# Economics 435
# The Financial System
(10/2/19)

Instructor: Prof. Menzie Chinn
UW Madison
Fall 2019

# Future Value and Present Value

- If the present value is \$100 and the interest rate is 5%, then the *future value* one year from now is:

$$\$100 + \$100(0.05) = \$105$$

- This also shows that the higher the interest rate, the higher the future value.
- In general:

$$FV = PV + PV(i) = PV(1 + i)$$

- And:

$$PV = \frac{FV}{(1+i)}$$

# Future Value and Compound Interest

- What if you leave your $100 in the bank for two years at 5% yearly interest rate?
- The future value is:

$$\$100 + \$100(0.05) + \$100(0.05) + \$5(0.05) = \$110.25$$

$$\$100(1.05)(1.05) = \$100(1.05)^2$$

- In general

$$FV_n = PV(1 + i)^n$$

$$PV = \frac{FV}{(1 + i)^n}$$

# Complications

- What if payments, $X_t$, occur all the way along until the end?
- What if the interest rate, $i_t$, is not constant?

$$PV_t = \left[ \frac{X_{t+1}}{(1+i_t)} + \frac{X_{t+2}}{(1+i_t)\times(1+i_{t+1})} + \ldots\ldots + \frac{X_{t+n}}{(1+i_t)\times(1+i_{t+1})\ldots(1+i_{t+n-1})} \right]$$

- But at time t, one doesn't know t+n information … so:

$$PV_t = \left[ \mathcal{E}_t \frac{X_{t+1}}{(1+i_t)} + \mathcal{E}_t \frac{X_{t+2}}{(1+i_t)\times(1+i_{t+1})} + \ldots\ldots + \mathcal{E}_t \frac{X_{t+n}}{(1+i_t)\times(1+i_{t+1})\ldots(1+i_{t+n-1})} \right]$$

# Bond Basics

- The most common type of bond is a coupon bond.
  - Issuer is required to make annual payments, called coupon payments.
  - The annual interest the borrower pays ($i_c$), is the coupon rate.
  - The date on which the payments stop and the loan is repaid (n), is the maturity date or term to maturity.
  - The final payment is the principal, face value, or par value of the bond.

# Bond Prices

**1. Zero-coupon or discount bond**

- Promise a single payment on a future date
- Example: Treasury bill

**2. Fixed-payment loan**

- Sequence of fixed payments
- Example: Mortgage or car loan

**3. Coupon bond**

- periodic interest payments + principal repayment at maturity
- Example: U.S. Treasury Bonds and most corporate bonds

**4. Consol**

- periodic interest payments forever, principal never repaid
- Example: U.K. government has some outstanding

# Zero-Coupon Bonds

- U.S. Treasury bills (T-bills) are the most straightforward type of bond.
  - Each T-bill represents a promise by the U.S. government to pay $100 on a fixed future date.
  - No coupon payments - zero-coupon bonds
  - Also called pure discount bonds (or discount bonds) since the price is less than face value - they sell at a discount.
- Price of $100 face value zero-coupon bond

$$= \frac{\$100}{(1+i)^n}$$

# Zero-Coupon Bonds

Assume $i = 5\%$

Price of a One-Year Treasury Bill

$$= \frac{100}{(1+0.05)} = \$95.24$$

Price of a Six-Month Treasury Bill

$$= \frac{100}{(1+0.05)^{1/2}} = \$97.59$$

# Zero-Coupon Bonds

- For a zero-coupon bond, the relationship between the price and the interest rate is the same as we saw on present value calculations.
- When the price moves, the interest rate moves with it, in the opposite direction.
- We can compute the interest rate from the price using the present value formula.

The price of a one-year T-bill is \$95.

$i$ = (\$100/\$95) - 1 = 0.0526 = 5.26%

# Fixed-Payment Loans

- Home mortgages and car loans are fixed-payment loans.
  - They promise a fixed number of equal payments at regular intervals.
  - Amortized loans - the borrower pays off part of the principal along with the interest for the life of the loan.
- Value of a Fixed Payment Loan =

$$\frac{FixedPayment}{(1+i)} + \frac{FixedPayment}{(1+i)^2} + \cdots + \frac{FixedPayment}{(1+i)^n}$$

- The sum of the present value of the payments.

# Coupon Bonds

- The issuer of a coupon bond promises to make a series of periodic interest payments (coupon payments), plus a principal payment at maturity.

Price of Coupon Bond =

$$P_{CB} = \left[ \frac{CouponPayment}{(1+i)^1} + \frac{CouponPayment}{(1+i)^2} + ..... + \frac{CouponPayment}{(1+i)^n} \right]$$

$$+ \frac{FaceValue}{(1+i)^n}$$

# Consols

- Consols or perpetuities, are like coupon bonds whose payments last forever.
- The borrower pays only interest, never repaying the principal.
- The U.S. government sold consols once in 1900, but the Treasury has bought them all back.
- The price of a consol is the present value of all future interest payments.

$$P_{Consol} = \frac{\text{Yearly Coupon Payment}}{i}$$

# Bond Yields

- We know how to calculate bond prices given an interest rate.
- We also need to be able to go in the other direction.
  - Calculate the return to an investment, implicit in the bond's price.
- We will combine information about the promised payments with the price to obtain the *yield*:
  - A measure of the cost of borrowing and the reward for lending.
  - We will use the terms *yield* and *interest rate* interchangeably.

# Yield to Maturity

- The most useful measure of the return on holding a bond is called the yield to maturity:
  - The yield bondholders receive if they hold the bond to its maturity when the final principal payment is made.

$$\text{Price of 1yr 5\% Coupon Bond} = \frac{\$5}{(1+i)} + \frac{\$100}{(1+i)}$$

- The value of $i$ that solves the equation is the yield to maturity.

# Current Yield

Example:

1 year, 5% coupon bond selling for \$99

$$\text{Current Yield} = \frac{5}{99} = 0.0505 \text{ , or } 5.05\%$$

Yield to maturity for this bond is 6.06 percent found as the solution to:

$$\frac{\$5}{(1+i)} + \frac{\$100}{(1+i)} = \$99$$

# Holding Period Returns

- The *one-year holding period return* is the sum of the yearly coupon payment divided by the price paid for the bond and the change in the price divided by the price paid.

$$= \frac{\text{Yearly Coupon Payment}}{\text{Price Paid}} + \frac{\text{Change in Price of the Bond}}{\text{Price of the Bond}}$$

$$= \text{Current Yield} + \text{Capital Gain (as a \%)}$$

# Data on "Treasury Notes and Bonds"

http://online.wsj.com/mdc/public/page/2_3020-treasury.html

**U.S. Treasury Quotes** — Tuesday, September 24, 2019

**Treasury Notes & Bonds** | Treasury Bills

Treasury note and bond data are representative over-the-counter quotations as of 3pm Eastern time. For notes and bonds callable prior to maturity, yields are computed to the earliest call date for issues quoted above par and to the maturity date for issues below par.

| MATURITY | COUPON | BID | ASKED | CHG | ASKED YIELD |
|---|---|---|---|---|---|
| 9/30/2019 | 1.000 | 99.3120 | 99.3160 | unch. | 1.570 |
| 9/30/2019 | 1.375 | 99.3120 | 99.3160 | -0.0020 | 1.943 |
| 9/30/2019 | 1.750 | 99.3120 | 99.3160 | -0.0040 | 2.315 |
| 10/15/2019 | 1.000 | 99.3000 | 99.3040 | 0.0040 | 1.858 |
| 10/31/2019 | 1.250 | 99.2900 | 99.2940 | -0.0020 | 2.048 |
| MATURITY | COUPON | BID | ASKED | CHG | ASKED YIELD |
| 8/15/2026 | 6.750 | 133.2740 | 133.2800 | 0.1820 | 1.547 |
| 8/31/2026 | 1.375 | 98.2240 | 98.2300 | 0.1600 | 1.571 |
| 11/15/2026 | 2.000 | 102.2840 | 102.2900 | 0.1780 | 1.568 |
| 11/15/2026 | 6.500 | 133.0840 | 133.0900 | 0.8700 | 1.556 |
| 2/15/2027 | 2.250 | 104.2140 | 104.2200 | 0.1800 | 1.575 |
| 2/15/2027 | 6.625 | 135.0420 | 135.0460 | 0.8860 | 1.568 |
| 5/15/2027 | 2.375 | 105.2060 | 105.2120 | 0.1720 | 1.585 |
| 8/15/2027 | 2.250 | 104.2700 | 104.2740 | 0.1940 | 1.592 |
| 8/15/2027 | 6.375 | 135.1420 | 135.1460 | 0.8960 | 1.577 |
| 11/15/2027 | 2.250 | 104.2860 | 104.2920 | 0.1920 | 1.603 |
| 11/15/2027 | 6.125 | 134.1800 | 134.1840 | 0.9100 | 1.580 |
| 2/15/2028 | 2.750 | 108.2840 | 108.2900 | 0.2020 | 1.611 |
| 5/15/2028 | 2.875 | 110.0100 | 110.0140 | 0.8640 | 1.624 |
| 8/15/2028 | 2.875 | 110.0740 | 110.0800 | 0.8700 | 1.631 |
| 8/15/2028 | 5.500 | 132.0460 | 132.0520 | 0.9160 | 1.603 |
| 11/15/2028 | 3.125 | 112.1840 | 112.1900 | 0.8740 | 1.636 |
| 11/15/2028 | 5.250 | 130.2500 | 130.2540 | 0.2360 | 1.612 |
| 2/15/2029 | 2.625 | 108.1760 | 108.1820 | 0.8720 | 1.636 |
| 2/15/2029 | 5.250 | 131.1760 | 131.1820 | 0.9260 | 1.613 |
| 5/15/2029 | 2.375 | 106.1720 | 106.1760 | 0.8800 | 1.637 |
| MATURITY | COUPON | BID | ASKED | CHG | ASKED YIELD |
| 2/15/2049 | 3.000 | 120.0120 | 120.0220 | 1.8620 | 2.083 |
| 5/15/2049 | 2.875 | 117.1160 | 117.1260 | 1.8440 | 2.085 |
| 8/15/2049 | 2.250 | 103.1640 | 103.1740 | 1.1320 | 2.090 |

Source: Tullett Prebon

Accessed
9/25/2019

# Data on Treasury Bills

**U.S. Treasury Quotes** Tuesday, September 24, 2019

Treasury Notes & Bonds | **Treasury Bills**

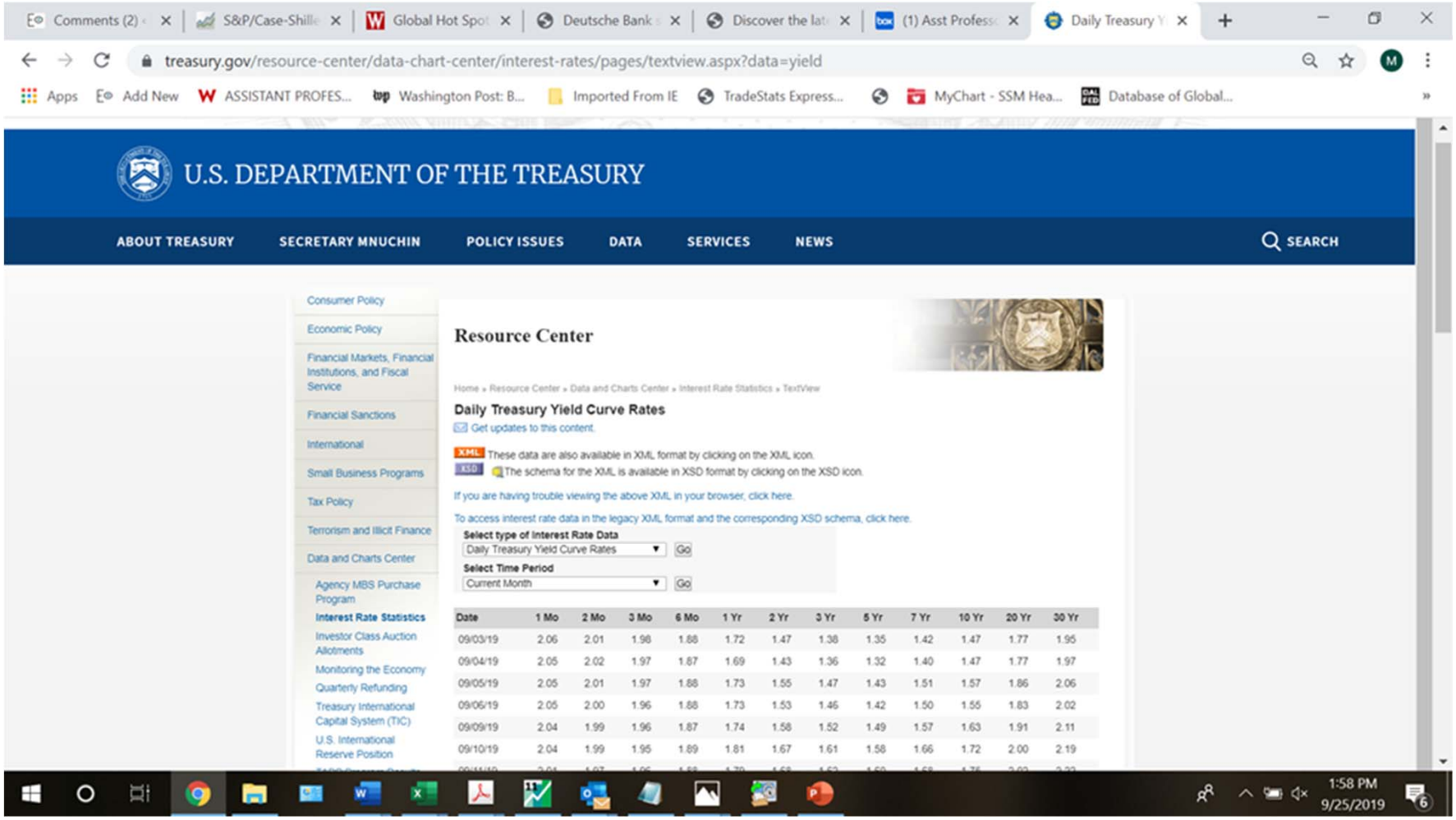
Treasury bill bid and ask data are representative over-the-counter quotations as of 3pm Eastern time quoted as a discount to face value. Treasury bill yields are to maturity and based on the asked quote.

| MATURITY | BID | ASKED | CHG | ASKED YIELD |
|---|---|---|---|---|
| 9/26/2019 | 1.773 | 1.763 | -0.010 | 1.792 |
| 10/1/2019 | 1.798 | 1.788 | -0.128 | 1.818 |
| 10/3/2019 | 1.773 | 1.763 | -0.053 | 1.7926 |
| 10/8/2019 | 1.855 | 1.845 | -0.068 | 1.877 |
| 2/6/2020 | 1.873 | 1.863 | -0.002 | 1.907 |
| 2/13/2020 | 1.865 | 1.855 | -0.002 | 1.900 |
| 2/20/2020 | 1.875 | 1.865 | -0.008 | 1.911 |
| 2/27/2020 | 1.863 | 1.853 | -0.008 | 1.893 |
| 3/5/2020 | 1.863 | 1.853 | -0.023 | 1.899 |
| 3/12/2020 | 1.870 | 1.860 | unch. | 1.908 |
| 3/19/2020 | 1.868 | 1.858 | unch. | 1.906 |
| 3/26/2020 | 1.863 | 1.853 | -0.002 | 1.901 |
| 4/23/2020 | 1.835 | 1.825 | -0.008 | 1.876 |
| 5/21/2020 | 1.818 | 1.808 | -0.003 | 1.860 |
| 6/18/2020 | 1.798 | 1.788 | -0.007 | 1.842 |
| 7/16/2020 | 1.780 | 1.770 | -0.015 | 1.826 |
| 8/13/2020 | 1.753 | 1.743 | -0.015 | 1.800 |
| 9/10/2020 | 1.740 | 1.730 | -0.018 | 1.789 |

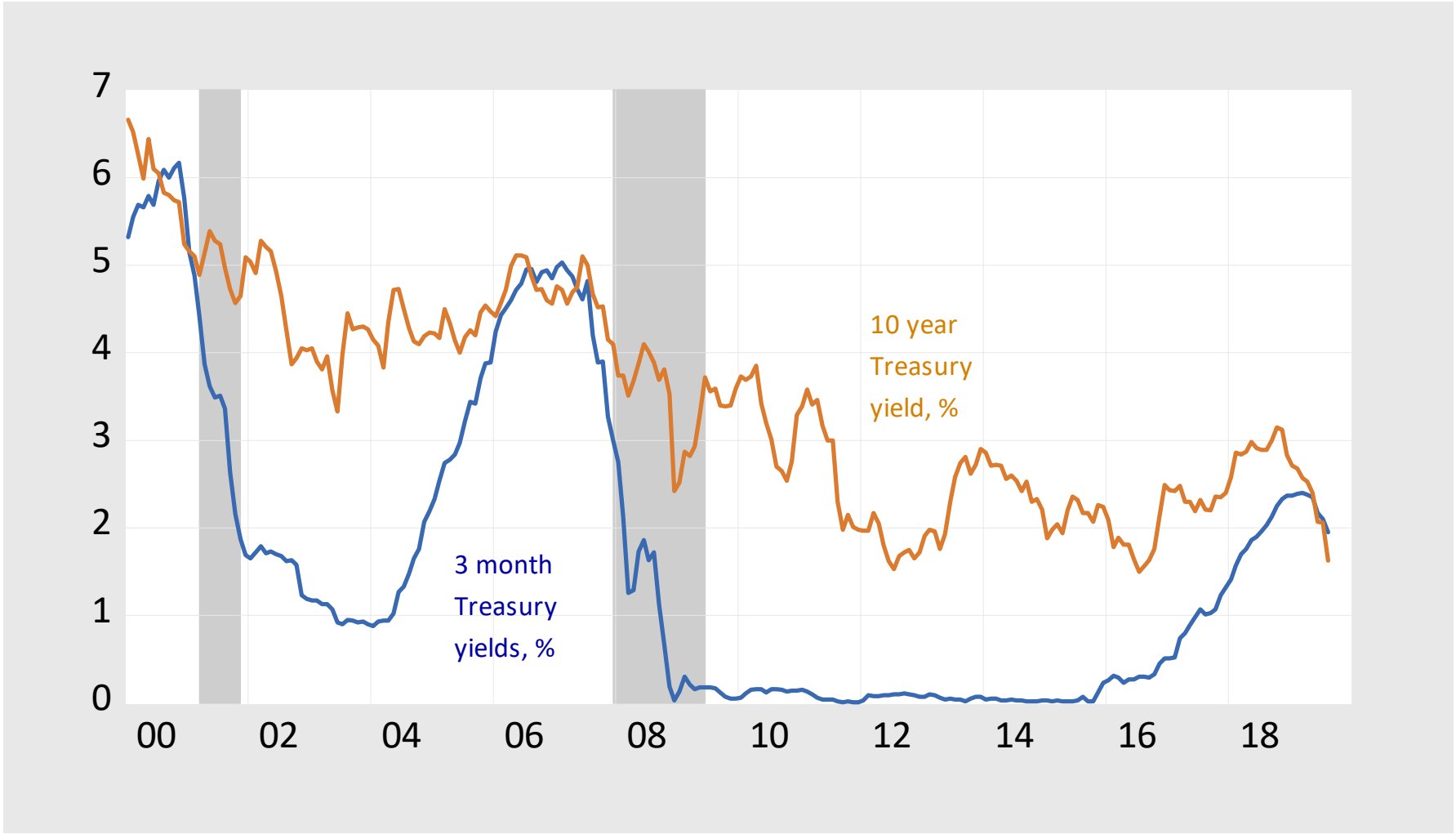
"On the run"

# Data on Treasurys

https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yield



# Secondary Market, Constant Maturity



# Real and Nominal Interest Rates

- The nominal interest rate you agree on ($i$) must be based on *expected inflation* ($\pi^e$) over the term of the loan plus the real interest rate you agree on ($r$).

$$i = r + \pi^e$$

- This is called the *Fisher Equation.*
- The higher expected inflation, the higher the nominal interest rate.

# Data on Treasury Inflation Protected Securities (TIPS)

http://online.wsj.com/mdc/public/page/2_3020-tips.html

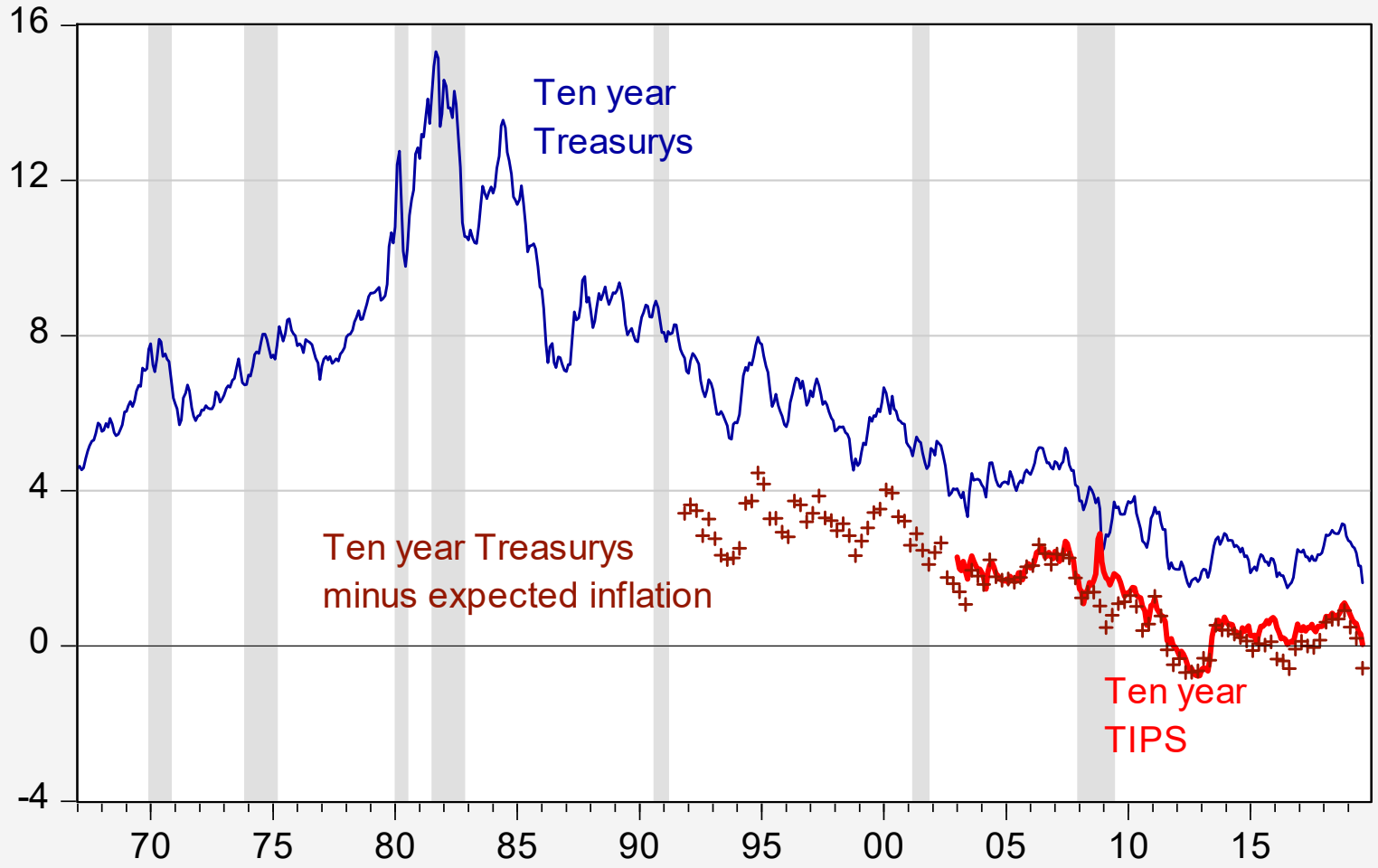
## Treasury Inflation-Protected Securities

Tuesday, October 02, 2018 Find Historical Data | WHAT'S THIS?

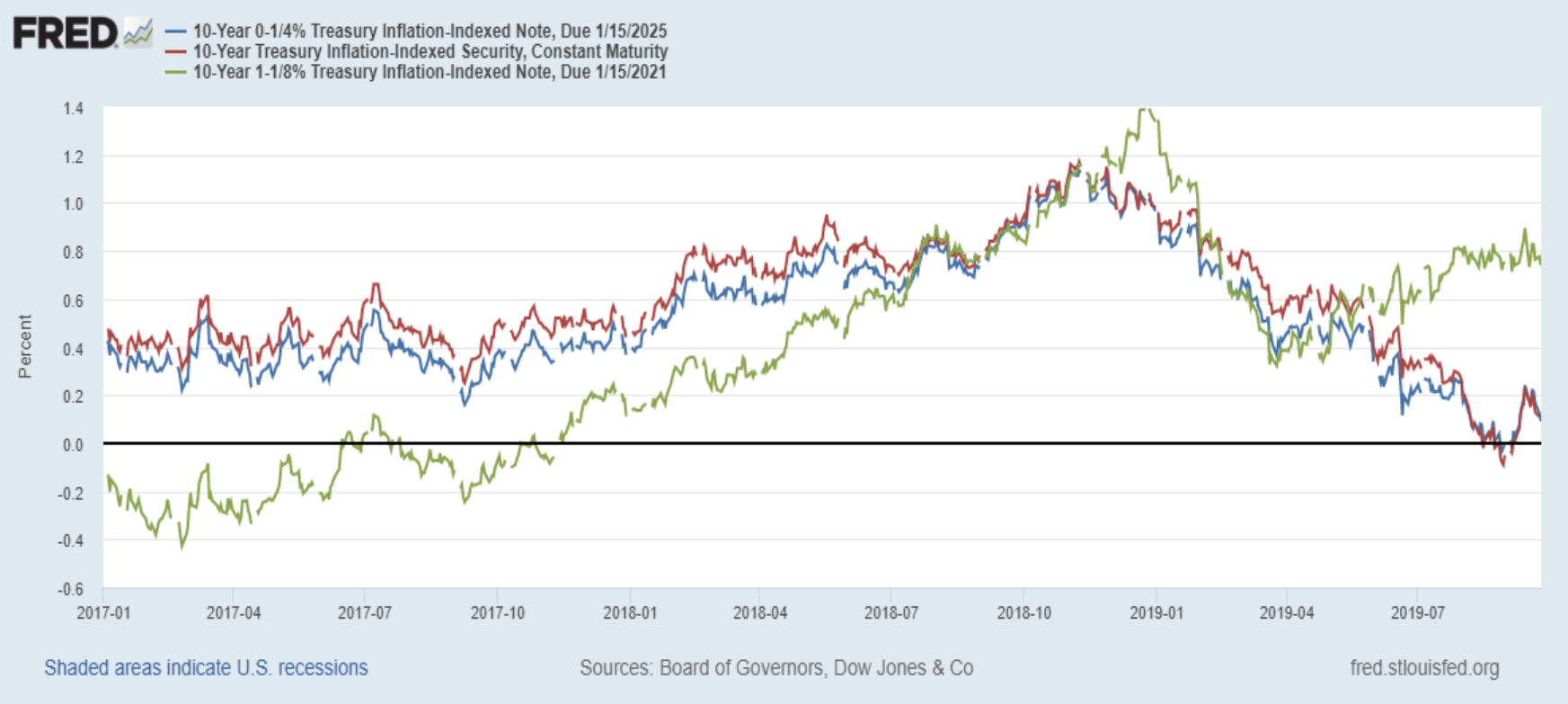
Treasury Inflation-Protected Securities, or TIPS, are securities whose principal is tied to the Consumer Price Index (CPI) . The principal increases with inflation and decreases with deflation. When the security matures, the U.S. Treasury pays the original or adjusted principal, whichever is greater. TIPS pay interest every six months. Figures after periods in bid and ask quotes represent 32nds; 101.26 means 101 26/32, or 101.8125% of 100% face value; 99.01 means 99 1/32, or 99.03125% of face value.

| Maturity | Coupon | Bid | Asked | Chg | Yield* | Accrued principal |
|---|---|---|---|---|---|---|
| 2019 Jan 15 | 2.125 | 100.10 | 100.12 | + 1 | 0.765 | 1173 |
| 2019 Apr 15 | 0.125 | 99.14 | 99.16 | unch. | 1.054 | 1075 |
| 2019 Jul 15 | 1.875 | 101.09 | 101.11 | unch. | 0.166 | 1180 |
| 2020 Jan 15 | 1.375 | 100.23 | 100.25 | - 1 | 0.755 | 1165 |
| 2020 Apr 15 | 0.125 | 98.23 | 98.25 | unch. | 0.922 | 1076 |
| 2020 Jul 15 | 1.250 | 101.05 | 101.07 | unch. | 0.562 | 1155 |
| 2021 Jan 15 | 1.125 | 100.19 | 100.21 | unch. | 0.831 | 1152 |
| 2021 Apr 15 | 0.125 | 97.31 | 98.01 | unch. | 0.910 | 1063 |
| 2021 Jul 15 | 0.625 | 99.24 | 99.26 | + 1 | 0.687 | 1118 |
| 2022 Jan 15 | 0.125 | 97.18 | 97.20 | + 1 | 0.863 | 1113 |
| 2022 Apr 15 | 0.125 | 97.06 | 97.08 | + 1 | 0.916 | 1036 |
| 2022 Jul 15 | 0.125 | 97.20 | 97.22 | + 1 | 0.748 | 1095 |
| 2023 Jan 15 | 0.125 | 96.27 | 96.29 | + 2 | 0.864 | 1091 |
| 2023 Apr 15 | 0.625 | 98.24 | 98.26 | + 2 | 0.895 | 1014 |
| 2023 Jul 15 | 0.375 | 98.02 | 98.04 | + 2 | 0.772 | 1082 |
| 2026 Jan 15 | 0.625 | 97.25 | 97.28 | + 3 | 0.926 | 1060 |
| 2026 Jan 15 | 2.000 | 107.16 | 107.19 | + 3 | 0.921 | 1269 |
| 2026 Jul 15 | 0.125 | 94.08 | 94.10 | + 4 | 0.881 | 1051 |
| 2027 Jan 15 | 0.375 | 95.13 | 95.16 | + 4 | 0.941 | 1043 |
| 2027 Jan 15 | 2.375 | 111.10 | 111.13 | + 4 | 0.941 | 1249 |
| 2027 Jul 15 | 0.375 | 95.15 | 95.18 | + 4 | 0.901 | 1030 |
| 2028 Jan 15 | 0.500 | 95.28 | 95.31 | + 4 | 0.954 | 1021 |
| 2028 Jan 15 | 1.750 | 106.30 | 107.02 | + 5 | 0.955 | 1202 |
| 2028 Apr 15 | 3.625 | 124.01 | 124.06 | + 4 | 0.963 | 1558 |
| 2028 Jul 15 | 0.750 | 98.10 | 98.16 | + 4 | 0.912 | 1003 |
| 2029 Jan 15 | 2.500 | 114.26 | 114.31 | + 4 | 0.967 | 1173 |
| 2029 Apr 15 | 3.875 | 128.21 | 128.26 | + 6 | 0.987 | 1533 |
| 2045 Feb 15 | 0.750 | 92.19 | 92.24 | + 14 | 1.067 | 1070 |
| 2046 Feb 15 | 1.000 | 98.09 | 98.14 | + 15 | 1.066 | 1063 |
| 2047 Feb 15 | 0.875 | 95.06 | 95.11 | + 14 | 1.066 | 1044 |
| 2048 Feb 15 | 1.000 | 98.08 | 98.13 | + 15 | 1.063 | 1021 |

# Nominal vs. Real

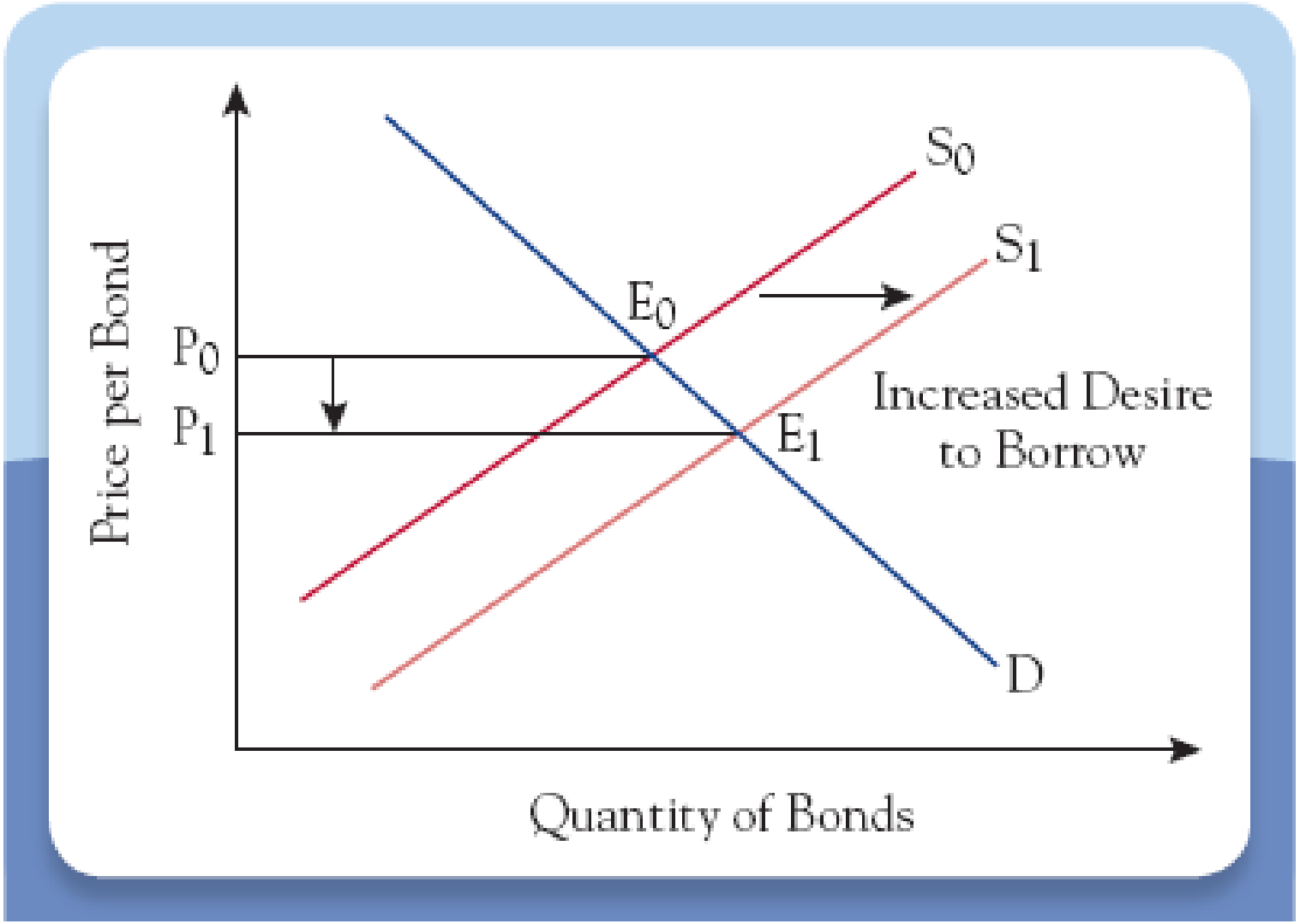


# Constant Maturity vs. On the Run vs. Off the Run



# Factors That Shift Bond Supply

**Figure 6.2** A Shift in the Supply of Bonds



# Factors That Shift Bond Demand

**Figure 6.3** A Shift in the Demand for Bonds