curve, solve for $di$, and set equal to zero.

$$
\frac{dY(1-Y_\rho \rho_Y) - Y_\rho \rho_Y dR}{(Y_\rho \rho_Y + Y_i)} = di = 0
$$

(4)

Then:

$$
dY\bigg|_{di=0} (1-Y_\rho \rho_Y) = Y_\rho \rho_Y dR
$$

(5)

and:

$$
dY\bigg|_{di=0} = \frac{Y_\rho \rho_Y dR}{(1-Y_\rho \rho_Y)}
$$

(6)

Notice that the LM curve shifts out because the increase in reserves increases the money supply, so that for any given income level, the equilibrating interest rate is lower. To see this, take equation (2), the LM curve, and solve for $di$, and set equal to 0.

$$
\frac{m(dR) - D_i dY}{D_i} = di = 0
$$

(2)

Then solve for $dY$.

3. Consider the implications of the Bernanke-Gertler-Gilchrist model. Using whatever equation(s) is relevant, show the implications for firm borrowing of the following developments:

Recall, from the paper:

$$
x_1 \leq a_0 f(x_0) + (q_1/r_1)K - r_0 b_0.
$$

(3)

Spending on the variable input cannot exceed the entrepreneur’s net worth, equal to the sum of gross cash flow $a_0 f(x_0)$ and net discounted assets, $(q_1/r_1)K - r_0 b_0$. If the entrepreneur’s net worth is less than the unconstrained optimal value of $x_1$, which satisfies $a_1 f(x_1) = r_i$, then the constraint (3) binds.

3.1 The interest rate rises.

This means $r_i$ is higher, so $q_1/r_1$ is smaller, so the collateral constraint binds more tightly leading to less borrowing, holding all else constant.

3.2 The relative price of capital falls.
This means $q_1$ is smaller, so the collateral constraint binds more tightly leading to less borrowing.

3.3 Technology improves.

If technology improves in period 1, nothing happens according to the model. If technology improved in period 0, then $a_0$ is higher, and hence cash flow from period 0 is higher, then it’s possible borrowing decreases. But if the constraint is binding in all periods, then there is no change in period 1 collateral constraint and no change in borrowing at least in this model.

4. If the Bernanke-Gertler-Gilchrist model is correct, after a monetary tightening and decrease in corporate earnings,

4.1 What should happen on average to borrowing by small firms relative to large firms? Why?

Since the collateral constraints will bind more tightly on small firms than large firms, large firm borrowing will rise relative to that of small firms.

4.2 What happens to commercial paper issuance relative to bank borrowing? Why?

Since large firms are able to issue commercial paper, and large firms are generally able to borrow after a decrease in corporate earnings (while small firms are less able), then issuance of commercial paper will typically rise relative to bank debt (which is the only debt small firms can access).