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Spring 2012 Social Sciences 7418
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Problem Set 1 Answers

Due ***in lecture*** on Monday, February 13th. Be sure to put your name on your problem set. Put "boxes" around your answers to the algebraic questions.

1. The CPI is calculated for a fixed market basket. It measures the change in the cost of the market basket from the base year until the current year. An index with the market basket fixed in the first year, like the CPI, is called a Laspeyres index. An alternative index, the Paasche Index, is based on a market basket in the end year. It measures the change in the cost of a market basket fixed in the end year. Suppose that the base is 2006, and further that the market basket contains only two items, wine and cheese, and the quantities consumed in 2006 and 2007 are

	wine	cheese
2006	50 bottles	100 wheels
2007	45 bottles	150 wheels

Suppose that the price of cheese increases from $1.00 per wheel of cheese in 2006 to $1.20 per wheel in 2007 and the price of wine increases from $0.50 per bottle to $2.00 per bottle.

1.1 Calculate the rate of inflation for the Laspeyres (CPI) index and the Paasche Index.

The Laspeyres price index for 2007 is 100*[(cost of base year basket in 2007) / (cost of base year basket in base year)]:

$$Laspeyres_{2007} = 100 \times \left(\frac{100 \times \$1.20 + 50 \times \$2}{100 \times \$1.00 + 50 \times \$.50} \right) = 176$$

Since 2006 is the base year, the Laspeyres price index is normalized to 100 for 2006. So, the rate of inflation using the Laspeyres index is:

$$Inflation = \frac{P_{new} - P_{old}}{P_{old}} = \frac{176 - 100}{100} = .76, \text{ or } 76\%$$

The Paasche price index for 2007 is 100*[(cost of 2007 basket in 2007) / (cost of 2007 basket in base year)]:

$$Paasche_{2007} = 100 \times \left(\frac{150 \times \$1.20 + 45 \times \$2}{150 \times \$1.00 + 45 \times \$.50} \right) = 156.5$$

Again, the index is normalized to 100 for the base year, so the rate of inflation using the Paasche index is:

$$Inflation = \frac{P_{new} - P_{old}}{P_{old}} = \frac{156.5 - 100}{100} = .565, \text{ or } 56.5\%$$

As usual, the Laspeyres index records more inflation than the Paasche index because it does not reflect substitution away from the goods whose prices increased the most.

1.2 Will inflation calculated using the Laspeyres index always exceed inflation calculated with the Paasche index? (Hint: Use standard indifference curve analysis.)

Yes, in general. Consumers will substitute away from goods that are relatively more expensive to goods that are relatively cheaper. Therefore, it will be more expensive to buy the original basket than the new basket and so Laspeyres will exceed Paasche when there is inflation.

1.3 Workers often receive an adjustment in their wages equal to only a fraction of inflation as calculated using the CPI. In view of the preceding analysis, explain why workers would likely be better off than they were before if they were fully compensated for inflation. Would this also be the case if inflation was calculated using the Paasche index?

> If we compensated workers for inflation using the Laspeyres index, the argument in part (b) shows that they'll get too much of an increase (i.e. they'll be *better* off than before). The Laspeyres is designed so that the consumer could afford the same basket as before, but he'd probably pick a basket with more of the cheaper good, which makes him better off. This is not true with a Paasche price index, since it's based on the *new* basket.

2. Chain-Weighting. Suppose that the agrarian economy of Simpsonia consists only of two sectors: private consumption and private investment. The following figures give total production and prices for both sectors in 2010 and 2011. The base year is 2010

CONSUMPTION

	CHEESE		**WINE**	
	Quantity	**Price**	**Quantity**	**Price**
2010	100	$6	150	$10
2011	100	$8	400	$2

INVESTMENT

	BULLDOZERS		**TRUCKS**	
	Quantity	**Price**	**Quantity**	**Price**
2010	4	$200	13	$50
2011	5	$260	15	$60

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2.1 Calculate nominal consumption, investment and GDP for 2010 and 2011

Answer: Nominal consumption for 2010 is
(100 cheese wheels)×$6 + (150 wine bottles)×$10 = $2100
Nominal consumption for 2011 is:
(100 cheese wheels)×$8 + (400 wine bottles)×$2 = $1600
Using similar calculations, nominal investment is $1450 in 2010 and $2200 in 2011. Nominal GDP, therefore, is $3550 in 2010 and $3800 in 2051.

2.2 Using the traditional method, calculate real consumption for 2011.

The traditional method values 2011 production at 2010 prices (the base year) to get a time-comparable real measure. So, 2011 real consumption:
(100 cheese wheels)*$6 + (400 wine bottles)*$10 = $4600

2.3 Using the traditional method, calculate real investment for 2011.

In the same way as 2.2, 2011 real investment is $1750.

2.4 Using the traditional method, calculate real GDP for 2011.

Real GDP is everything in 2011 production, but valued at 2150 prices. This is $6350, and is equal to the sum of the two components of real GDP.

2.5 Does 2011 real GDP equal the sum of real consumption and real investment in 2011 when calculated using the traditional method?

Yes.

2.6 Using the chain-weighted method, calculate real consumption in 2011.

Real quantity of cheese wheels grew 0% between 2010 and 2011. Real quantity of wine bottles grew (400-150)/150 = 1.67, or 167% between 2010 and 2011.
Now, using 2011 prices, $800 was spent on consumption in 2011; of this $800 was on cheese and $800 on wine. So, cheese represent proportion .5 of expenditures, as does wine. The weighted average growth rate (where weights are determined by expenditure shares) is:

=weight_cheese*growth_cheese + weight_wine*growth_wine
=.5*0 + .5*1.67 = .835

So, cumulated real consumption in 2011 using the chain-weighted method is an 83.5% increase over real consumption in 2010 (note that real 2010 consumption equals nominal 2010 consumption from (a), since 2010 is the base year):

Real_2011 = $2100*(1+.835) = $3853.5

2.7 Using the chain-weighted method, calculate real investment in 2011.

Using similar methods to 2.6, quantity of bulldozers grew 25% and trucks grew 15.4% between 2010 and 2011. $2200 was spent on investment in 2011. Of this, $1300 – 59.1% - was on bulldozers, and the other 40.9% was spent on truckss. Therefore, the weighted average growth rate is:

=weight_bulldozers*growth_bulldozers + weight_trucks*growth_truckss
=.591*.25 + .409*.154 = .211

Consequently, cumulated real investment in 2011 is 21.1% higher than in 2010:

Real_2011 = $1450*(1+.211) = $1755.95

2.8 Using the chain-weighted method, calculate real GDP in 2011 (note: develop weights for *all four* goods and take a weighted average of the growth rates).

If we look at the whole economy in 2011 ($3800), cheese wheels account for $800 – or 21.1%, wine bottles accounts for 21.1%, bulldozers for 34.2% and trucks for 23.7%.
Using the growth rates for each good from 2.6 and 2.7, the weighted average growth rate for the whole economy (all 4 goods) is:

=.211*0 + .211*1.67 + .342*.25 + .237*.154 = .474

So, real GDP in 2011 is 47.4% higher than in 2010, using the chain weighted method.

Real_2011 = 3550(1+.474) = $5232.7

2.9 Does 2011 real GDP equal the sum of real consumption and real investment in 2011 when using the chain-weighted method? Explain why or why not.

Notice that if we sum chain-weighted real consumption and real investment, calculated separately for each sector in 2.6 and 2.7, we get $5609.45. This is not the same as the answer we got by chain-weighting the whole economy in 2.8.

This is one caveat when using figures adjusted for inflation with chain-weighted methods. Chain-weighted GDP does NOT generally equal the sum of the chain-weighted figures for each sector separately. However, this property IS true for real GDP calculated the traditional way (see 2.4). Why is this? Unlike the traditional method, where goods are aggregated in a linear way, the aggregation across goods for chain-weighted GDP is nonlinear. Consequently, there is no reason to expect it to be additively separable across sectors.

3. This problem requires obtaining data from various sources. You can access the latest GDP data from the BEA at http://www.bea.gov/newsreleases/national/gdp/2012/pdf/gdp4q11_adv.pdf (January 2012 release). The Consumer Price Index figures can be obtained from the St. Louis Fed website at http://research.stlouisfed.org/fred2/categories/9 and http://research.stlouisfed.org/fred2/categories/32424 .

3.1. Calculate the *annualized* quarterly growth rate of real GDP in each of the last four quarters. Is the economy expanding or contracting? **Show your work!**

Table 3. Gross Domestic Product and Related Measures: Level and Change From Preceding Period

	Billions of current dollars						Billions of chained (2005) dollars					
		Seasonally adjusted at annual rates						Seasonally adjusted at annual rates				
	2011	2010	2011				2011	2010	2011			
		IV	I	II	III	IV		IV	I	II	III	IV
Gross domestic product	15,087.7	14,755.0	14,867.8	15,012.8	15,176.1	15,294.3	13,313.4	13,216.1	13,227.9	13,271.8	13,331.6	13,422.4

Use the formula:

$$annualizedgrowthrate = \left(\frac{Z_t}{Z_{t-1}}\right)^4 - 1$$

$$\left(\frac{13227.9}{13216.1}\right)^4 - 1 = 0.00358 \Rightarrow 0.04\%$$

$$\left(\frac{13271.8}{13227.9}\right)^4 - 1 = 0.01334 \Rightarrow 1.3\%$$

$$\left(\frac{13331.6}{13271.8}\right)^4 - 1 = 0.01815 \Rightarrow 1.8\%$$

$$\left(\frac{13422.4}{13331.6}\right)^4 - 1 = 0.02752 \Rightarrow 2.8\%$$

which matches up with the reported statistics in Table 1 of the BEA's GDP release:

Table 1. Real Gross Domestic Product and Related Measures: Percent Change From Preceding Period

	2009	2010	2011	Seasonally adjusted at annual rates															
				2008				2009				2010				2011			
				I	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV
Gross domestic product (GDP)	−3.5	3.0	1.7	−1.8	1.3	−3.7	−8.9	−6.7	−0.7	1.7	3.8	3.9	3.8	2.5	2.3	0.4	1.3	1.8	2.8

3.2. Calculate the annual rate of change of the GDP deflator, and the Personal Consumption Expenditure deflator, from the fourth quarter of 2010 to the fourth quarter of 2011. **Show your work!** Are they the same value?

From Table 6, the price indices are:

	Seasonally adjusted				
	2010	2011			
	IV	I	II	III	IV
Gross domestic product	111.699	112.390	113.091	113.811	113.935
Personal consumption expenditures (PCE)	111.673	112.747	113.666	114.324	114.524

Use the formula:

$$annual\_growth\_rate = \left(\frac{Z_t}{Z_{t-4}}\right) - 1$$

For GDP price index,

$$\left(\frac{113.935}{111.699}\right) - 1 = 0.0200 \Rightarrow 2.0\%$$

For the Personal Consumption expenditures price index,

$$\left(\frac{114.524}{111.673}\right) - 1 = 0.0255 \Rightarrow 2.6\%$$

They are not exactly the same value. In general the two inflation rates will differ since the baskets of goods and services differ.

3.3 Calculate the annual rate of change in the Consumer Price Index - All, and the Consumer Price Index excluding food and energy, from December 2010 to December 2011 (using seasonally adjusted data). **Show your work!** Are the rates identical?

obs	CPIAUCSL	CPILFESL
2010M12	***220.1860***	***222.2100***
2011M01	221.0620	222.5870
2011M02	222.2700	223.0290
2011M03	223.4900	223.3310
2011M04	224.4330	223.7450
2011M05	224.8040	224.3870
2011M06	224.3040	224.9580
2011M07	225.4250	225.4630
2011M08	226.2680	226.0140
2011M09	226.9550	226.1370
2011M10	226.7630	226.4440
2011M11	226.7200	226.8360
2011M12	***226.7470***	***227.1660***

For CPI-All,

$$\left(\frac{226.747}{220.186}\right) - 1 = 0.0298 \Rightarrow 2.98\%$$

For the Core CPI,

$$\left(\frac{227.166}{222.210}\right)-1=0.0223 \Rightarrow 2.23\%$$

4. Consider the following economy.

Eq.No.	Equation	Description
(1)	$Y = Z$	Output equals aggregate demand, an equilibrium condition
(2)	$Z \equiv C + I + G$	Definition of aggregate demand
(3)	$C = c_o + c_1 Y_D$	Consumption function, c_0 = 2000, c_1 = 0.8
(4)	$Y_D \equiv Y - T$	Definition of disposable income
(5)	$T = t_0 + t_1 Y$	Tax function; t_0 = -800, t_1 = 0.25
(6)	$I = b_0$	Investment function, b_0 = 800
(7)	$G = GO_0$	Government spending, GO_0 = 1400

4.1 Express, in algebraic symbols, the equilibrium level of income (***Y_0***) in this economy. Show your work.

$Y = AD = C + I + G$; substitute in for C, I, G

$Y = c_0 + c_1 Y_D + b_0 + GO_0$; substitute in for Y_D

$Y = c_0 + c_1(Y - T) + b_0 + GO_0$; substitute in for tax, transfers functions

$Y = c_0 + c_1(Y - t_0 - t_1 Y) + b_0 + GO_0$; bring the "$Y$" terms to left hand side.

$Y - c_1(Y - t_1 Y) = Y(1 - c_1(1-t_1)) = c_0 - c_1 t_0 + b_0 + GO_0$

divide both sides by $(1 - c_1(1-t_1))$, let $\Lambda_0 = c_0 - c_1 t_0 + b_0 + GO_0$

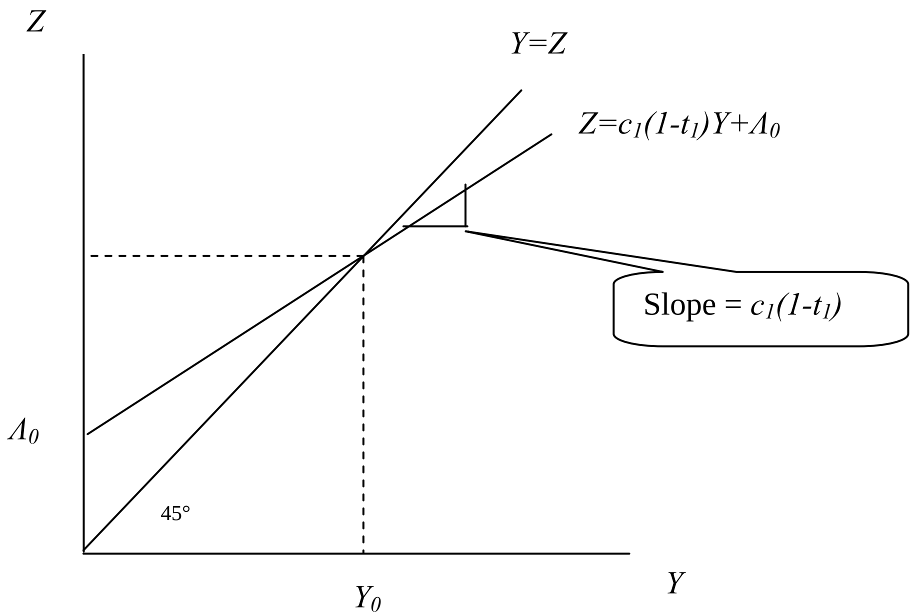
let $\bar{\gamma} = \frac{1}{[1 - c_1(1 - t_1)]}$

4.2. Substituting in the numerical values given above, indicate the numerical value of equilibrium income (in this and future subsequent numerical answers, round off your answer at two decimal places).

$$Y_0 = \left(\frac{1}{1 - 0.8(1 - 0.25)}\right) \times [2000 - (0.80) \times (-800) + 800 + 1400]$$

$$\boxed{Y_0 = 12100}$$

4.3. Using the Keynesian Cross diagram, illustrate your answer in part (4.1), with all relevant curves, intercepts and slopes indicated clearly.



4.4. Once again, using algebraic symbols, calculate the government spending multiplier in this economy. What is the multiplier for lump sum taxes (recall that a government transfer is the opposite of taxes)? Why does a reduction in lump sum have a different size impact from an increase in government spending on goods and services?

Recall that equilibrium income is:

$$\boxed{Y_0 = \bar{\gamma}\Lambda_0}$$

Taking the total differential yields:

$$\Delta Y = \bar{\gamma}[\Delta c_0 - c_1 \Delta t_0 + \Delta b_0 + \Delta GO]$$

To obtain the government spending multiplier, set all the Δ terms, except for Δ*GO*, equal to zero, and solve for Δ*Y*/Δ*GO*). To obtain the government transfers multiplier, recall that transfers are the negative of taxes. Set all the Δ terms, except for Δ*TA*, equal to zero, and solve for ΔY/Δt_0. This yields:

$$\Delta Y = \bar{\gamma}\Delta GO \Rightarrow \frac{\Delta Y}{\Delta GO} = \bar{\gamma}$$

$$\Delta Y = -\bar{\gamma}c_1\Delta t_0 \Rightarrow \frac{\Delta Y}{\Delta t_0} = -\bar{\gamma}c_1$$

Note that if we broke out transfers (*Tr*), they would be the negative of lump sum taxes.

$$\Delta Y = \bar{\gamma}c_1\Delta Tr \Rightarrow \frac{\Delta Y}{\Delta Tr} = \bar{\gamma}c_1$$

The two multipliers are different because in the case of the government spending multiplier, the initial spending (on goods and services) is immediately counted in GDP. However, in the case of government transfers, the initial effect is to increase income, of which only $c_1(1\text{-}t_1)$ is directed to consumption spending, which is then counted as part of GDP.

4.5. Using the answer to part (4.2), what is the level of consumption spending in this economy?

Recall $\boldsymbol{C = c_0 + c_1(Y - T) = c_0 + c_1 (1\text{-}t_1)Y - c_1\, t_0}$ = 2000 + (0.8)(.75)(8900) + 0.8(800); hence,

$$\boxed{C_0 = 9900}$$

4.6 If the level of investment spending were to rise to 1000, what would be the equilibrium level of income?

There are two ways of solving this. One way is to just substitute the value of $\boldsymbol{b_0}$ = 800, instead of 1000, into the expression for part 4.2. The other way is to recognize that the multiplier for changes in autonomous investment, *viz.*, $\boldsymbol{\Delta Y/\Delta b_0}$, is $\bar{\gamma}$, so the new income level is:

$$Y_1 = Y_0 + \bar{\gamma}\Delta b_0 = 12100 + 2.5 \times (1000 - 800)$$

$$\boxed{Y_1 = 12600}$$

5. Using the same economy as described in question 4, answer the following, given that the budget surplus is:

$$BuS \equiv T - G = t_0 + t_1 Y - GO_0$$

Assuming there is no government debt.

5.1. What is the value of the budget surplus when investment spending is 800?

$BuS \equiv (T - G)$; substituting in the functions given in question 4, one finds:

$$BuS = t_0 + t_1 Y - GO_0 = -800 + 0.25 \times 12100 - 1400$$

$$\boxed{BuS_0 = 825}$$

5.2. What is the budget surplus when I rises to $b_0 = 1200$?

The new income level corresponding to I = 1200 is 13100:

$$BuS_1 = -800 + 0.25 \times 13100 - 1400$$

$$\boxed{BuS_1 = 1075}$$

5.3. What accounts for the change in the budget surplus from part (5.1) to (5.2)?

The budget surplus is endogenous. When income changes (in this case because investment spending changes) then tax receipts fall, and so too does the budget surplus. In symbols:

$$\Delta BuS = \Delta t_0 + t_1(\Delta Y) - \Delta GO = t_1(\bar{\gamma}\Delta b_0) = 0.25 \times (2.5 \times 400) = 250$$

5.4. Suppose potential GDP (or "full-employment GDP") Y_n is 9000. What is the full-employment, or structural, budget surplus, BuS_n, when I = 800? 1000?

$BuS_n = t_0 + t_1 Y_n - GO_0$ Substituting in the given numerical values, one finds:

$BuS_0^* = -800 + 0.25 \times 9000 - 1400$

$\boxed{BuS_0^* = 50}$

5.5. Can you write out what the *BuS* depends upon, *algebraically* (i.e., using the symbols rather than the numbers)? What variables affect *BuS*? What variables affect the full-employment budget surplus, Bus_n?

This is answered in parts 5.3 and 5.4 above.

6. National savings identity and the Keynesian Model

Suppose we add equations (8) and (9') to the model in problem (4), and redefine aggregate demand (2') to the open economy, as in the *Notes on the Keynesian Model of Income Determination*.

(1)	$Y = Z$	Output equals aggregate demand, an equilibrium condition
(2')	$Z \equiv C + I + G + X - IM$	Definition of aggregate demand
(3)	$C = c_o + c_1 Y_D$	Consumption function, c_1 is the marginal propensity to consume
(4)	$Y_D \equiv Y - T$	Definition of disposable income
(5)	$T = t_0 + t_1 Y$	Tax function; t_0 is lump sum taxes, t_1 is marginal tax rate.
(6)	$I = b_0$	Investment function, exogenous
(7)	$G = GO_0$	Government spending on goods and services, exogenous
(8)	$X = x_0$	Exports, exogenous
(9')	$IM = m_0 + m_1 Y$	Imports, endogenous

6.1. Solve for the impact of an increase in investment on the trade balance or net exports, algebraically.

First, one has to re-solve the model, using equation (2') instead of (2). This results in:

$\boxed{Y_0 = \bar{\gamma}\tilde{\Lambda}_0}$ where $\bar{\gamma} = \dfrac{1}{[1 - c_1(1 - t_1) + m_1]}$ and $\tilde{\Lambda}_0 \equiv [c_0 - c_1 t_0 + b_0 + GO_0 + x_0 - m_0]$

The trade balance is X-IM, so the change in the trade balance is:

$\Delta X - \Delta IM = \Delta x_0 - \Delta m_0 - m_1 \Delta Y = -m_1 \Delta Y$

But one knows that the change in income resulting from a change in investment is given by:

$\Delta Y = \bar{\gamma} b_0$

Substituting this into the first expression yields:

$\Delta X - \Delta IM = -m_1 \times (\bar{\gamma} b_0)$

$$\boxed{\frac{\Delta(TB)}{\Delta b_0} = -m_1 \bar{\gamma} < 0}$$

6.2. Using the definition of the budget surplus in problem 5, solve for the impact of an increase in investment on the budget balance, algebraically.

$\Delta BuS = \Delta t_0 + t_1(\Delta Y) - \Delta GO$ holding government spending constant.

$\Delta BuS = \Delta t_0 + t_1(\bar{\gamma}\Delta b_0) = \bar{\gamma}\Delta b_0 t_1$

$$\boxed{\frac{\Delta BuS}{\Delta b_0} = \bar{\gamma} t_1 > 0}$$

6.3. Will the budget and trade balances move in the same direction in response to a tax increase?

No, the trade balance and the budget balance will move in the opposite direction in response to an investment increase

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