

Uncovered Interest Parity and Monetary Policy Near and Far from the Zero Lower Bound

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Abstract Relying upon a standard New Keynesian DSGE, we propose an explanation for two empirical findings in the international finance literature. First, the unbiasedness hypothesis — the proposition that ex post exchange rate depreciation matches interest differentials — is rejected much more strongly at short horizons than at long. Second, even at long horizons, the unbiasedness hypothesis tends to be rejected when one of the currencies has experienced a long period of low interest rates, such as in Japan and Switzerland. Using a calibrated New Keynesian dynamic stochastic general equilibrium model, we show how a monetary policy rule can induce the negative (positive) correlation between depreciation and interest differentials at short (long) horizons. The tendency to reject unbiasedness for Japan and Switzerland even at long horizons we attribute to the interaction of the monetary reaction function and the zero lower bound.

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1 Introduction

One of the most enduring empirical regularities in the international finance literature is the rejection of the joint hypothesis of uncovered interest rate parity (UIP) and rational expectations. According to the unbiasedness hypothesis, the interest rate differential between two countries should be an unbiased predictor of the change in the exchange rate between them. Chinn and Meredith (2004) document the continued failure of the unbiasedness hypothesis to hold for horizons of up to a year. But they also document that at horizons of greater than a year, the unbiasedness hypothesis holds better, so that at the ten year horizon, the hypothesis cannot be rejected.

Chinn and Quayyum (2013) update the results in Chinn and Meredith, using data ranging up to 2012. They confirm the finding that unbiasedness holds better at longer horizons than at short.

Chinn and Meredith attribute their finding to the greater endogeneity of the short term rate, given a Taylor-rule type central bank reaction function. They demonstrate that a Keynesian model incorporating sticky prices and model consistent expectations – and importantly a random shock in the uncovered interest parity relation – can replicate the pattern of unbiasedness rejections.

One key difference between the findings using the shorter and longer sample periods is that in the sample encompassing the Great Recession, and the encounter with the zero lower bound (ZLB), the tendency to reject the unbiasedness, even at the long horizon, increases. Even before the onset of the financial crisis, several countries had short term interest rates near the zero interest bound. Perhaps more importantly for our results, long term interest rates were particularly quite low in Japan and Switzerland, two currencies for which the estimated β is noticeably low.

At first glance, there is no particular reason why the unbiasedness hypothesis should not hold when short and/or long term rates strike the zero interest bound, as expected exchange rate changes could adjust to make the parity condition hold. However, it seems worthwhile to investigate the sensitivity of the estimates to inclusion of the low-interest rate period. We arbitrarily define 1996Q1 as the low interest rate period for these two countries. Estimating the Fama regression on long horizon data, allowing for a slope break at 1996Q1, indicates that post-1996 the unbiasedness relationship breaks down, with the slope coefficient significantly different – and negative – in the latter period.

The advent of low long term interest rates in Japan and Switzerland is associated with the attenuation of evidence for the unbiasedness hypothesis at the long horizon – for currency pairs involving these countries. However, it is not an explanation, as uncovered interest parity is an arbitrage condition.

There are two key motivations for this paper. The first is to show that a conventional New Keynesian dynamic stochastic general equilibrium (DSGE) model can

yield the implication that under normal conditions, the long horizon Fama coefficient is positive, and greater than the short horizon coefficient (which is typically negative) with only a minor modification. The second is to demonstrate that when there is a substantial likelihood of encountering the zero lower bound (ZLB), the long horizon Fama coefficient will be lower.

In the next section we briefly review unbiasedness hypothesis and new empirical results. In Section 3, the model derivation is presented, and summarized in Section 3.11. In Section 4, the model calibration is shown. Sections 5 and 6 present, respectively, the calibration results without and with the zero lower bound. Section 7 concludes.

2 Theory and Literature

The UIP hypothesis states, if investors are risk neutral, and there are no barriers to cross-border capital flows, then the return from investing a unit amount in a risk free bond in the home currency for period k , $i_{t,k}$, should equal the expected return from converting the unit amount into foreign currency and investing in risk free bond in foreign currency for period k , and converting the foreign currency return back into home currency after period k .

$$E_t s_{t+k} - s_t = (i_{t,k} - i_{t,k}^*) \quad (1)$$

Obviously, this hypothesis is difficult to test since the market's subjective expectation of the exchange rate is unobservable. The typical recourse adopted by researchers is to adopt the rational expectations hypothesis, so that the subjective market expectation equals the mathematical (conditional) expectation. Under the rational expectations hypothesis, the actual value of the future spot rate equals the expected value of future spot rate plus an error term,

$$s_{t+k} = E_t s_{t+k} + \varepsilon_{t+k} \quad (2)$$

where ε_{t+k} is an expectation error that is orthogonal to everything that is known at time t . This yields:

$$s_{t+k} - s_t = (i_{t,k} - i_{t,k}^*) + \varepsilon_{t+k} \quad (3)$$

The resulting unbiasedness hypothesis is usually tested using the following specification, often referred to as the "Fama regression" (Fama 1984):

$$s_{t+k} - s_t = \alpha + \beta(i_{t,k} - i_{t,k}^*) + \zeta_{t+k} \quad (4)$$

where the expected value of β according to the hypothesis is 1. The maintained hypothesis is the null hypothesis, and under the unbiasedness null, the error ζ_{t+k} is merely the expectations error, and is orthogonal to the interest rate differential, $i_{t,k} - i_{t,k}^*$. (Under the alternative, ζ_{t+k} could be an error term that does not fulfill the rational expectations criteria, and/or a time varying risk premium, possibly correlated with the interest differential.) In the regressions reported in this paper, the exact formulas are used instead of the log approximations.

Note that the intercept α may be non zero while testing for UIP using Eq. (4). A non zero α may reflect a constant risk premium (hence, we are really testing for a



Fig. 1 Panel Fama coefficients 1980Q1-2007Q4, unbalanced, with fixed effects¹

time-varying risk premium, rather than risk neutrality per se) and/or approximation errors stemming from Jensen's Inequality from the fact that expectation of a ratio (the exchange rate) is not equal to the ratio of the expectation

Most of the studies fall into the category employing the rational expectations hypothesis; in our lexicon, that means they are tests of the unbiasedness hypothesis. Estimates of Eq. (4) using values for k that range up to one year typically reject the unbiasedness restriction on the slope parameter. The widely cited 1990 survey by Froot and Thaler (1990) finds an average estimate for β of -0.88 .² Bansal and Dahlquist (2000) provide more mixed results, when examining a broader set of advanced and emerging market currencies. They also note that the failure of unbiasedness appears to depend upon whether the US interest rate is above or below the foreign.³ Frankel and Poonawala (2010) show that for emerging markets more generally, the unbiasedness hypothesis coefficient is typically more positive.⁴

¹Includes Canada, Germany, Euro, Japan, Switzerland, and UK; excludes Euro for 5 and 10 year horizons.

²Similar results are cited in surveys by Macdonald and Taylor (1992) and Isard (1995). Meese and Rogoff (1983) show that the forward rate is outpredicted by a random walk, which is consistent with the failure of the unbiasedness hypothesis.

³Flood and Rose (1996, 2002) note that including currency crises and devaluations, one finds more evidence for the unbiasedness hypothesis.

⁴There is an alternative approach that involves using survey-based data to measure exchange rate expectations. Chinn and Frankel (1994) document that it is difficult to reject UIP for a broad set of currencies when using survey based forecasts. Similar results are obtained by Chinn (2014), when extending the data up to 2009. These results echo early findings by Froot and Frankel (1989). While we do not pursue this avenue in this paper (long horizon expectations being difficult to obtain), it is useful to recall these findings when interpreting rejections of the joint hypothesis of UIP and rational expectations.

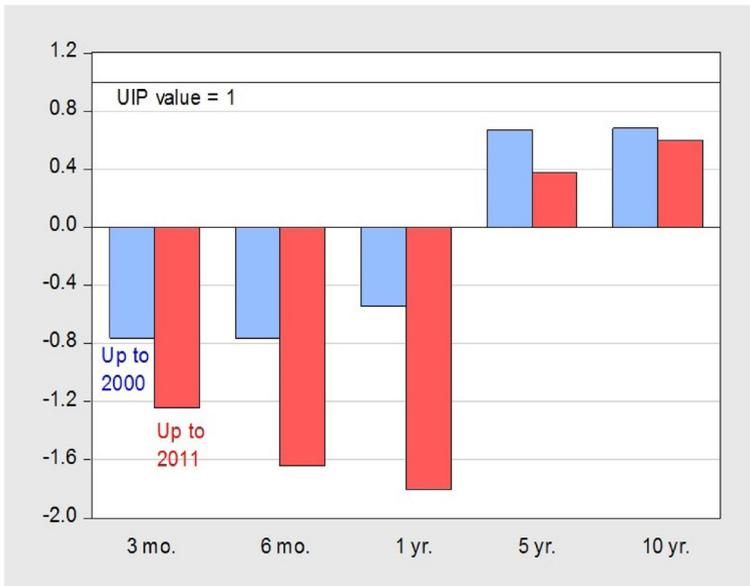


Fig. 2 Panel Fama coefficients⁵

The earliest studies that explored the UIP hypothesis over long horizons include Flood and Taylor (1997) and Alexius (2001). Flood and Taylor (1997) explores the UIP relationship at three year horizon pooling 21 currencies of mostly developed countries, with the U.S. Dollar as the base currency. They estimate β to be 0.596 with a standard error of 0.195. However, the bond yields used are heterogeneous, with varying maturities, so are difficult to interpret. Alexius (2001) finds evidence of UIP holding at long horizon using 14 long term bonds of varying maturities, using a sample from 1957-97. As the sample encompasses the Bretton Woods era and the associated capital control regime, those findings are perhaps not applicable in current times.

Chinn and Meredith (2004) tested the UIP hypothesis at five year and ten year horizons for the Group of Seven (G7) countries, and found greater support for the unbiasedness hypothesis holding at these long horizons than in shorter horizons of three to twelve months. The estimated coefficient on the interest rate differentials were positive and were closer to the value of unity than to zero in general. The sample used in that paper extended up to 2000. Figure 1 depicts the indicated pattern — negative coefficients at short horizons, and positive coefficients at longer horizons, updated to 2007Q4, so as to exclude the effects of the global financial crisis and ensuing period of near zero interest rates.

⁵Up to 12 months, panel estimates using eurodeposit rates against US\$. For 5 and 10 year, panel estimates using constant yields to maturity. Early sample extends up to 2000Q4, full sample to 2011Q4. Early sample includes mark, yen, pound, franc, lira, and Canadian dollar against dollar for up to 12 months, and excludes lira and franc for 10 year, and also yen for 5 year. Full sample replaces euro legacy currencies with euro and adds in Swiss franc.

In a sample extending up to 2011, Chinn and Quayyum (2013) find that the relationship between exchange rate changes and interest differentials has altered noticeably both at short and long horizons; the coefficients of different maturities for major currencies have declined in the wake of the Great Recession. This point is highlighted in Fig. 2.

3 The Model

In this section, we develop a standard small open-economy New Keynesian model following Galí (2008) and Walsh (2010).

The world economy constitutes of a continuum of small open economies represented by the unit interval.⁶ Since each economy is of measure zero, its performance does not have any impact on the rest of the world. In addition, we assume symmetry, that is, all countries share the same preference and technology.⁷

3.1 Households

The representative household maximizes the following expected discounted sum of utilities over possible paths of consumption and labor, subject to the budget constraint:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \right\} \quad (5)$$

where the period utility is defined as

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} \quad (6)$$

N_t is the amount of labor provided, and C_t is a composite index of domestic and imported bundles of goods. The consumption preference largely follows Walsh (2010). The detailed consumption preference set-up is relegated to Appendix A.1.

For any given state of the world, the following set of efficiency conditions must hold:

$$C_t^{-\sigma} = \beta E_t[(1+i_t) \left(\frac{P_t^c}{P_{t+1}^c} \right) C_{t+1}^{-\sigma}] \quad (7)$$

$$\frac{N_t^\eta}{C_t^{-\sigma}} = \frac{W_t}{P_t^c} \quad (8)$$

where P_t^c is the price of the composite good, and W_t is the wage rate. Equation (7) takes the form of a familiar consumption Euler equation. Notice that following large part of the recent literature, we do not introduce money explicitly.

⁶In order to simplify the notation, variables without a k -index are used to refer to the small open economy being modeled. i.e., the Home country. Variables with a $k \in [0, 1]$ subscript refer to economy k other than the Home country, one among the continuum of economies making up the world economy. Finally, variables with an asterisk superscript * correspond to the world economy as a whole.

⁷For conciseness, in this paper we only specify technology for the Home country.

3.2 Production and Price Setting

A typical firm j in the Home economy produces a differentiated good with a linear technology represented by the production function

$$Y_t(j) = e^{\epsilon_t} N_t(j)$$

Let $Y_t \equiv \left[\int_0^1 Y_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$ represent an index for aggregate domestic output, analogous to the introduced for consumption. Hence, notice that

$$N_t \equiv \int_0^1 N_t(j) dj = \frac{Y_t Z_t}{e^{\epsilon_t}}$$

where $Z_t \equiv \int_0^1 \frac{Y_t(j)}{Y_t} dj$. Galí and Monacelli (2005) proved that equilibrium variations in $z_t \equiv \ln Z_t$ around the perfect foresight steady state are of second order. Thus, up to a first order approximation, we have an aggregate relationship

$$y_t = n_t + \epsilon_t \quad (9)$$

Each firm faces a constant elasticity demand curve for its output, The firm's real marginal cost is equal to

$$MC_t = \frac{W_t / P_t^h}{e^{\epsilon_t}}$$

where $\frac{W_t}{P_t^h}$ is the real product wage and e^{ϵ_t} is the marginal product of labor. In terms of percentage deviations around the steady state, this expression becomes

$$mc_t = w_t - p_t^h - \epsilon_t \quad (10)$$

We assume that firms set prices in a staggered fashion, as in Calvo (1983). Hence, a measure $1 - \omega$ of (randomly selected) firms sets new prices each period, with an individual firm's probability of re-optimizing in any given period being independent of the time elapsed since it last reset its price.

3.3 The Law of One Price, Terms of Trade, and the Real Exchange Rate

For the rest of the world, the distinction between the CPI and the price of domestic production is ignored, both denoted as P_t^* . That is, the rest of the world is large relative to the home country,⁸ so changes in the price of home-produced goods have little impact on the consumer price index for the rest of the world. Because of asymmetry, all imported goods sell for the same price P_t^f at home country. That is, $P_{k,t}(j) = P_{k,t} = P_t^f$ for any $j, k \in [0, 1]$. Assume that the law of one price holds. This implies

$$P_t^f = S_t P_t^* \quad (11)$$

⁸The home country is of measure zero.

The terms of trade is the relative price of imported goods:

$$\Delta_t \equiv \frac{P_t^f}{P_t^h} = \frac{S_t P_t^*}{P_t^h} \tag{12}$$

while the real exchange rate is defined as

$$Q_t = \frac{S_t P_t^*}{P_t^c} = \left(\frac{P_t^h}{P_t^c}\right)\Delta_t \tag{13}$$

With these new definitions, we can log-linearize (58), (59), (60), (7), and (8), and get (lowercase letters denote percentage deviation around the steady states of the corresponding uppercase letters)

$$c_t^f = -a\delta_t + c_t^h \tag{14}$$

$$p_t^c = (1 - \gamma)p_t^h + \gamma p_t^f = p_t^h + \gamma\delta_t \tag{15}$$

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}^c) \tag{16}$$

$$\eta n_t + \sigma c_t = w_t - p_t^h - \gamma\delta_t \tag{17}$$

where $\pi_t^c = p_t^c - p_{t-1}^c$. Defining inflation in the prices of domestically produced goods as $\pi_t^h = p_t^h - p_{t-1}^h$, inflation in the consumer price index equals

$$\begin{aligned} \pi_t^c &= p_t^c - p_{t-1}^c \\ &= (p_t^h + \gamma\delta_t) - (p_{t-1}^h + \gamma\delta_{t-1}) \\ &= \pi_t^h + \gamma(\delta_t - \delta_{t-1}) \end{aligned} \tag{18}$$

3.4 The Rest of the World

Since the home country is of zero measure, variables of the rest of the world is actually equivalent to the whole world. This means that it is unnecessary to distinguish between domestic output and consumption for the rest of the world. i.e. $C_t^* = Y_t^*$. Similar to (59), it is easy to derive

$$C_t^{h*} = \gamma \Delta_t^a Y_t^*$$

or

$$c_t^{h*} = a\delta_t + y_t^* \tag{19}$$

3.5 Complete Asset Markets and International Risk Sharing

The agents around the world have access to a complete set of internationally traded securities. The international risk sharing guarantees:

$$C_t = Q_t^{\frac{1}{\sigma}} C_t^* \tag{20}$$

and its log-linearized version

$$c_t = c_t^* + \left(\frac{1}{\sigma}\right) q_t = c_t^* + \left(\frac{1-\gamma}{\sigma}\right) \delta_t \tag{21}$$

The last equation employs the fact that

$$\begin{aligned} q_t &= s_t + p_t^* - p_t^c \\ &= (s_t + p_t^* - p_t^h) + (p_t^h - p_t^c) \\ &= \delta_t + (p_t^h - p_t^c) \\ &= \delta_t - \gamma \delta_t \\ &= (1-\gamma)\delta_t \end{aligned} \tag{22}$$

Detailed derivation of (21) is relegated to Appendix A.2.

3.6 Uncovered Interest Parity

We model the uncovered interest parity equation following McCallum (1994) and Engel (2015), that the market’s expectation of the currency depreciation differs from the rational expectation by a mean-zero, i.i.d. random variable u_t :

$$i_t - i_t^* = E_t s_{t+1} - s_t + u_{it} \tag{23}$$

The presence of this shock term is crucial, and yet at the same time intuitively plausible. This term could be motivated by the real-world observation of deviations from rational expectations (Chinn and Frankel 1994), perhaps due to agents having bounded rationality. It could also be interpreted as an idiosyncratic risk premium term associated with the institutional structure of the foreign exchange market, and not driven by broader factors in the broader economy.

Using the definition of real exchange rate, we re-write the interest parity equation

$$i_t - i_t^* = E_t q_{t+1} - q_t + E_t \pi_{t+1}^c - E_t \pi_{t+1}^* + u_{it} \tag{24}$$

3.7 Market Clearing Condition

Market clearing requires that Home production equal the consumption of the Home produced good. Since the Home produced good is consumed by both Home residents and by residents of the rest of the world, equilibrium requires

$$Y_t = (1-\gamma) \left(\frac{P_t^h}{P_t^c}\right)^{-a} C_t + \gamma \left(\frac{P_t^h}{S_t P_t^*}\right)^{-a} Y_t^* \tag{25}$$

Using (20) $C_t = Q_t^{\frac{1}{\sigma}} C_t^* = Q_t^{\frac{1}{\sigma}} Y_t^*$ to eliminate Y_t^* , and employing the definition of the real exchange rate, the good market equilibrium condition can be written as

$$Y_t = \left(\frac{P_t^h}{P_t^c}\right)^{-a} [(1-\gamma) + \gamma Q_t^{(a-\frac{1}{\sigma})}] C_t \tag{26}$$

Log-linearize this equation yields:

$$\begin{aligned}
 y_t &= -a(p_t^h - p_t^c) + \gamma\left(a - \frac{1}{\sigma}\right)q_t + c_t \\
 &= \gamma a \delta_t + \gamma\left(a - \frac{1}{\sigma}\right)q_t + c_t \\
 &= \frac{\gamma a}{1 - \gamma}q_t + \gamma\left(a - \frac{1}{\sigma}\right)q_t + c_t \\
 &= c_t + \frac{\gamma}{\sigma(1 - \gamma)}[a\sigma + (a\sigma - 1)(1 - \gamma)]q_t \tag{27}
 \end{aligned}$$

$$\begin{aligned}
 &= c_t^* + \frac{1}{\sigma}q_t + \frac{\gamma}{\sigma(1 - \gamma)}[a\sigma + (a\sigma - 1)(1 - \gamma)]q_t \\
 &= y_t^* + \frac{1}{\sigma}q_t + \frac{\gamma}{\sigma(1 - \gamma)}[a\sigma + (a\sigma - 1)(1 - \gamma)]q_t \\
 &= y_t^* + \frac{1}{\sigma_\gamma(1 - \gamma)}q_t \tag{28}
 \end{aligned}$$

where $\sigma_\gamma \equiv \frac{\sigma}{1 - \gamma(1 - a\sigma)(2 - \gamma)}$

3.8 Inflation Adjustment and the Phillips Curve

In the small open economy, the dynamics of domestic inflation in terms of real marginal cost are described by an equation analogous to the one associated with a closed economy.⁹ Hence,

$$\pi_t^h = \beta E_t \pi_{t+1}^h + k \cdot mc_t \tag{29}$$

where $k \equiv \frac{(1 - \omega)(1 - \beta\omega)}{\omega}$. From (10), (17), (27), and (28), we have

$$\begin{aligned}
 mc_t &= w_t - p_t^h - \epsilon_t \\
 &= \eta n_t + \sigma c_t + \gamma \delta_t - \epsilon_t \\
 &= \eta(y_t - \epsilon_t) + \sigma c_t + \gamma \delta_t - \epsilon_t \\
 &= \eta y_t + \sigma c_t + \frac{\gamma}{1 - \gamma}q_t - (1 + \eta)\epsilon_t \\
 &= \eta y_t + \sigma \left\{ y_t - \frac{\gamma}{\sigma(1 - \gamma)}[a\sigma + (a\sigma - 1)(1 - \gamma)]q_t \right\} + \frac{\gamma}{1 - \gamma}q_t - (1 + \eta)\epsilon_t \\
 &= (\eta + \sigma)y_t + \frac{\sigma_\gamma - \sigma}{\sigma_\gamma(1 - \gamma)}q_t - (1 + \eta)\epsilon_t \\
 &= (\eta + \sigma)y_t + (\sigma_\gamma - \sigma)(y_t - y_t^*) - (1 + \eta)\epsilon_t \\
 &= (\eta + \sigma_\gamma)y_t + (\sigma - \sigma_\gamma)y_t^* - (1 + \eta)\epsilon_t \tag{30}
 \end{aligned}$$

⁹See Galí and Monacelli (2005) for derivation.

Thus (29) can be re-written as:

$$\pi_t^h = \beta E_t \pi_{t+1}^h + k[(\eta + \sigma_\gamma)y_t + (\sigma - \sigma_\gamma)y_t^*] + u_{ct} \tag{31}$$

where $u_{ct} \equiv -k(1 + \eta)\epsilon_t$ is defined as the cost/productivity shock. A positive u_{ct} indicates a decrease in Home productivity.

3.9 Monetary Policy Rule for the Small Open Economy

Assume Home monetary authority adopts a stylized Taylor-type rule, where the domestic interest rate respond systematically to (expected) domestic inflation, output, and expected real exchange rate depreciation:

$$\begin{aligned} i_t &= \delta_\pi E_t \pi_{t+1}^c + \delta_y y_t + u_{mt} \\ &= \delta_\pi E_t \pi_{t+1}^h + \delta_y y_t + \frac{\delta_\pi \gamma}{1 - \gamma} E_t \Delta q_{t+1} + u_{mt} \end{aligned} \tag{32}$$

where u_{mt} is an exogenous shock to the monetary policy. The second equation follows from (18).

3.10 Term Structure of Interest Rates

Intertemporal substitution between an N-period maturity bond (with interest rate denoted by $i_t^l(N)$) and a one-period maturity bond requires

$$\frac{U'(C_t)}{P_t} = E_t \left[\frac{\beta^N U'(C_{t+N})(1 + i_t^l(N))^N}{P_{t+N}} \right] \tag{33}$$

$$\frac{U'(C_t)}{P_t} = E_t \left[\frac{\beta^N U'(C_{t+N}) \prod_{j=0}^{N-1} (1 + i_{t+j})}{P_{t+N}} \right] \tag{34}$$

Consequently, we have

$$E_t \left[\frac{C_{t+N}^{-\sigma} (1 + i_t^l(N))^N}{P_{t+N}} \right] = E_t \left[\frac{C_{t+N}^{-\sigma} \prod_{j=0}^{N-1} (1 + i_{t+j})}{P_{t+N}} \right] \tag{35}$$

Log-linearization of the above equation leads to the expectation hypothesis of the term structure (EHTS)

$$i_t^l(N) = \frac{\sum_{j=0}^{N-1} E_t i_{t+j}}{N} \tag{36}$$

That is, the long-term interest rate is determined as the average of the current short-term interest rate and its expected value over the subsequent periods.

There is an important asymmetry in our treatment of the household’s maximization problem with respect to the intertemporal dimension versus the cross-country dimension; in the latter we have appended a disturbance term, while none such exists in the EHTS just described. This implicit segmentation is consistent with the evidence provided by Alexius and Sellin (2012), showing that parity between yields on

short term instruments fails to hold while parity cannot be rejected on holding period returns on long term instruments.

3.11 Model Summary

The model is summarized by the following dynamic system:

- Interest Parity (From (24))

$$i_t - i_t^* = E_t q_{t+1} - q_t + E_t \pi_{t+1}^c - E_t \pi_{t+1}^* + u_{it} \tag{37}$$

- Phillips Curve (From (31))

$$\pi_t^h = \beta E_t \pi_{t+1}^h + k[(\eta + \sigma_\gamma)y_t + (\sigma - \sigma_\gamma)y_t^*] + u_{ct} \tag{38}$$

- Market Clear (From (28))

$$y_t = y_t^* + \frac{1}{\sigma_\gamma(1 - \gamma)} q_t \tag{39}$$

- Taylor Rule (From (32))

$$i_t = \delta_\pi E_t \pi_{t+1}^h + \delta_y y_t + \frac{\delta_\pi \gamma}{1 - \gamma} E_t \Delta q_{t+1} + u_{mt} \tag{40}$$

- Relationship between π_t^c and π_t^h (From (18) and (76))

$$\pi_t^c = \pi_t^h + \frac{\gamma}{1 - \gamma} (q_t - q_{t-1}) \tag{41}$$

- Term Structure of Interest Rates (From (36))

$$i_t^l(N) = \frac{\sum_{j=0}^{N-1} E_t i_{t+j}}{N} \tag{42}$$

- Nominal Exchange Rate Depreciation (From (23))

$$E_t s_{t+1} - s_t = i_t - i_t^* - u_{it} \tag{43}$$

The intertemporal substitution Eq. (7) is implied by international risk sharing (21), market clear condition (39) and term structure of interest rates (42), so we do not include it here.

4 Model Calibration

In this section, we calibrate the model and show that the calibrated New Keynesian model captures the pre-ZLB (1975Q1-1995Q4) volatility of Japanese 3-month T-bill rate and depreciation of nominal exchange rate (with U.S. dollar as the base currency). We focus on interest rate and depreciation of nominal exchange rate because

Table 1 Variance and C.I. of Japanese major economic variables

	y_t	π_t^c	$i_t - i_t^*$	$s_{t+1} - s_t$
Simulated	0.006756	0.003117	0.000690	0.004547
Actual data	0.001114	0.001363	0.000690	0.003860
90% C.I.	[0.000848,0.001380]	[0.001021,0.001705]	[0.000523,0.000857]	[0.002928,0.004793]

this paper is about whether interest rate ZLB would affect the coefficient of Fama regression.⁹

4.1 Empirical Moments

The Japanese data come from the IMF's International Financial Statistics (IFS) database. We use quarterly observations on Japanese 3-month T-bill rate and depreciation of (log) nominal exchange rate, starting from 1975Q1 to 1995Q4. The motivation for choosing such sample period is that it starts from the flexible exchange rate regime, and ends before the advent of the ZLB interest rate regime.¹⁰ We also use U.S. 3-month T-bill rate from Federal Reserve Board's H.15 release as the world interest rate. The variance and 90% confidence intervals are summarized in the 2nd and 3rd row of Table 1.¹¹

4.2 Theoretical Predictions

We impose the following additional assumptions:

1. There is no structure change during the period under consideration (1975Q1-1995Q4).
2. Without loss of generality, foreign/world economic variables (output, interest rate, etc.) are assumed to be constant.
3. All shocks are assumed to be either white noise or stationary AR(1) process:

$$u_{it} = \rho_i u_{it-1} + e_{it}$$

$$u_{ct} = \rho_c u_{ct-1} + e_{ct}$$

$$u_{mt} = \rho_m u_{mt-1} + e_{mt}$$

where $\rho_i = 0$. That is, the interest parity shock u_{it} is an i.i.d white noise, which is consistent with our assumption in Section 3.6.

⁹We can also match the moments for other macroeconomic variables; e.g., output by imposing habit persistence structure. We do not impose such structure because it brings unnecessary complication without helping us understand the nature of ZLB.

¹⁰Following Chinn and Meredith (2004), Chinn and Quayyum (2013).

¹¹Both the depreciation rate and the interest rate are expressed at annual rate.

Table 2 Calibration of pre-ZLB economy

$\omega = 0.89$	probability of not being able to reset price in each period
$\beta = 0.98$	domestic discount factor
$k = 0.0158$	$k \equiv \frac{(1-\omega)(1-\beta\omega)}{\omega}$
$\gamma = 0.5$	foreign bias
$a = 1.5$	elasticity of substitution between domestic and foreign good
$\eta = 3, \sigma = 2$	CRRA coefficient in utility
$\sigma_\gamma \equiv \frac{\sigma}{[1-\gamma(1-a\sigma)(2-\gamma)]}$	
$\delta_\pi = 1.5, \delta_y = 0.8$	coefficient of Taylor rule
$\rho_i = 0$	AR(1) coefficient of u_i
$\rho_c = 0.65$	AR(1) coefficient of u_c
$\rho_m = 0.8$	AR(1) coefficient of u_m
$e_{it} \sim N(0, 0.02^2)$	white noise
$e_{ct} \sim N(0, 0.002^2)$	white noise
$e_{mt} \sim N(0, 0.002^2)$	white noise
$\bar{i} = 6.7 * 10^{-5}$	steady state of domestic interest rate

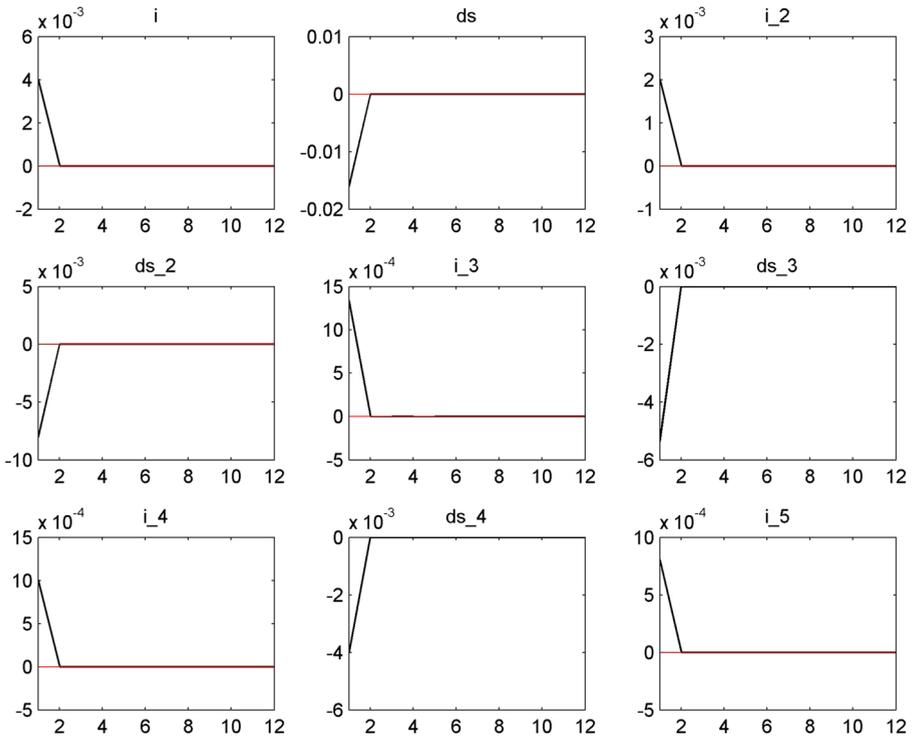


Fig. 3 The impulse response function of e_{it} on macroeconomic variables

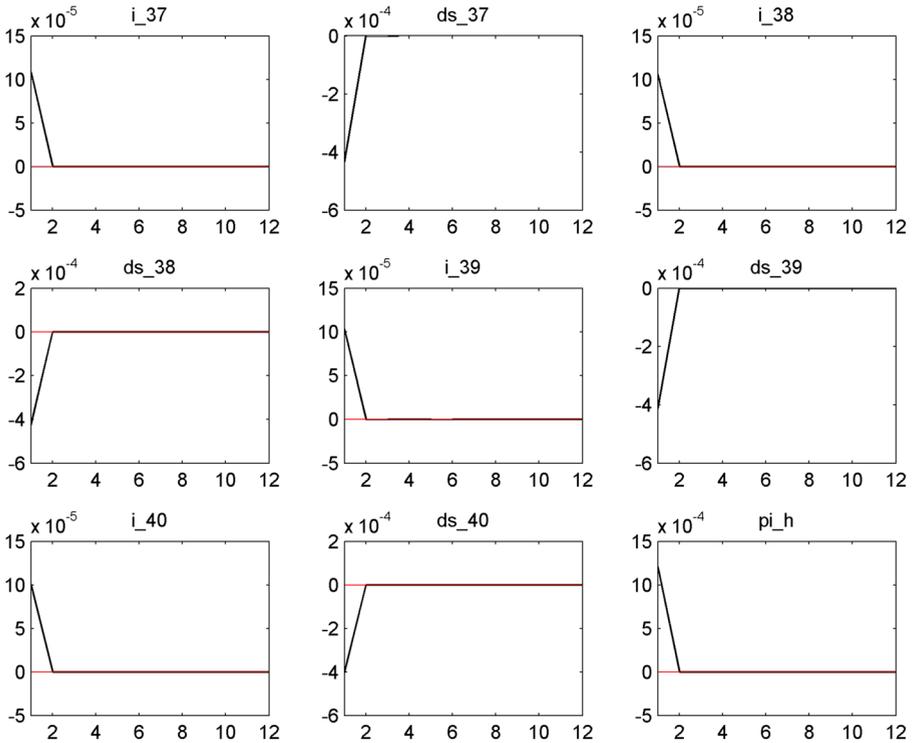


Fig. 4 The impulse response function of e_{it} on macroeconomic variables

In our open-economy New Keynesian model, one period is defined as one quarter. We calibrate and summarize the parameter values in Table 2.¹² The parameter values are standard and mostly follow Sanchez (2005) and Braun and Körber (2011). For instance, we set Calvo-pricing parameter $\omega = 0.89$, indicating that Japanese firms reset prices every 9 quarters, which is consistent with Sanchez (2005)’s estimation that Japan’s ω varies from 0.58 to 0.92.

The variances of major economic variables are presented in the 1st row of Table 1. We can see that the exchange rate depreciation and interest differential implied by the theoretical model are 0.004547 and 0.000690 respectively, which lie within the 90% confidence interval derived from the actual data, and are comparable to the variance estimated from the actual data. It should also be noted that the simulated output gap and inflation are more volatile than the actual data. This is a common and known feature of standard open-economy New Keynesian models. We can match the moments of these two variables by introducing additional mechanism; e.g., habit

¹²Because of symmetry, θ and ξ do not enter the VAR system in Section 3.11. Hence we do not specify their values in Table 2. We will discuss \bar{i} in Section 6, when the interest rate is near ZLB.

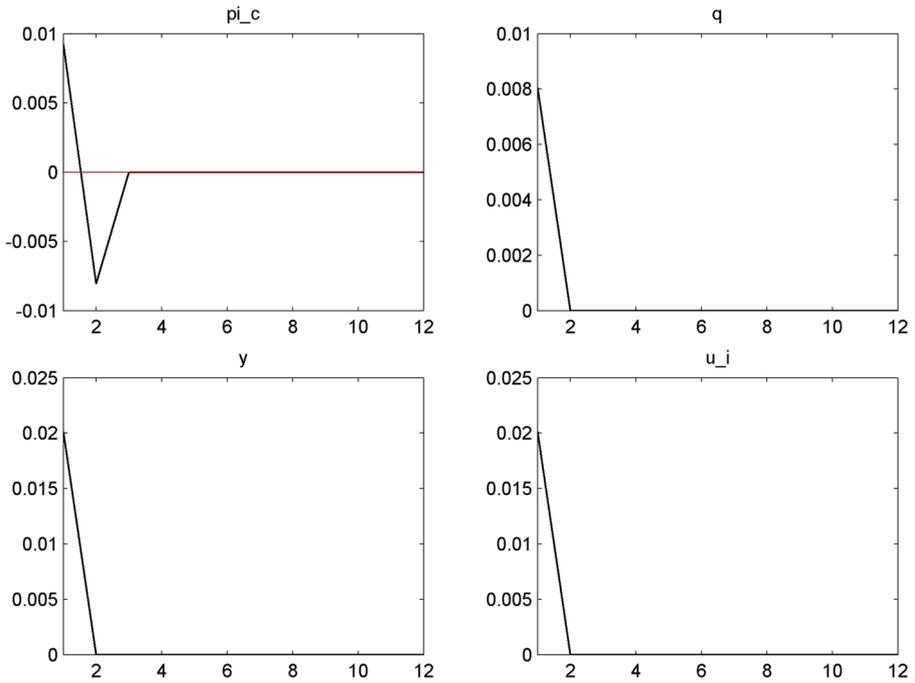


Fig. 5 The impulse response function of e_{it} on macroeconomic variables

persistence structure. However, we do not impose such structure because it brings unnecessary complication without helping us understand the nature of ZLB.

5 Uncovered Interest Parity far from the ZLB

5.1 Closed-Form Solution

Given the VAR system summarized in Sections 3.11 and 4.2, our model has a closed-form solution that $i_t - i_t^*$, $E_t s_{t+1} - s_t$, q_t , π_t^h , etc, are linear combinations of the AR(1) shocks u_{it} , u_{ct} , and u_{mt} . Without loss of generality, we shall write the solution as:

$$i_t - i_t^* = c_1 u_{it} + c_2 u_{ct} + c_3 u_{mt} \tag{44}$$

$$E_t s_{t+1} - s_t = -(1 - c_1) u_{it} + c_2 u_{ct} + c_3 u_{mt} \tag{45}$$

To illustrate the model's properties, Figs. 3, 4 and 5 shows impulse responses for one standard deviation innovations in u_{it} (e_{it}) for the nominal exchange rate

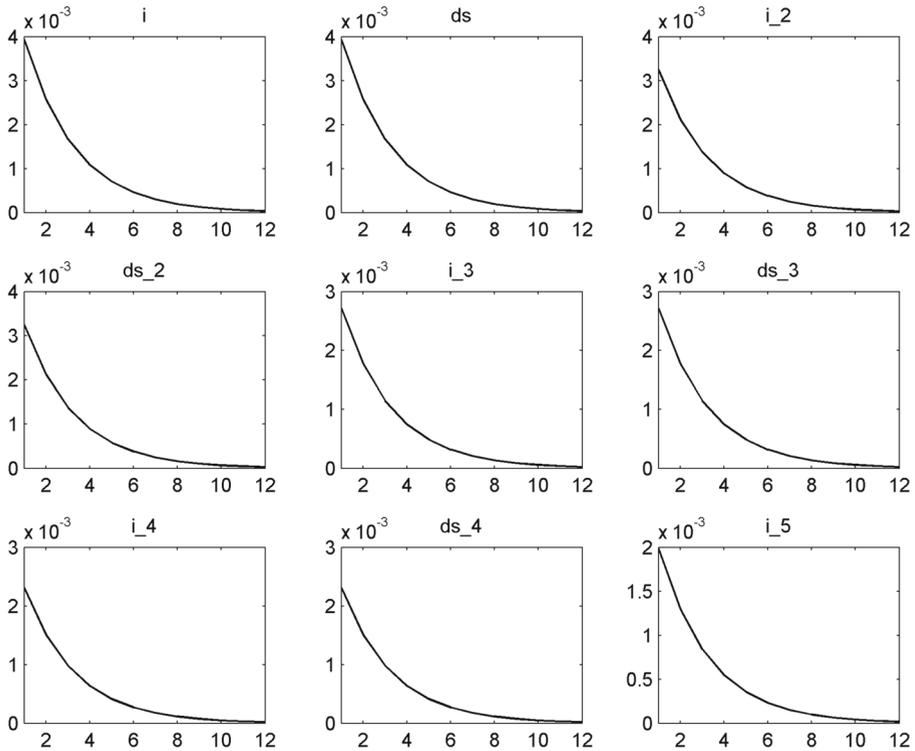


Fig. 6 The impulse response function of e_{ct} on macroeconomic variables

depreciation, and interest rates over different horizons.^{13, 14} Our simulation indicates $c_1 = 0.2$.¹⁵

Figures 6, 7, 8, 9, 10 and 11 shows impulse responses for standardized innovations in u_{ct} (e_{ct}) and u_{mt} (e_{mt}) for the nominal exchange rate depreciation over different horizons, and interest rates with different maturities. All the impulse response functions are intuitive. A positive cost shock e_{ct} indicates a decrease in Home productivity. Home output decreases and inflation increases - this is the well-known phenomenon of stagflation. After trade-off between inflation and unemployment, the central bank will set a higher interest rate to eliminate inflation. This also brings an expected depreciation of Home currency. A positive monetary shock e_{mt} has a similar effect. It implies the central bank increases interest rate unexpectedly. Obviously

¹³In those graphs, i_2 refers to interest rate with 6-month (2-quarter) maturity. ds_2 refers to depreciation rate over two quarters. Similar definitions apply for other variables.

¹⁴For conciseness, we do not include IRFs for all horizons.

¹⁵From Fig. 3, it is straightforward to see $\frac{\partial(i_t - i_t^*)}{\partial e_{it}} \text{std}(e_{it}) = 0.004$. Since $\text{std}(e_{it}) = 0.02$, it is straightforward to see $c_1 = \frac{\partial(i_t - i_t^*)}{\partial e_{it}} = 0.2$.

it increases interest rate and expected depreciation instantly. Our simulation indicates $c_2 = 1.95$ and $c_3 = -0.0655$.¹⁶ The closed-form solution also provides a convenient way to calculate the theoretical moments.¹⁷

5.2 The Fama Coefficient at the Short Horizon

Recall that u_{it} (e_{it}) is a deviation from rational expectation, econometric theory suggests the coefficient of Fama regression satisfy:

$$\begin{aligned}
 \text{plim } \hat{\beta} &= \frac{\text{cov}(s_{t+1} - s_t, i_t - i_t^*)}{\text{var}(i_t - i_t^*)} \\
 &= \frac{\text{cov}(E_t s_{t+1} - s_t, i_t - i_t^*)}{\text{var}(i_t - i_t^*)} \\
 &= \frac{\text{cov}(i_t - i_t^* - u_{it}, i_t - i_t^*)}{\text{var}(i_t - i_t^*)} \\
 &= \frac{\text{var}(i_t - i_t^*) - \text{cov}(u_{it}, i_t - i_t^*)}{\text{var}(i_t - i_t^*)} \\
 &= 1 - \frac{\text{cov}(u_{it}, i_t - i_t^*)}{\text{var}(i_t - i_t^*)} \\
 &= 1 - \frac{\text{cov}(u_{it}, c_1 u_{it} + c_2 u_{ct} + c_3 u_{mt})}{\text{var}(i_t - i_t^*)} \\
 &= 1 - \frac{\text{cov}(u_{it}, c_1 u_{it})}{\text{var}(i_t - i_t^*)} \\
 &= 1 - \frac{c_1 \text{var}(u_{it})}{\text{var}(i_t - i_t^*)} \\
 &= 1 - \frac{c_1 \text{var}(u_{it})}{c_1^2 \text{var}(u_{it}) + c_2^2 \text{var}(u_{ct}) + c_3^2 \text{var}(u_{mt})} \tag{46}
 \end{aligned}$$

According to our calibration, u_{it} (e_{it}) is more volatile than u_{ct} (e_{ct}) and u_{mt} (e_{mt}). At short horizon, u_{it} (e_{it}) dominates. Hence at short horizon Fama regression generally yields a negative coefficient. From our derivation (46), it is straightforward to see the endogeneity caused by u_{it} makes this estimator deviate from unity, and sometimes being negative.

¹⁶ Figures 6 and 9 show $\frac{\partial(i_t - i_t^*)}{\partial e_{ct}} \text{std}(e_{ct}) = 0.0039$ and $\frac{\partial(i_t - i_t^*)}{\partial e_{mt}} \text{std}(e_{mt}) = -0.000131826$. Since $\text{std}(e_{ct}) = \text{std}(e_{mt}) = 0.002$, we have $c_2 = \frac{\partial(i_t - i_t^*)}{\partial e_{ct}} = 1.95$ and $c_3 = \frac{\partial(i_t - i_t^*)}{\partial e_{mt}} = -0.0655$.

¹⁷Note in our simulation, the depreciation rate and interest rate are quarterly rate, i.e., percent per quarter. Hence, variances derived from Eqs. (44) and (45) should be multiplied by 16 to compare with the empirical moments in Table 1.

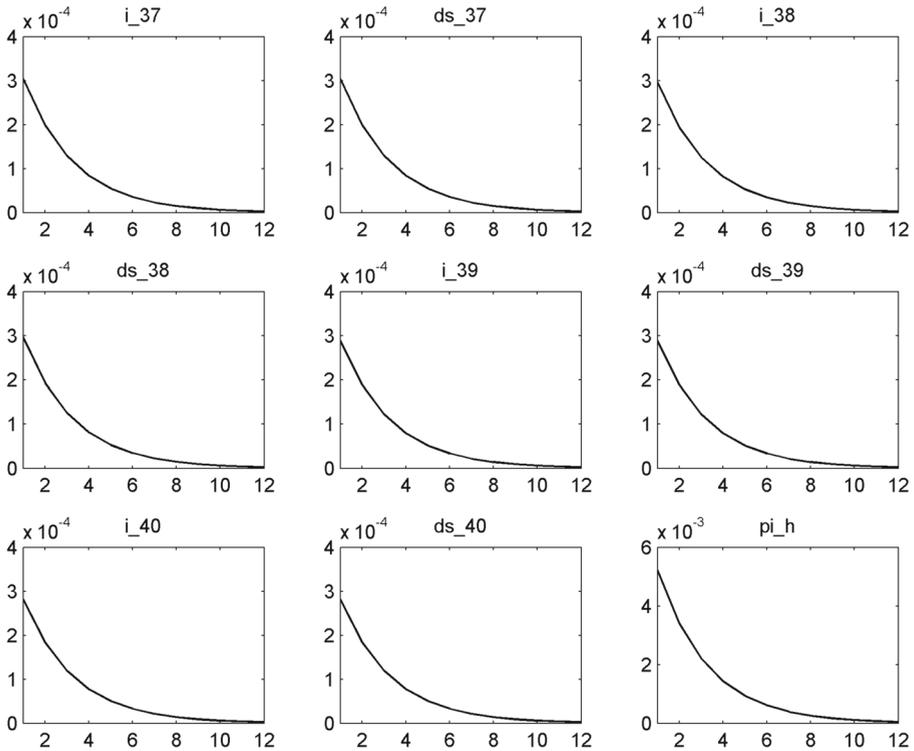


Fig. 7 The impulse response function of e_{ct} on macroeconomic variables

5.3 The Long Horizon

In the long run, $plim\hat{\beta}(N)$ ¹⁸ becomes

$$\begin{aligned}
 plim\hat{\beta}(N) &= \frac{cov(\sum_{j=0}^{N-1} (s_{t+j+1} - s_{t+j}), \sum_{j=0}^{N-1} E_t(i_{t+j} - i_{t+j}^*))}{var(\sum_{j=0}^{N-1} E_t(i_{t+j} - i_{t+j}^*))} \\
 &= \frac{cov(\sum_{j=0}^{N-1} E_t(s_{t+j+1} - s_{t+j}), \sum_{j=0}^{N-1} E_t(i_{t+j} - i_{t+j}^*))}{var(\sum_{j=0}^{N-1} E_t(i_{t+j} - i_{t+j}^*))} \\
 &= 1 - \frac{cov(\sum_{j=0}^{N-1} E_t u_{it+j}, \sum_{j=0}^{N-1} E_t(i_{t+j} - i_{t+j}^*))}{var(\sum_{j=0}^{N-1} E_t(i_{t+j} - i_{t+j}^*))} \\
 &= 1 - \frac{c_1 var(u_{it})}{c_1^2 var(u_{it}) + c_2^2 var(\sum_{j=0}^{N-1} E_t u_{ct+j}) + c_3^2 var(\sum_{j=0}^{N-1} E_t u_{mt+j})} \quad (47)
 \end{aligned}$$

At long horizon, since the cost shock u_{ct} and monetary shock u_{mt} are more persistent than u_{it} (AR(1) v.s. white noise), as horizon increases, they tend to make

¹⁸ N is the horizon/maturity of the long term bond.

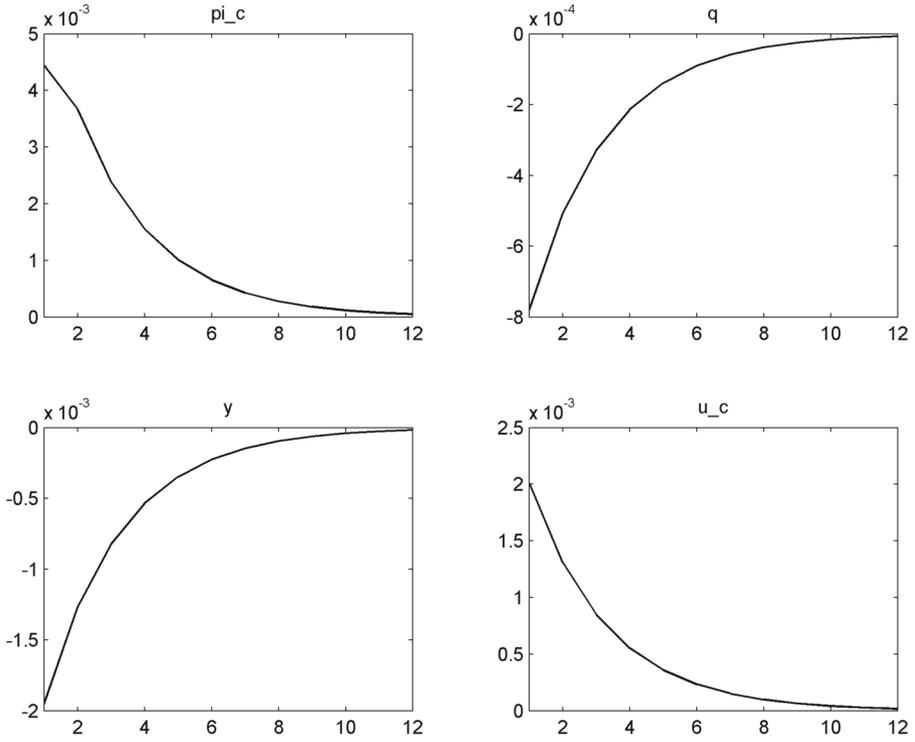


Fig. 8 The impulse response function of e_{ct} on macroeconomic variables

greater contribution to $cov(\sum_{j=0}^{N-1} (s_{t+j+1} - s_{t+j}), \sum_{j=0}^{N-1} E_t(i_{t+j} - i_{t+j}^*))$. That is, in (47), $var(\sum_{j=0}^{N-1} E_t u_{ct+j})$ and $var(\sum_{j=0}^{N-1} E_t u_{t+j})$ increases as N grows, so we would get a larger Fama regression coefficient estimate at longer horizon. Hence, we expect to find unbiasedness hypothesis holds better at long horizons than at short in our simulated data. Strictly, we have the following proposition:

Proposition 1 $plim \hat{\beta}(N)$ is monotonically increasing in N .

Proof Straightforward. □

5.4 Simulation Results

In this sub-section, stochastic simulations were performed on the model. Each simulation was performed over a 100 periods. This process was repeated 1000 times to generate a hypothetical population of 1000 such draws. For each draw, standard UIP regression were run, using horizons varying from 1 to 40 quarters. The average parameters as well as their confidence intervals are shown in Fig. 12.

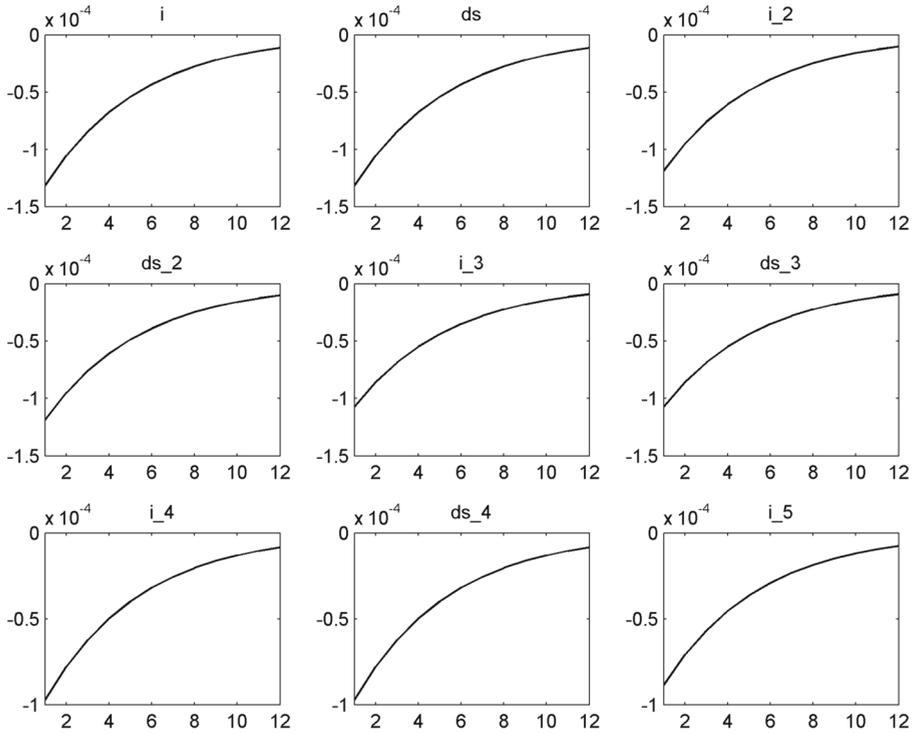


Fig. 9 The impulse response function of e_{mt} on macroeconomic variables

The simulated results are consistent with our analysis in Sections 5.2 and 5.3, as well as Chinn and Meredith (2004).

6 Uncovered Interest Parity near the ZLB

When the interest rate is near the ZLB, Taylor rule (40) has to be re-written as

$$i_t = \max\{0, \delta_\pi E_t \pi_{t+1}^h + \delta_y y_t + \frac{\delta_\pi \gamma}{1 - \gamma} E_t \Delta q_{t+1} + u_{mt}\} \tag{48}$$

This brings about two important changes to the economy:

(i) **The Transmission Mechanism**

When the interest rate is positive, u_{it} and u_{ct} affect central bank’s monetary policy decisions and enter i_t via $E_t \pi_{t+1}^h$, y_t and $E_t \Delta q_{t+1}$, and u_{mt} affects i_t directly. Then the interest rate affects household intertemporal substitution decisions, and determines demand, which in turn, affects inflation, output and

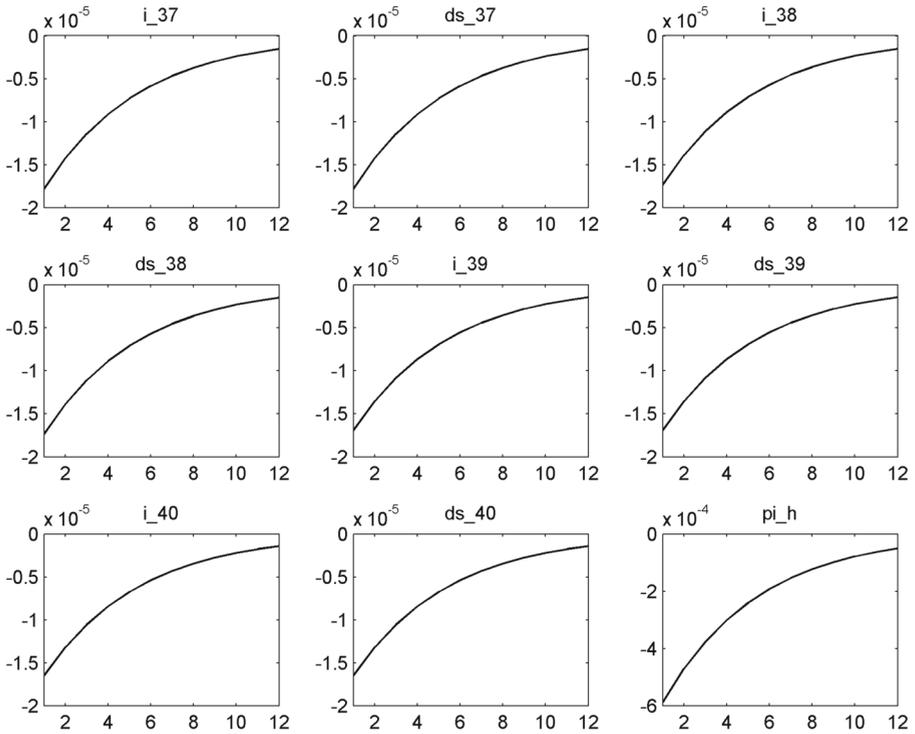


Fig. 10 The impulse response function of e_{mt} on macroeconomic variables

the exchange rate. However, if the ZLB is binding, such a transmission mechanism is shut down. When the shocks u_{it} , u_{ct} , and u_{mt} enter in the determination of i_t , i_t will remain at zero, and these disturbances are blocked.

(ii) **Household Expectations and Intertemporal Substitution**

When the ZLB is binding, i_t will remain at zero despite the realization of u_{it} , u_{ct} , and u_{mt} . The households are able to anticipate this situation ahead of time, so they will adjust their intertemporal substitution decisions accordingly.¹⁹

With these changes, a closed-form solution no longer exists, and we cannot derive $plim \hat{\beta}$ as in Sections 5.2 and 5.3. Nonetheless, in our model, both $s_{t+1} - s_t$ and $i_t - i_t^*$ (as well as i_t) are driven by three shocks u_{it} (e_{it}), u_{ct} (e_{ct}), and u_{mt} (e_{mt}), where u_{it} is volatile but transitory, and the other two disturbances are less volatile but more persistent.

¹⁹A common mistake is to write the solution as $i_t - i_t^* = \max\{0, c_1 u_{it} + c_2 u_{ct} + c_3 u_{mt}\}$ and $E_t s_{t+1} - s_t = i_t - i_t^* - u_{it}$. This is wrong because it does not take into account the fact that the households are able to anticipate ZLB ahead of time. Actually, a closed-form solution does not exist when the interest rate is near ZLB.

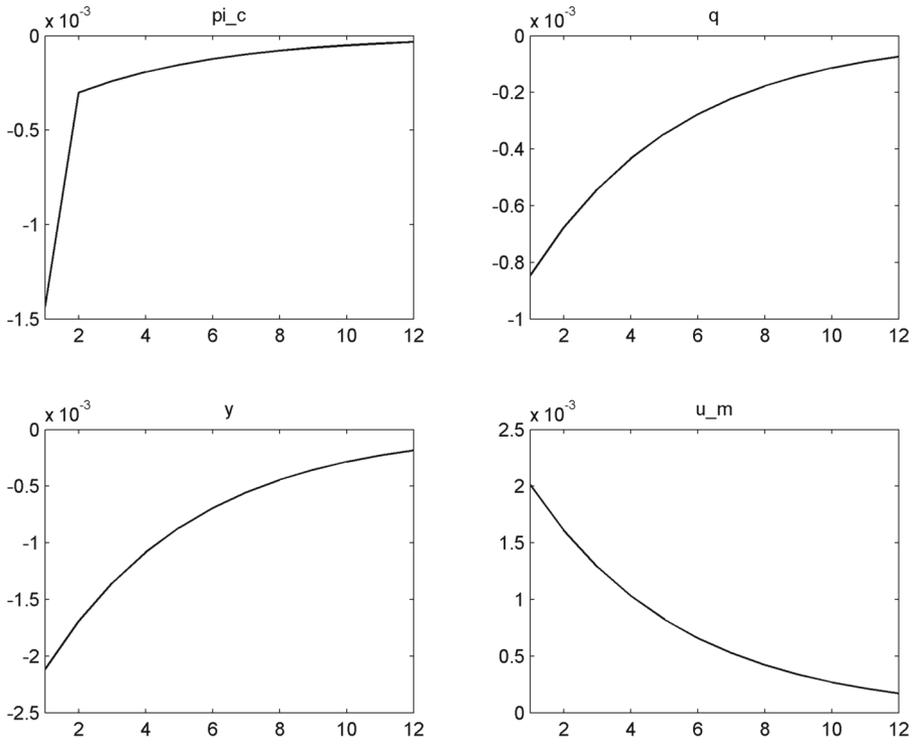


Fig. 11 The impulse response function of e_{mt} on macroeconomic variables

\bar{i} — the steady state level of i_t ²⁰ plays an important role with occasionally binding ZLB. If \bar{i} is large, then ZLB does not bind very often. When it does bind, it is most likely triggered by a negative u_{it} (e_{it}), and the possibility of binding ZLB is generally random. This approach is denoted by “*Random ZLB*”. However, if \bar{i} is small, then ZLB binds more frequently. When it binds, it is often triggered by a negative u_{ct} (e_{ct}) or u_{mt} (e_{mt}), and the interest rate series are segmented. i.e., if $i_t = 0$, then it is very likely that its adjacent leads and lags are also zero. If $i_t > 0$, then it is very likely that its adjacent leads and lags are also positive. This is determined by the different degree of persistence and volatility of different shocks. This approach is denoted by “*Persistent ZLB*”.

Again, we perform stochastic simulations for our model with ZLB constraint. Our simulation (with ZLB) method follows Holden and Paetz (2012), as well as Erceg

²⁰The so-defined \bar{i} also includes other constants omitted by log-linearization. Hence it does not necessarily equal $\frac{1}{\beta} - 1$. For conciseness, it is still referred to as “the steady state level of i_t ”.

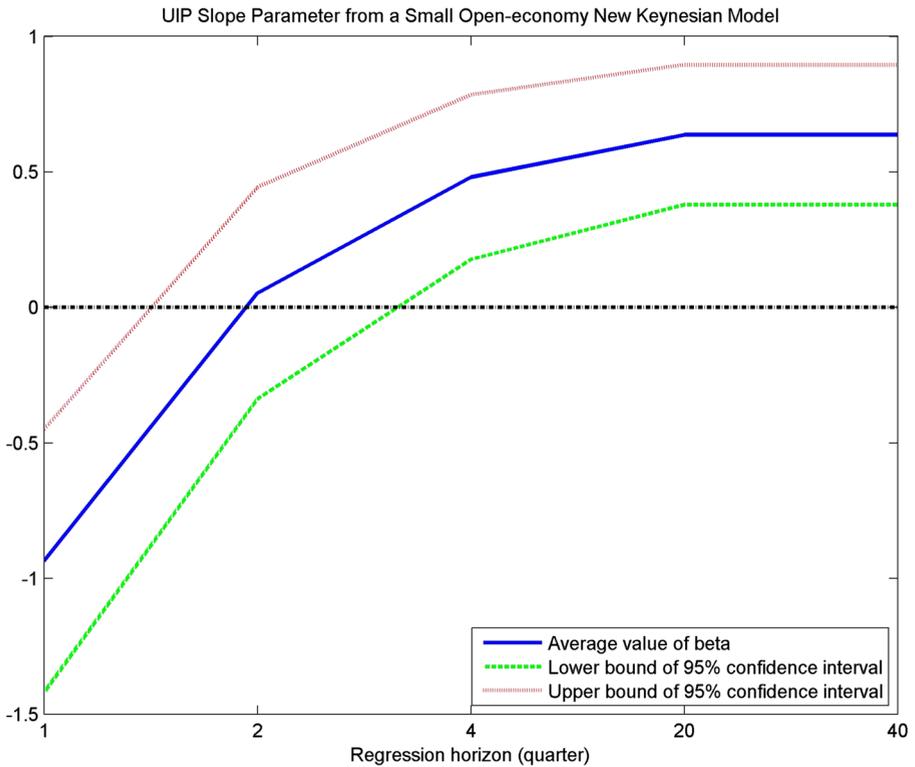


Fig. 12 UIP slope parameter from model simulations

and Lindé (2014). Each simulation was performed over a 100 periods. This process was repeated 1000 times to generate a hypothetical population of 1000 such draws. In our simulation, about half the time (49.99%) the interest hits ZLB. Figure 13 is one such random draw of 100-period simulation, where the interest ZLB constraint follows “*Persistent ZLB*” approach. This indicates our simulation is consistent with the fact that Japan (starting from 1996) set their short term interest rate (close to) zero for several consecutive years.

With “*Persistent ZLB*”, most binding ZLB periods are triggered by a negative u_{ct} (e_{ct}) or u_{mt} (e_{mt}). That is, the shocks u_{ct} (e_{ct}) and u_{mt} (e_{mt}) are suppressed under ZLB constraints. Recall in Section 5.2, we showed volatility of u_{ct} (e_{ct}) and u_{mt} (e_{mt}) are critical to offset the endogeneity caused by u_{it} (e_{it}). Once u_{ct} (e_{ct}) and u_{mt} (e_{mt}) are suppressed by ZLB constraints, the endogeneity problem deteriorates, hence we would expect the constrained Fama regression coefficient to be less positive. That is, unbiasedness hypothesis holds worse when interest rate is near ZLB. Such property is summarized in Fig. 14, and it is consistent with Fig. 2, and Chinn and Quayyum (2013).

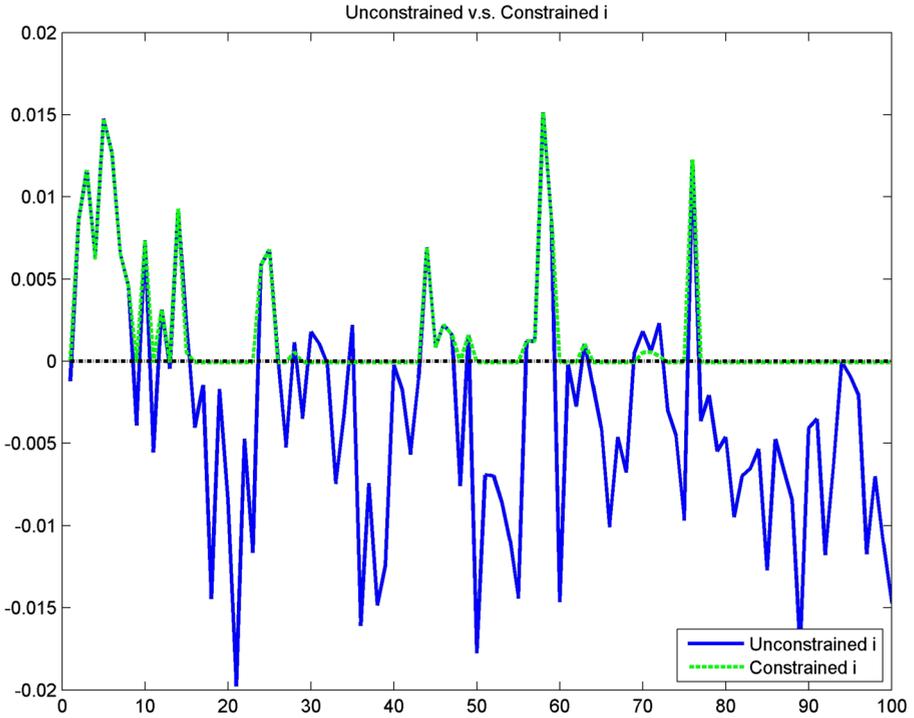


Fig. 13 A sample simulation of 100 periods with ZLB

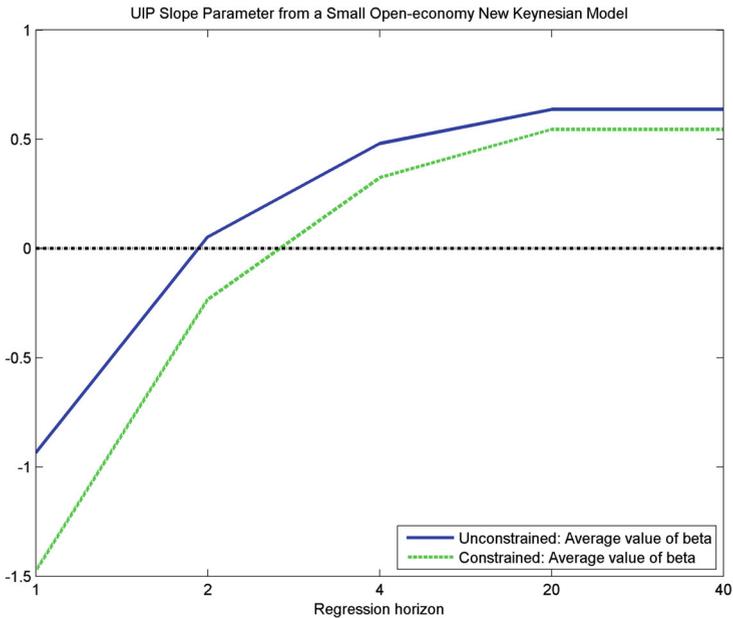


Fig. 14 Unconstrained v.s. constrained UIP slope parameters from model simulations

7 Conclusion

Two empirical findings underpin the exercises conducted in this paper. First, interest differentials at longer horizons tend to correlate positively with exchange rate depreciation, while those at short horizons tend to correlate negatively. Second, long horizon interest differentials tend to reject the unbiasedness hypothesis more strongly for exchange rates that involve a currency that has experienced low interest rates for an extended period. To the extent that low interest rates are likely to be a part of the global economic landscape for some time, it is of interest to know why this finding obtains.

Using a standard New Keynesian DSGE, incorporating sticky prices, we show how these empirical findings can be rationalized. The long horizon/short horizon dichotomy stylized fact is shown using a random (non-persistent) shock to the UIP relationship, along with persistent aggregate supply and monetary policy shocks. The attenuation in the long horizon unbiasedness in the era of low interest rates is replicated by simulating the NK DSGE in the presence of a zero lower bound.

Appendix

A.1 Households

Consumption preferences in the Home economy are described by the following composite index of domestic and imported bundles of goods:

$$C_t \equiv [(1 - \gamma)^{\frac{1}{a}} (C_t^h)^{\frac{a-1}{a}} + \gamma^{\frac{1}{a}} (C_t^f)^{\frac{a-1}{a}}]^{\frac{a}{a-1}} \quad (49)$$

where $a > 1$ is the elasticity of substitution between domestically produced and imported goods. C_t^h (and C_t^f) are Dixit-Stiglitz aggregates of domestically produced and imported goods.

$$C_t^h \equiv \left[\int_0^1 C_t^h(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad (50)$$

where $j \in [0, 1]$ denotes the good variety. C_t^f is an index of imported goods given by

$$C_t^f \equiv \left[\int_0^1 (C_{k,t})^{\frac{\xi-1}{\xi}} dk \right]^{\frac{\xi}{\xi-1}} \quad (51)$$

where $C_{k,t}$ is, in turn, an index of the quantity of goods imported from country k and consumed by domestic households. It is given by an analogous CES function

$$C_{k,t} \equiv \left[\int_0^1 C_{k,t}(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad (52)$$

The optimal allocation of any given expenditure within each category of goods yields the demand functions

$$C_t^h(j) = \left[\frac{P_t^h(j)}{P_t^h} \right]^{-\theta} C_t^h \tag{53}$$

$$C_{k,t}(j) = \left[\frac{P_{k,t}(j)}{P_{k,t}} \right]^{-\theta} C_{k,t} \tag{54}$$

for all $j, k \in [0, 1]$, where P_t^h is the Home price index (i.e., the index of prices of domestically produced goods) defined as

$$P_t^h \equiv \left[\int_0^1 P_t^h(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \tag{55}$$

and $P_{k,t}$ is a price index for goods imported from country k (expressed in Home currency) for all $k \in [0, 1]$

$$P_{k,t} \equiv \left[\int_0^1 P_{k,t}(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \tag{56}$$

We derive the price index for imported goods (also expressed in Home currency)

$$P_t^f \equiv \left[\int_0^1 P_{k,t}^{1-\xi} dk \right]^{\frac{1}{1-\xi}} \tag{57}$$

The Home households's relative demand for C_t^h and C_t^f depends on their relative prices P_t^h and P_t^f . It is easy to derive:

$$C_t^h = (1 - \gamma) \left(\frac{P_t^h}{P_t^c} \right)^{-a} C_t \tag{58}$$

$$C_t^f = \gamma \left(\frac{P_t^f}{P_t^c} \right)^{-a} C_t \tag{59}$$

where P_t^c is the composite consumer price index defined as

$$P_t^c \equiv [(1 - \gamma)(P_t^h)^{1-a} + \gamma(P_t^f)^{1-a}]^{\frac{1}{1-a}} \tag{60}$$

The representative household maximizes the following expected discounted sum of utilities over possible paths of consumption and labor:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \right\} \tag{61}$$

where the period utility is defined as

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} \tag{62}$$

By considering the optimal expenditure conditions (58) and (59), the sequence of budget constraints assumes the following form:

$$C_t + B_t = \frac{P_t^h}{P_t^c} Y_t + B_{t-1} \frac{1 + i_{t-1}}{1 + \pi_t^c} + T_t \tag{63}$$

where B_t is the bond holdings of representative home household,²¹ and T_t is the lump-sum profit transfer from the firms.

The representative household chooses processes $\{C_t, N_t\}_{t=0}^\infty$ and asset portfolio to maximize (61) subject to (63). For any given state of the world, the following set of efficiency conditions must hold:

$$C_t^{-\sigma} = \beta E_t[(1 + i_t) \left(\frac{P_t^c}{P_{t+1}^c}\right) C_{t+1}^{-\sigma}] \tag{64}$$

$$\frac{N_t^\eta}{C_t^{-\sigma}} = \frac{W_t}{P_t^c} \tag{65}$$

Equation (64) takes the form of a familiar consumption Euler equation. Notice that following large part of the recent literature, we do not introduce money explicitly.

A.2 Asset Markets

The household is subject to a complete set of assets against all sorts of risks. For any state s of the economy at Period $t+1$, there exists a corresponding Arrow security with price v_t that pays one unit of Home currency in period $t+1$ if and only if state s occurs.²²

Assume state s occurs with probability $\tau(s)$. Then for the Home households, intertemporal substitution yields:

$$\frac{v_t U'(C_t)}{P_t^c} = \frac{\beta U'(C_{t+1}(s)) \tau(s)}{P_{t+1}^c(s)} \tag{66}$$

where s in the parentheses means it is state dependent.

Substituting $U(\cdot)$ with its functional form, and a little tranformation, we can get

$$\beta \left[\frac{C_{t+1}(s)^{-\sigma}}{C_t^{-\sigma}} \cdot \frac{P_t^c}{P_{t+1}^c(s)} \right] = \frac{v_t}{\tau(s)} \tag{67}$$

The foreign households also have access to this Arrow security, and their intertemporal substitution yields:

$$\frac{v_t U'(C_t^*)}{S_t P_t^*} = \frac{\beta U'(C_{t+1}^*(s)) \tau(s)}{S_{t+1}(s) P_{t+1}^*(s)} \tag{68}$$

²¹Since our model has a complete asset market (see Section 3.5), this budget constraint is an abbreviated version. We only use (63) to derive the IS curve. In fact, the household is subject to a complete set of assets against all sorts of risks.

²²If such an Arrow security does not exist, we can always construct an identical portfolio from existing assets. This is guaranteed by the definition of complete asset markets.

That is,

$$\beta \left[\frac{C_{t+1}^*(s)^{-\sigma}}{C_t^{*- \sigma}} \cdot \frac{S_t P_t^*}{S_{t+1}(s) P_{t+1}^*(s)} \right] = \frac{v_t}{\tau(s)} \tag{69}$$

i.e.,

$$\beta \left[\frac{C_{t+1}^*(s)^{-\sigma}}{C_t^{*- \sigma}} \cdot \frac{Q_t}{Q_{t+1}(s)} \cdot \frac{P_t^c}{P_{t+1}^c(s)} \right] = \frac{v_t}{\tau(s)} \tag{70}$$

Comparing (67) and (70), we can get

$$\frac{C_{t+1}(s)^{-\sigma}}{C_t^{-\sigma}} = \frac{C_{t+1}^*(s)^{-\sigma}}{C_t^{*- \sigma}} \cdot \frac{Q_t}{Q_{t+1}(s)} \tag{71}$$

i.e.,

$$\frac{C_{t+1}(s)^{-\sigma}}{C_{t+1}^*(s)^{-\sigma}} Q_{t+1}(s) = \frac{C_t^{-\sigma}}{C_t^{*- \sigma}} Q_t \tag{72}$$

The right hand side of the above equation is independent of state s , hence for any state s , we must have

$$\frac{C_{t+1}^{-\sigma}}{C_{t+1}^{*- \sigma}} Q_{t+1} = \frac{C_t^{-\sigma}}{C_t^{*- \sigma}} Q_t \tag{73}$$

This equation indicates $\frac{C_t^{-\sigma}}{C_t^{*- \sigma}} Q_t$ is a constant for any t . For convenience, normalize this constant to be unity. This is consistent with a symmetric initial condition with zero net foreign asset holdings. Hence we get

$$C_t = Q_t^{\frac{1}{\sigma}} C_t^* \tag{74}$$

and its log-linearized version

$$c_t = c_t^* + \left(\frac{1}{\sigma} \right) q_t = c_t^* + \left(\frac{1 - \gamma}{\sigma} \right) \delta_t \tag{75}$$

The last equation employs the fact that

$$\begin{aligned} q_t &= s_t + p_t^* - p_t^c \\ &= (s_t + p_t^* - p_t^h) + (p_t^h - p_t^c) \\ &= \delta_t + (p_t^h - p_t^c) \\ &= \delta_t - \gamma \delta_t \\ &= (1 - \gamma) \delta_t \end{aligned} \tag{76}$$

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