Notes on the Okun’s Law, the Phillips Curve, and AD (rev’d 3/29)

Summary: This note focuses on the key equation in Chapter 9, combining the relationship between output growth and the change in unemployment (Okun’s Law) with the inflation/unemployment relationship (the Phillips Curve) from Chapter 8, and a dynamic version of the Aggregate Demand curve (focused on money).

**Okun’s Law**

This is an empirical observation, or “stylized fact”, relating the rate of change of the unemployment rate to the growth rate of GDP relative to the trend or “natural” rate of growth.

\[ u_t - u_{t-1} = -\beta (g_{yt} - \bar{g}_y) \]  

(9.3) “Okun’s Law”

Where \( g_{yt} \equiv (Y_t - Y_{t-1}) / Y_{t-1} \)

Empirically, \( \beta = 0.4 \), and \( \bar{g}_y = 0.03 \) for the United States.

**Figure 9-1**: Okun’s Law, 1970 onward.

**The Phillips Curve**

The expectations augmented Phillips curve is:

\[ \pi_t = \pi^e_t - \alpha (u_t - u_n) \]  

(9.4, 8.9) “Expectations augmented Phillips curve”

For simplicity, use the “accelerationist hypothesis” version of the expectations augmented Phillips curve:

\[ \pi_t - \pi_{t-1} = -\alpha (u_t - u_n) \]  

(9.5, 8.10) “Accelerationist hypothesis/Phillips curve”

**Aggregate Demand in Growth Rates**

Recall aggregate demand from Chapter 7 can be written as:

\[ Y_t = Y \left( \frac{M_t}{P_t}, G_t, T_t \right) \]  

(7.3) “Aggregate Demand”

Where \( Y(.) \) is a function of real money balances, government spending, and taxes (>0, >0, <0). Holding fiscal policy constant, and linearizing (7.3) yields:
\[ Y_t = \gamma \left( \frac{M_t}{P_t} \right) \]  
\[ \text{(9.6)} \]

Where \( \gamma > 0 \). If equation (9.6) holds, then (from Proposition 8, textbook Appendix 2, page A-8), the growth rate of \( M/P \) can be expressed as the difference in growth rates of \( M \) and \( P \):

\[ g_{yt} = (g_{mt} - g_{pt}) = (g_{mt} - \pi_t) \]  
\[ \text{(9.7)} \]

where \( g_{pt} \equiv \frac{P_t - P_{t-1}}{P_{t-1}} \equiv \pi_t \) and \( g_{mt} \equiv \frac{M_t - M_{t-1}}{M_{t-1}} \)

**Analyzing an Increase in (Trend) Nominal Money Growth**

One can see the short run implications of an increase in money growth by substituting the AD curve into Okun’s Law (9.7 into 9.3):

\[ u_t - u_{t-1} = -\beta(g_{yt} - \bar{\gamma}_y) \]

becomes

\[ u_t - u_{t-1} = -\beta((g_{mt} - \pi_t) - \bar{\gamma}_y) \]

One can rewrite this expression in the form of an equation of a line:

\[ \pi_t = \frac{1}{\beta} (u_t - u_n) - \frac{1}{\beta} (u_{t-1} - u_n) + (g_{mt} - \bar{\gamma}_y) \]

The graphical depiction is below. At any given instant, inflation and unemployment is given by the growth rate of money at time \( t \), lagged unemployment, the natural or “structural” rate of unemployment.

Increases in the growth rate of money induce increases in inflation, and decreases in the unemployment rate. Notice that equilibrium, conditional on lagged variables, is given by:

\[ \pi_t = \frac{1}{1 + \alpha \beta} \left[ \pi_{t-1} - \alpha (u_{t-1} - u_n) + \alpha \beta (g_{mt} - \bar{\gamma}_y) \right] \]

\[ u_t = \frac{1}{1 + \alpha \beta} \left[ u_{t-1} - \beta (g_{mt} - \pi_{t-1} - \bar{\gamma}_y) - \alpha \beta u_n \right] \]

Notice in the medium run \( g_{yt} = \bar{\gamma}_y \) so the AD equation becomes \( \bar{\gamma}_y = \bar{\gamma}_m - \pi \) so \( \pi = \bar{\gamma}_m - \bar{\gamma}_y \)