Midterm Exam 1 Answers

You have 65 minutes to complete this 60 minute exam. Be sure to “box in” your answers. Show your work (so that partial credit can be granted if the final answer is incorrect).

1. [20 minutes total] Suppose the economy is described by the following equations:

**Real Sector**

1. \( Y = Z \) Output equals aggregate demand, an equilibrium condition
2. \( Z = C + I + G \) Definition of aggregate demand
3. \( C = c_o + c_1Y_D - c_2i \) Consumption fn, \( c_1 \) is the marginal propensity to consume
4. \( Y_D = Y - T \) Definition of disposable income
5. \( T = t_iY \) Tax function; \( t_i \) is marginal tax rate.
6. \( I = b_0 + b_1Y - b_2i \) Investment function
7. \( G = GO_o \) Government spending on goods and services, exogenous

**Asset Sector**

\[
i = \frac{\mu_0}{h} - \left( \frac{1}{h} \left( \frac{M_0}{P_0} \right) \right) + \left( \frac{1}{h} \right) Y
\]

1.1 (4 minutes) Solve for the IS curve (\( Y \) as a function of \( i \)).

\[
Y = Z = C + I + G
\]

\[
Y = c_o + c_1Y_D - c_2i + b_o + b_1Y - b_2i + GO_o
\]

substitute in for \( C, I, G \)

\[
Y = c_o + c_1 (Y - T) - c_2i + b_o + b_1Y - b_2i + GO_o
\]

substitute in for tax, transfers functions

\[
Y = c_o + c_1 (Y - t_iY) - c_2i + b_o + b_1Y - b_2i + GO_o
\]

bring the "Y" terms to left hand side.

\[
Y - b_1(Y - t_iY) = Y(1-b_1(1-t_i)) = c_o + c_1 TR_0 + b_o + GO_o - (c_2 + b_2) i
\]

divide both sides by \((1-c_1 (1-t_i)-b_1)\) and let \( \Lambda_0 \equiv c_o + b_o + GO_o \)

\[
\Lambda_0 \equiv \bar{\gamma}(L_a - (c_2 + b_2)i) \quad \text{let} \quad \bar{\gamma} = \frac{1}{[1-c_1(1-t_i)-b_1]}
\]

1.2 (4 minutes) Solve for equilibrium income.

To solve for the equilibrium value of income, substitute the LM into the IS equation in the answer to 3.1:

\[
Y = \left( \frac{1}{1-c_1(1-t_i)-b_1} \right) \times \left[ \Lambda_0 - (b_2 + c_2) \left( \frac{\mu_0}{h} - \frac{1}{h} \frac{M_0}{P_0} + \left( \frac{1}{h} \right) Y \right) \right]
\]

Move the term in parentheses (.) and the \((b_2+c_2)/h)Y\) term to the LHS; factoring out the \(Y\)s on the LHS yields:
\[
Y \left( 1 - c_1(1 - t_1) - b_1 + \frac{b_2 + c_2}{h} \right) = \left[ \Lambda_0 - (b_2 + c_2) \left( \frac{M_0}{P_0} - \frac{1}{h} \right) \right]
\]

Dividing both sides by the term in the parentheses yields:

\[
Y_0 = \hat{\gamma} \left[ \Lambda_0 - \left( \frac{b_2 + c_2}{h} \right) \mu_0 + \left( \frac{b_2 + c_2}{h} \right) \left( \frac{M_0}{P_0} \right) \right] \quad \text{where} \quad \hat{\gamma} \equiv \frac{1}{1 - c_1(1 - t_1) - b_1 + \left( b_2 + c_2 \right) / h}
\]

1.3 (4 minutes) Calculate the government spending multiplier.

Take the total differential of your answer to 3.2:

\[
\Delta Y = \hat{\gamma} \left[ \Delta \Lambda - \frac{\left( b_2 + c_2 \right) \mu_0}{h} + \left( \frac{b_2 + c_2}{h} \right) \Delta \left( \frac{M}{P} \right) \right]
\]

To find the government spending multiplier, set the changes in real money to zero and the money constant, set \( \Delta \Lambda = \Delta GO \) and divide both sides by \( \Delta GO \):

\[
\Delta Y = \hat{\gamma} \Delta GO \Rightarrow \frac{\Delta Y}{\Delta GO} = \hat{\gamma} \equiv \frac{1}{1 - c_1(1 - t_1) - b_1 + \left( b_2 + c_2 \right) / h}
\]

1.4 (4 minutes) Is the government spending multiplier smaller than in the standard model covered in class? Why or why not?

The government spending multiplier is smaller than in the standard model by virtue of the \( c_2 \) parameter. The reason is that consumption is now also interest sensitive, so there are two channels by which expansionary fiscal policy crowds out components of aggregate demand.

\[
\frac{1}{1 - c_1(1 - t_1) - b_1 + \left( b_2 + c_2 \right) / h} < \frac{1}{1 - c_1(1 - t_1) - b_1 + b_2 / h} \equiv \hat{\alpha}
\]

New multiplier \(<\) standard multiplier

1.5 (4 minutes) Is monetary policy more or less powerful than in the standard model covered in class? Why or why not?

To find the monetary policy multiplier, set the changes in government spending to zero divide both sides by \( \Delta (M/P) \):

\[
\Delta Y = \left( \frac{b_2 + c_2}{h} \right) \hat{\gamma} \Delta (M/P) \Rightarrow \frac{\Delta Y}{\Delta (M/P)} = \frac{\left( b_2 + c_2 \right)/h}{1 - c_1(1 - t_1) - b_1 + \left( b_2 + c_2 \right)/h}
\]

It’s hard to tell mathematically if the monetary policy multiplier is more or less powerful. However, intuitively, since there are now two channels by which lower interest rates affect aggregate demand, then monetary policy must be more powerful than in the standard formulation.

2. [15 minutes total] Suppose the yield on a five year bond is 1%, and the yield on a ten year bond is 1.5%.
2.1 (10 minutes) Solve for the stock price assuming expectations are rational and there are no bubbles.

In general, for the pure expectations hypothesis :

\[ i_{m} = \left( i_{1t} + i_{t+1} + \ldots + i_{t+n-1} \right) / n \]

So for the ten year yield:

\[ i_{10t} = \left( i_{1t} + i_{t+1} + i_{t+2} + \ldots + i_{t+9} \right) / 10 \]

Rearranging:

\[ 10 \times i_{10t} = \left( i_{1t} + i_{t+1} + i_{t+2} + \ldots + i_{t+9} \right) \]

Notice that the term in the square bracket is 5 times the five year yield, viz.:

\[ 5 \times i_{5t} = \left( i_{1t} + i_{t+1} + i_{t+2} + i_{t+3} + i_{t+4} \right) \]

Substituting in, one finds:

\[ 10 \times i_{10t} = 5 \times i_{5t} \]

Hence:

\[ i_{5t+5} = 2 \times i_{5t} \]

or 2%.

2.2 (5 minutes) Can you solve for what the 5 year bond yield is expected to be five years from now?

According to the expectations hypothesis of the term structure, for a two year bond,

\[ i_{t+1} = 2i_{t} - i_{t-1} \]

The ten year/five year structure is analogous. So treat the periods as 5 years long. Then substituting in 0.015 and 0.01 for the two period and one period bond yields results in:

\[ i_{5t+5} = 2 \times 0.015 - 0.01 = 0.02 \]

or 2%

3. [15 minutes total] Asset prices. Suppose:

\[ P_{t} = \frac{D_{t+1}}{1 + rf + rp} + \frac{E_{t}P_{t+1}}{1 + rf + rp} \]

(2)

3.1 (7 minutes) Solve for the stock price assuming expectations are rational and there are no bubbles.

**Answer:** Recursively substituting, one obtains:

\[ P_{t} = \frac{D_{t+1}}{1 + rf + rp} + \frac{E_{t}D_{t+2}}{(1 + rf + rp)^{2}} + \ldots + \frac{E_{t}D_{t+w}}{(1 + rf + rp)^{w}} + \sum_{n=1}^{w} \frac{E_{t}D_{t+n}}{(1 + rf + rp)^{n}} + \frac{E_{t}P_{t+\infty}}{(1 + rf + rp)^{\infty}} \]
This means there is always a price term. Only if this last term is near zero will dividends only matter, viz.,
\[
\lim_{n \to \infty} \frac{E_i P_{i+n}}{(1 + k_e)^n} = 0
\]
\[
P_i = \frac{D_{i+1}}{1 + rf + rp} + \frac{E_i D_{i+2}}{(1 + rf + rp)^2} + \frac{E_i D_{i+3}}{(1 + rf + rp)^3} + \ldots + \frac{E_i D_{i+n}}{(1 + rf + rp)^n} + \ldots = \sum_{n=1}^{\infty} \frac{E_i D_{i+n}}{(1 + rf + rp)^n}
\]
In other words, if there is no bubble, then the price today is a PDV of expected dividends.

3.2 (4 minutes) Calculate the price of a share of stock, assuming dividends are expected to be constant at \(D_0 = 1\) and \((rf + rp)\) is also expected to be constant at 0.10. Show your algebraic work.

Answer:
\[
P_i = \frac{D_{i+1}}{1 + rf + rp} + \frac{E_i D_{i+2}}{(1 + rf + rp)^2} + \frac{E_i D_{i+3}}{(1 + rf + rp)^3} + \ldots + \frac{E_i D_{i+n}}{(1 + rf + rp)^n} + \ldots = \sum_{n=1}^{\infty} \frac{E_i D_{i+n}}{(1 + rf + rp)^n}
\]
\[
P_i = \frac{D}{1 + rf + rp} + \frac{D}{(1 + rf + rp)^2} + \frac{D}{(1 + rf + rp)^3} + \ldots + \frac{D}{(1 + rf + rp)^n} + \ldots = \frac{D}{rf + rp}
\]
\[
P_i = \frac{1}{0.1} = 10
\]

Some might have memorized the expression from the book:
\[
P_i = \frac{D_i (1 + g)}{(i - g)} \text{ where } g = 0, \text{ and } rf + rp = i \text{ so}
\]
\[
P_i = \frac{D}{(i)} = 1 / .10 = 10
\]

3.3 (4 minutes) Suppose that you revise your expectations regarding \((rf + rp)\) downward by 4 percentage points. What immediately happens to the price of the share of stock? Once again, show your work.

\[
P_i = \frac{1}{0.06} = 1 / 0.06 \approx 16.67
\]

4. [10 minutes total] Suppose we have the AD-AS model outlined in lecture. You can assume expected inflation is always zero.
4.1 (3 minutes) Assume a tax cut in period 1 that shift out the AD curve by 10% at price level $P_0$. That is $t_1$ falls in $T = t_0 + t_1Y$. Show what happens in period 1

Shown above. AD shifts out (light gray arrow). Output rises to $Y_1'$ in period 1 when the tax cut occurs, $Y_1' = 1.1 \times Y_{FE}$. Since the Phillips Curve (not augmented with expected inflation) is:

$$P_t = P_{t-1} + P_{t-1} \times f \left( \frac{Y_{t-1} - Y^*}{Y^*} \right)$$

And output equaled $Y_{FE}$ or $Y^*$ in period 0, price line does not rise in period 1.

4.2 (3 minutes) Show what happens over time.

Shown below. In period 2, price rises to $P_2$ (medium gray arrow) because output gap in period 1 is above 0, and Phillips Curve says price should rise:

$$P_t = P_{t-1} + P_{t-1} \times f \left( \frac{Y_{t-1} - Y^*}{Y^*} \right)$$

Output falls as higher price erodes real money supply, $(M/P)$. In period 3, price rises again (dark gray arrow) because output gap in period 2 is positive. Output falls again. This process repeats over and over again (black arrow) until output gap is zero, so price level converges to $P_F$. 

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4.3 (4 minutes) Assume that the tax cut in period 1 increases work effort and the capital effectiveness, so $Y^{FE}$ increases immediately by 10%. Show what happens in period 1, and thereafter.

Shown below. $AS_{LR}$ shifts out (white arrow) so $Y^{FE} = 1.1 \times Y^{FE}$. Since AD has shifted out same distance as $AS_{LR}$, then output gap remains zero and there is no inflationary or deflationary pressure.