

Problem Set 3 Answers

Due *in Canvas* on Thursday, November 4. Be sure to put your name on your problem set. Put “boxes” around your answers to the algebraic questions.

1. Suppose the price change of a stock is given by:

$$P_{t+1} - P_t = (E_t P_{t+1} - P_t) + \left[\frac{D_{t+2} - E_t D_{t+2}}{(1+rp+rf)} + \frac{E_{t+1} P_{t+2} - E_t P_{t+2}}{(1+rp+rf)} \right]$$

Assume no news regarding dividends is coming out between t and $t+1$ (e.g., each period is one day).

1.1 Why how might changes in expectations from t to $t+1$ regarding events at $t+2$ have an impact on the price change from t to $t+1$? Be explicit about the channel.

Note that under the assumptions:

$$P_{t+1} - P_t = (E_t P_{t+1} - P_t) + \left[\frac{E_{t+1} P_{t+2} - E_t P_{t+2}}{(1+rp+rf)} \right]$$

Going from t to $t+1$, expectations of the stock price at $t+2$ are revised in response to events in $t+1$, and this feeds back to the present price. That is,

$$E_{t+1} P_{t+2} \neq E_t P_{t+2}$$

1.2 Should the change in the stock price be a completely uncorrelated random error? Show why or why not.

There are two ways to answer this question. The first (preferred) is to use the equation above, imposing the assumption of no-dividends being paid, no news regarding dividends. Then:

$$P_{t+1} - P_t = (E_t P_{t+1} - P_t) + \left[\frac{E_{t+1} P_{t+2} - E_t P_{t+2}}{(1+rp+rf)} \right]$$

The item in parentheses (.) is known to market participants (though not necessarily to you and me), but that in the square brackets [.] is unknown. If expectations are rational, then [.] is truly random, and is not predictable on the basis of past information that was known at time t .

The second uses the approximation in the handout.

$$P_t = \frac{E_t P_{t+1}}{1+(rf+rp)}$$

Assume log-normality of the error term:

$$p_t = E_t p_{t+1} - \ln(1 + rf + rp) \rightarrow p_t + \ln(1 + rf + rp) = E_t p_{t+1}$$

Use the assumption of rational expectations:

$$p_{t+1} = E_t p_{t+1} + \tilde{u}_{t+1}$$

Substitute in the expression for log price:

$$p_{t+1} = p_t + (rf + rp) + \tilde{u}_{t+1}$$

Re-express in terms of growth rate of the stock price:

$$p_{t+1} - p_t = (rf + rp) + \tilde{u}_{t+1}$$

Then the change in the stock price equals an uncorrelated random error and a constant (if the interest rate is constant; otherwise it could be nonconstant but predictable).

1.3 Suppose P refers to the price of Bitcoin. Further suppose that overnight the US government announced a ban on the use of Bitcoin to take effect one year from today. What would happen to the price of Bitcoin going from today to tomorrow (i.e., what would $P_{t+1} - P_t$ look like)? You can assume that no other information comes in overnight.

Consider the given expression.

$$P_{t+1} - P_t = (E_t P_{t+1} - P_t) + \left[\frac{D_{t+2} - E_t D_{t+2}}{(1+rp+rf)} + \frac{E_{t+1} P_{t+2} - E_t P_{t+2}}{(1+rp+rf)} \right]$$

Bitcoin does not pay a dividend, but it provides a service that could have a monetary value. Think of D as being that service. The announcement going from period t to $t+1$ implies that in $t+365$ (i.e., in one year), D will go to zero. Since the price tomorrow should depend on the discounted stream of services D from now until $t+365$, and D at $t+365$ and onward is going to zero, then the price going from t to $t+1$ should go down discretely. In other words, once the news comes out about something happening in the future (and is believable, i.e., credible), then the price will drop at $t+1$.

2. Consider a Bank that has the following balance sheet:

2.1 Suppose the bank has the following structure:

Assets		Liabilities	
Reserves	\$50M	Checkable Deposits	\$230M
Securities	\$25M		
Govt Securities	\$25M		
Loans	\$150M	Bank Capital	\$20M

Bank capital is the equity of the owners (shareholders) of the bank. ABS stands for asset backed securities.

Under the Basel II guidelines, government securities would have zero weight in assets; calculate the capital ratio for this bank. Show your work. (Note also reserves carry zero weight in the calculation of risk weighted assets.)

The capital ratio is $20/(0 \times 25 + 1 \times 25 + 1 \times 150) = 20/175 = 11.43\%$

2.2 Suppose the government securities are actually as risky as non-government securities. Calculate the true capital ratio.

The true capital ratio is $20/(1 \times 25 + 1 \times 25 + 1 \times 150) = 20/200 = 10.00\%$

3. Leverage, liquidity, and bank balance sheets

3.1 Consider two banks, H (high bank capital) and L (low bank capital).

High Bank Capital				Low Bank Capital			
Assets		Liabilities		Assets		Liabilities	
Reserves	\$9M	Deposits	\$90M	Reserves	\$10M	Deposits	\$96M
Loans	\$71M	Bank Capital	\$10M	Loans	\$70M	Bank Capital	\$4M
ABS	\$20M			ABS	\$20M		

Bank capital is the equity of the owners (shareholders) of the bank. ABS stands for asset backed securities.

Calculate the return on equity (ROE) for each bank, if the rate of return on loans is 5%, and 10% on ABS, and the interest rate on deposits is 2%.

For H bank:
 $((0.05 \times 71 + 0.10 \times 20) - (0.02 \times 90)) / 10 = 3.75 / 10 = 0.375 = 37.5\%$

For L bank:
 $((0.05 \times 70 + 0.10 \times 20) - (0.02 \times 96)) / 4 = 3.63 / 4 = 0.895 = 89.5\%$

3.2 Show what happens to each of the bank balance sheets when the asset backed securities lose 25% of their value.

High Bank Capital				Low Bank Capital			
Assets		Liabilities		Assets		Liabilities	
Reserves	\$9M	Deposits	\$90M	Reserves	\$10M	Deposits	\$96M
Loans	\$71M	Bank Capital	\$10M	Loans	\$70M	Bank Capital	\$4M
ABS	\$20M			ABS	\$20M		

Bank capital is the equity of the owners (shareholders) of the bank. ABS stands for asset backed securities.

High Bank Capital		Low Bank Capital	
Assets	Liabilities	Assets	Liabilities
Reserves \$9M	Deposits \$90M	Reserves \$10M	Deposits \$95M
Loans \$71M	Bank Capital \$5M	Loans \$70M	Bank Capital \$0M
ABS \$15M		ABS \$15M	

3.3 Now consider two banks, one which borrows a nothing short term, and one that borrows a lot on short term money markets.

Bank Deposit Based		Money Market Based	
Assets	Liabilities	Assets	Liabilities
Reserves \$6M	Deposits \$60M	Reserves \$3M	Deposits \$30M
Loans \$74M	Short term \$30M	Loans \$77M borrowing	Short term \$60M borrowing
ABS \$20M	Bank Capital \$10M	ABS \$20M	Bank Capital \$10M

Calculate the return on equity (ROE) for each bank, if the rate of return on loans is 5%, and 10% on ABS, and the interest rate on deposits is 2%, and the interest rate on short term borrowing is 1%.

For Deposit-based bank:

$$((0.05 \times 74 + 0.10 \times 20) - (0.02 \times 60 + 0.01 \times 30)) / 10 = 4.2 / 10 = 0.42 = \mathbf{42\%}$$

For Money market based bank:

$$((0.05 \times 77 + 0.10 \times 20) - (0.02 \times 30 + 0.01 \times 60)) / 10 = 46.5 / 10 = 0.465 = \mathbf{46.5\%}$$

3.4 Show what each bank must do when short term money markets freeze, so that the banks cannot continue to borrow short term.

For Deposit-based bank, either deposits must be increased by \$30M (and reserves raised by \$3M), or Loans and ABS fall by \$30M. For the Money market-based bank, deposits must be increased by \$60M (and reserves increased by \$6M), or Loans and ABS fall by \$60M.