The Formation of Expectations, Inflation, and the Phillips Curve

Olivier Coibion, Yuriy Gorodnichenko, and Rupal Kamdar*

This paper argues for a careful (re)consideration of the expectations formation process and a more systematic inclusion of real-time expectations through survey data in macroeconomic analyses. While the rational expectations revolution has allowed for great leaps in macroeconomic modeling, the surveyed empirical micro-evidence appears increasingly at odds with the full-information rational expectation assumption. We explore models of expectation formation that can potentially explain why and how survey data deviate from full-information rational expectations. Using the New Keynesian Phillips curve as an extensive case study, we demonstrate how incorporating survey data on inflation expectations can address a number of otherwise puzzling shortcomings that arise under the assumption of full-information rational expectations. (JEL D04, E24, E27, E31, E37)

1. Introduction

Macro economists have long recognized the central role played by expectations: nearly all economic decisions contain an inter-temporal dimension such that contemporaneous choices depend on agents’ perceptions about future economic outcomes. How agents form those expectations should therefore play a central role in macroeconomic dynamics and policy making. While full-information rational expectations (FIRE) have provided the workhorse approach for modeling expectations for the past few decades, the increasing availability of detailed micro-level survey-based data on subjective expectations of individuals has revealed that expectations deviate from FIRE in systematic and quantitatively important ways including forecast-error predictability and bias.

How should we interpret these results from survey data? In this paper, we tackle this question by first reviewing the rise of the FIRE assumption and some of the successes that it has achieved. We then discuss the literature testing the FIRE assumption, focusing particularly on more recent work exploiting detailed micro-level survey data that has become increasingly available. This growing body of work documents pervasive departures from the FIRE assumption, especially when looking at the beliefs of households or

*Coibion: University of Texas at Austin and NBER. Gorodnichenko: University of California, Berkeley and NBER. Kamdar: University of California, Berkeley.
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managers. Given these differences between the traditional assumption of FIRE and the empirical evidence on how agents form their expectations, we then review the range of theoretical models that have been proposed to account for the observed deviations from FIRE, as well as some of the empirical evidence specifically testing these models.

We focus, in particular, on inflation expectations and their role in the Phillips curve. Our emphasis on inflation expectations, rather than expectations of other macroeconomic variables, reflects both their greater availability in survey data as well as their unique importance in macroeconomics. The crucial role played by inflation expectations on aggregate outcomes and policy decisions was highlighted by former Fed Chairman Alan Greenspan.1 “I am not saying what [inflation expectations] is a function of. We know it’s a very difficult issue, but that is the key variable. It’s important, but just because we can’t make a judgment as to what these driving forces are in an econometric sense doesn’t mean that it’s not real.” [italics added]

The role of expectations in the context of the Phillips curve has, of course, long been emphasized, going back to Friedman (1968) and Phelps (1967). While the Phillips curve began as an empirical correlation between wage inflation and unemployment in Phillips (1958), today the workhorse version of the relationship is the micro-founded New Keynesian Phillips curve (NKPC) that characterizes current inflation as a function of firms’ expectations about future inflation and economic slack. Over the years, research on the NKPC has identified a number of shortcomings such as the need for ad hoc lags in estimation to generate persistence in inflation, instability, or a flattening of the curve, missing disinflation during the Great Recession, inferior forecasting relative to naïve alternatives, and sensitivity to the slack variable employed; see Mavroeidis, Plagborg-Møller, and Stock (2014) for a recent survey. These puzzles have resulted in declarations of the death of the Phillips curve (e.g., Hall 2013).

However, the prognosis for the Phillips curve may be less grim after allowing for deviations from FIRE. That is, employing subjective expectations gathered from surveys in the estimation of expectations-augmented Phillips curves alleviates many of the previously identified puzzles 2.

The remainder of the paper proceeds as follows. The next section will discuss the development of FIRE from Muth (1961), the rational-expectations “revolution,” and the assumption’s current proliferation both inside and outside of macroeconomics. Section 3 reviews evidence that tests the null of FIRE using survey data. In section 4, models of expectation formation are discussed that may account for the deviations between survey expectations and rational expectations. Section 5 provides a detailed case study on the importance of careful consideration of the expectation formation in the case of the Phillips curve. We discuss the strengths and empirical limitations of the FIRE-based Phillips curve. Then wide-ranging evidence, inclusive of our own empirical analysis, is presented to demonstrate how conditioning on the real-time expectations of economic agents (based on survey measures) can address many of the documented limitations of the Phillips curve. We conclude in section 6 with a call for careful consideration of expectation formation processes, additional measurement of expectations to address the shortcomings of currently available survey data, and an increase in usage of survey data in future research.


2 Crump et al. (2015) similarly note that conditioning on survey data of households’ inflation expectations helps address puzzles associated with typical estimates of consumption Euler equations.
2. *Let There Be FIRE*

The expectations of agents are of integral importance in many macroeconomic models and have been emphasized as far back as Keynes’ *General Theory* (2018 [1936]), where he provided a motivation for how and why expectations may affect macroeconomic variables. Over the years, economists have continued to incorporate expectations into their models and early attempts to model the expectation formation process yielded alternatives such as adaptive expectations (expectations based on lagged experience) and rational expectations (expectations are “model-consistent”). Today, the workhorse expectation process assumed by macroeconomists is that of FIRE.

Muth (1961) was the first to suggest that expectations are the same as the appropriate economic theory, or “model consistent.” Muth’s proposal was not met with great excitement, and many continued to use adaptive expectations. It was not until a decade later that the rational-expectations “revolution” began.

Keynesian models of the 1960s typically implied that policies could forever be used to achieve lower unemployment and higher output at the cost of higher inflation. The stagflation experience of the 1970s, however, led many to conclude that a complete rethinking of macroeconomic models was needed. Lucas was at the forefront of this task and the rational-expectations revolution. He began with a paper in 1972 (Lucas 1972) in which an islands model was proposed where policy makers are unable to systematically exploit the Phillips curve relationship to control the real economy. Then, Lucas (1976) developed what is now known as the “Lucas critique”: using Keynesian models with parameters calibrated to past experience is an invalid way to evaluate changes in government policy. In particular, if policy is altered, the way expectations are formed changes, and if expectations affect economic outcomes, then outcomes estimated using a model calibrated with a different policy regime are inaccurate. Finally, Lucas and Sargent (1979) forcefully argued that Keynesian economic models should be abandoned in favor of equilibrium models characterized by agents with rational expectations, reacting to policy changes in a way that optimizes their personal interests so that analysis is not subject to the Lucas critique.

From relative obscurity, the rational-expectations assumption has become ubiquitous in macroeconomic models. Examples include the efficient markets hypothesis, the permanent income theory of consumption, and housing investment and price appreciation models. Furthermore, policy makers have often relied on versions of rational expectations in modeling expectations. For instance, variants of macroeconomic models employed at the Federal Reserve Board, the Bank of Canada, and the International Monetary Fund have used rational expectations (Brayton et al. 1997 and Dorich et al. 2013).

One early and enduring use of rational expectations has been in the Phillips curve that summarizes a relationship between nominal and real quantities in the economy. The curve is a central ingredient in macroeconomic models used by researchers and policy makers. In general, models with short-run trade-offs implied by the Phillips curve help generate monetary non-neutralities as documented in the empirical literature (e.g.,

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3 There are also variants of these macroeconomic models that employ other expectation formation processes. The FRB/US used by the Federal Reserve can either use rational expectations or expectations formed using a small vector autoregression (VAR) (Brayton et al. 1997). The Bank of Canada’s Terms-of-Trade Economic Model (ToTEM) originally modeled firms and households as rational. In 2011, the Bank of Canada updated to ToTEM II, which allows for rule-of-thumb firms and households (Dorich et al. 2013).

4 For a thorough history of the Phillips curve, see King (2008) and Gordon (2011). The former focuses on the use of the Phillips curve in policy and the latter highlights the different schools of thought on the Phillips curve post-1975.

The development of the curve began with Phillips (1958), which described an empirical relationship between wage rates and unemployment in the United Kingdom. Samuelson and Solow (1960), soon after Phillips, documented a similar finding for the United States. The relationship was later extended to the more general, overall price level, and other slack variables were employed (e.g., output gap, labor income share).

The Phillips curve trade-off was assumed to be continuously exploitable by many; however, others were unconvinced. Friedman (1968) and Phelps (1967) both argued for the natural rate hypothesis suggesting a vertical long-run Phillips curve relationship. Their analyses highlighted the importance of expectations in the Phillips curve. If agents are not surprised, monetary expansion may have no real effects. Solow (1969) and Gordon (1970) set out to empirically assess if the Phillips curve allowed for long-run trade-offs. They estimated expectations-augmented Phillips curves under the assumption of adaptive expectations. Their findings suggested that although policies that maintain low unemployment lead to higher inflation and inflation expectations, these policies could be sustainable. It was not until the stagflation of the 1970s and the Lucas (1972) and Sargent (1971) critiques of the Solow and Gordon tests, that the long-run trade-off beliefs were abandoned and the importance of inflation expectations accepted.

After Lucas (1972), which relied on imperfect information, macroeconomists set out to incorporate sticky prices and wages into rational expectation models. Some assumed prices or wages were set in a prior period and chosen so that the expectation of demand equaled the expectation of supply. Fischer (1977), Gray (1976), and Phelps and Taylor (1977) take this expected market-clearing approach. A shortcoming of this method—as well as Lucas’ island model—is that the persistence of macroeconomic shocks could only be as long as the longest lead at which prices or wages were being set. Others utilized staggered contract models that better capture the stylized facts of firm price-setting behavior (price and wage setting is staggered with not all firms changing simultaneously, and prices and wages are fixed for long periods of time). Taylor (1979, 1980) developed the staggered pricing model with fixed duration. Firms in his models pick their prices for \( N > 1 \) periods, also known as the contract period. In each period, \( 1/N \) firms pick their new price, a function of past and future price choices of other firms. The backward-looking component of the price choice of firms is able to generate persistence. As an alternative to constant-duration staggered pricing, Calvo (1983) introduced random-duration staggered pricing. He assumed that a firm faced a constant probability of being allowed to change prices in a given period. This results in i.i.d. contract lengths across firms, greatly simplifying the algebra required in staggered price-setting models.

As a result of these theoretical efforts, the purely forward-looking NKPC emerged as the dominant framework. It is micro-founded from the optimization problem of monopolistically competitive firms subject

\[ \text{\textsuperscript{6}} \text{Research has also explored state-dependent pricing, where firms can change prices whenever desired, but to do so must pay a fixed cost. This approach leads to Ss pricing decisions, which are generally difficult to aggregate (e.g., Golosov and Lucas 2007). Gertler and Leahy (2008) analytically develop a state-dependent pricing model with idiosyncratic shocks to firm productivity. The resulting Phillips curve is a variant of the one derived under Calvo pricing, with the main variation being the parameterization of the coefficient on output.} \]
to a friction limiting their price-changing ability. The most common friction imposed today is that of time-dependent sticky prices as in Calvo (1983); however, other pricing frictions, such as fixed-duration contracts, also suffice. Similar to earlier versions of expectations-augmented Phillips curves, the NKPC underscores the prominent role of inflation expectations in determining current inflation. However, in contrast to earlier work, the canonical NKPC traces the coefficients in the Phillips curve to structural parameters—hence making the NKPC immune to the Lucas critique—and enshrines the FIRE framework, thus completing the research agenda laid out in the 1970s. Given the prominence of the NKPC as an application of rational expectations, we use it as the primary example in our discussion henceforth.

3. Measuring Expectations: From Skepticism to Increasing Acceptance

The proliferation and dominance of FIRE in macroeconomic models is due in large part to the fact that it allows for policy analysis not subject to the Lucas critique, as well as relative ease of optimization in comparison to more complicated expectation formation processes. However, are rational expectations consistent with micro-level evidence provided by survey data? There is a vast literature that tests the null hypothesis of FIRE using a number of different procedures and data sets. In this section, we focus on findings related to inflation expectations in order to guide our analyses on the Phillips curve.

Although surveys can provide valuable information to answer this question, many macroeconomists have been uncomfortable with relying on these data to inform their choice or calibration of models. Skepticism toward survey expectations can be traced back to papers from the 1940s to 1960s that critiqued survey methodologies (e.g., Machlup 1946) and found survey data not useful in predicting individual behavior (e.g., National Bureau of Economic Research 1960; Juster 1964). Others argued that only theories, not assumptions, could be empirically tested. Prescott (1977) forcefully expressed this view: “Like utility, expectations are not observed, and surveys cannot be used to test the rational expectations hypothesis. One can only test if some theory, whether it incorporates rational expectations or, for the matter, irrational expectations, is or is not consistent with observations” [underlining his]. Thus, it was commonplace for economists to view the use of survey expectations as suspect.

This perspective has become increasingly uncommon, however. Zarnowitz (1984) and Lovell (1986) argued against the premise that assumptions should not be tested using micro data. Manski (2004) concluded that the hostility toward surveys is based on meager evidence and suggested that survey expectations provide a viable way to test models of the expectation formation process. If survey evidence consistently and forcefully rejects FIRE, one may be more inclined to discount models relying on the assumption.

A number of papers have used a battery of econometric tests to investigate if survey-based expectations are in line with FIRE. In the literature consistently finds

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7 For a thorough review, see Pesaran and Weale (2006).

8 See Dominitz and Manski (1997) and Manski (2004) for a history of the controversy surrounding the use of surveys as a measure for expectations.

9 As reviewed in Shefrin (1996), there are four popular tests of rationality: (1) unbiasedness—surveys should provide an unbiased predictor of the relevant variable; (2) efficiency—the survey expectation should use past observations of the variable in the same way that the variable actually evolves over time; (3) forecast-error unpredictability—the difference between the survey expectations and actual realizations should be uncorrelated with all information available at the time of forecast; (4) consistency—given forecasts made at different times for some variable
that survey-based expectations deviate from FIRE. Jonung and Laidler (1988) exploit a Swedish survey on contemporaneous inflation perceptions, i.e. agents’ beliefs about current or past inflation rates, to assess rationality. In contrast to inflation expectations (which are about the future), inflation perceptions are not subject to the “peso problem.”

They find that although unbiased, errors made by households are serially correlated. Roberts (1998) suggests that inflation expectations from the Livingston Survey and the Michigan Survey of Consumers (MSC) have an “intermediate” level of rationality, that is, they are neither rational nor do they follow a simple autoregressive model. Croushore (1998) notes that, over time with longer time series of survey data, survey expectations have become more in line with the predictions of rational expectations; however, expectations still do not pass all tests for optimality and at times are biased. Mankiw, Reis, and Wolfers (2003) use inflation expectations gathered from various surveys and demonstrate that each survey meets and fails some of the requirements of rationality. Croushore (1993, 1997) provides an overview of rationality tests that have used the Survey of Professional Forecasters (SPF) and Livingston data.

In addition to the canonical econometric tests, one can also assess if FIRE hold in subsamples of the population. Of course, while this does not invalidate the possibility that FIRE may hold in the aggregate, it does provide qualitative evidence that agents may not be fully informed. Some have noted the existence of demographic biases in inflation expectations. Bryan and Venkatu (2001) note that women tend to have higher inflation expectations even after controlling for race, education, marital status, income, and age. Souleles (2004) finds consumer demographics are correlated with inflation forecast errors in the MSC. Bruine de Bruin et al. (2010) highlight how inflation expectations are higher among those with lower financial literacy. Similarly, experiences, and therefore age, may also affect inflation expectations. Malmendier and Nagel (2016) document that learning from experience, that is, overweighting the inflation experienced during one’s own lifetime, appears to occur in the MSC.

One striking feature of survey data is that it reveals dramatic differences across individuals in terms of their perceived (past) inflation rates. For example, Jonung (1981) found, in a survey of Swedish households, that differences in households’ perceptions about recent inflation were almost as large as differences in their expectations of future inflation, and that households’ beliefs about recent inflation were a strong predictor about their beliefs over future inflation. Kumar et al. (2015) document similar patterns for households and firms managers in New Zealand. Large differences in perceptions of recent inflation across economic agents are striking because they are strongly at odds with the common assumption of fully informed agents. Subsequent work has documented properties of inflation perceptions and how these relate to differences in inflation expectations.

Building on Jonung’s (1981) finding that women had a higher perceived past inflation rate than men in 1977 because women purchased a larger share of food and food price inflation in 1977 was higher than that of general inflation, others have shown that an individual’s consumption basket affects his or her perception of inflation. Georganas, Healy, and Li (2014) conduct a financially incentivized experiment among consumers and find that perceived inflation rates are biased

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\(^{10}\)Suppose one is forming expectations about the value of the Mexican peso to the US dollar, and there is a positive probability of a peso devaluation. FIRE agents would incorporate the devaluation probability into their expectations. Assume the devaluation does not occur. Then ex post, the ex ante inflation expectations would appear to have a systematic error despite agents having FIRE.
toward goods frequently bought. Ranyard et al. (2008) note that the expenditure weighting in the Consumer Price Index (CPI) calculation results in measured inflation being a better representation of inflation experienced by households in the upper percentiles of the expenditure distribution than those that are less wealthy. Coibion and Gorodnichenko (2015b) find that households in groups that purchase gasoline more frequently adjust their inflation forecasts by more when oil prices change than do other households. Cavallo, Cruces, and Perez-Truglia (2017) document that consumers use their memories of supermarket prices when forming their inflation expectations. Johannsen (2014) finds that low-income households experience more dispersion in changes of their cost of living and also display more heterogeneity in their inflation forecasts.

Others have discussed how the accuracy and dispersion of inflation expectations may vary over time. Some have noted that the accuracy of inflation expectations varies with the business cycle. Carvalho and Nechio (2014) find that many households form their forecasts in a way that is consistent with a Taylor rule on the part of monetary policy makers, but that this is primarily true during downturns. Coibion and Gorodnichenko (2015a) also find that deviations from FIRE in the United States decline during downturns, as do Loungani, Stekler, and Tamirisa (2013) in a much wider cross-section of countries. Others have empirically demonstrated and modeled how the amount of disagreement in inflation expectations may vary over time. Cukierman and Wachtel (1979) and Mankiw, Reis, and Wolfers (2003) empirically demonstrate that a high dispersion of inflation expectations is positively correlated with a high level of inflation and a high variance in recent inflation.\footnote{11 Capistrán and Timmermann (2009) offer an alternative explanation within the FIRE framework. They develop a model of asymmetric costs to over- and under-predicting, heterogeneous loss functions amongst agents, and a constant loss component to try to fit the observed characteristics of SPF inflation forecasts without deviating from FIRE. However, Coibion and Gorodnichenko (2012) show that this type of model makes counterfactual predictions for the dynamic responses of forecast errors to shocks.}

**Recap:** Early suspicion of directly measuring expectations has subsided over the years, and economists are increasingly conducting surveys and relying on survey data. Survey-based expectations have been used to test the assumption of FIRE, and the literature consistently finds deviations from FIRE. Surveys reveal demographic biases across gender and age; perceived inflation is affected by an individual’s consumption basket; and the accuracy and dispersion of expectations may vary systematically over time. These characteristics suggest that assuming agents hold FIRE may be too strong. At the same time, Croushore (2010) finds that while departures from rational expectations over short periods of time can be frequent and large, these departures tend to dissipate over longer periods. Thus, expectations appear to converge to FIRE over time. Coming to grips with these different empirical findings requires developing models of the expectations formation process that go beyond FIRE.\footnote{\textsuperscript{12} The evidence discussed in this section need not invalidate the usage of FIRE in all cases when one is interested in aggregate outcomes and arbitrage opportunities are not costly. For example, consider financial markets where some traders are rational while others are not. With sufficient resources and an appropriate institutional environment, rational agents could arbitrage away market outcomes that are not rational resulting in the aggregates being effectively set using FIRE.}

4. **Alternatives to FIRE**

As documented in the previous section, the empirical evidence generally rejects the FIRE assumption. How should one interpret these deviations? Do they imply that expectations are irrational? Or do they reflect constraints on the information processing
capacity of economic agents? Does it matter? And, importantly, how should we model the expectation formation process? As Shiller (1978) noted early in the rational-expectations revolution, “Even when we do have survey data or other data which purport to represent expectations, if these expectations are endogenous in our model then we still must model the determination of these expectations.”

Most recent work in this direction has emphasized possible deviations from full information due to information rigidities while maintaining the assumption of rational expectations. One such approach is the sticky information approach of Mankiw and Reis (2002), in which agents update their information sets infrequently but when they do so, they acquire FIRE. Carroll (2003) helps rationalize the sticky information approach by suggesting that information is transferred from professional forecasters to consumers over time via the news. An alternative approach, often called noisy information or rational inattention, is motivated by information processing constraints of agents. The information constraints are modeled as agents either receiving noisy signals (agents observe the true values with some error) or agents rationally choosing what information to pay attention to subject to some information constraint. Woodford (2002) takes the first approach and Sims (2003) and Maćkowiak and Wiederholt (2009) take the latter.

Sticky information, noisy information, and rational inattention models make some common predictions. First, the mean forecast across agents of a macroeconomic variable will under-respond relative to the actual response of the variable after a macroeconomic shock. For example, if a shock raises inflation for a number of periods, the mean forecast of inflation in both models will not rise by as much as actual inflation. In sticky information, this is because some agents will be unaware that the shock has occurred and will not change their forecast at all. In noisy information models, agents will receive signals pointing to higher inflation, but they will adjust their forecasts only gradually because of their initial uncertainty as to whether the higher signals represent noise or true innovations. In rational inattention models, some agents will not be paying complete attention to incoming inflation data and will not sufficiently increase their forecasts. Coibion and Gorodnichenko (2012) document that, consistent with models of information rigidities, survey forecasts of inflation under-respond to different macroeconomic shocks. These results obtain for a variety of surveys, including the SPF, the Livingston Survey, Federal Open Market Committee (FOMC) member forecasts, and the MSC. Furthermore, the implied levels of information rigidity are economically large, pointing to important deviations from FIRE.

Another common prediction from these models is that mean ex post forecast errors across agents will be predictable on average using ex ante revisions in mean forecasts, in contrast to the prediction from FIRE that ex post forecast errors should be unpredictable. In sticky information, this reflects the fact that some agents do not update their information, and so their forecasts remain unchanged, anchoring the mean forecast to the previous period’s mean forecast. In noisy information, agents update their forecasts only gradually because of the noise in the signal, again anchoring current forecasts to previous forecasts. In rational inattention models, current mean forecasts will be anchored to past mean forecasts because some agents will not be paying complete attention to the relevant variable. These mechanisms imply a gradual adjustment in mean forecasts and therefore predictability in mean forecast errors. Coibion and Gorodnichenko (2015a) test this prediction and find robust evidence for predictability of ex post forecast errors from ex ante mean
forecast revisions, consistent with these models of information rigidity. Once again, the implied deviations from FIRE are economically large and can be found in a variety of different surveys, such as the SPF, the MSC, financial market forecasts, and Consensus Economics forecasts for different countries.

In addition to models that emphasize information rigidities, research has considered an array of other possible departures from FIRE. These models fall broadly into two, sometimes overlapping, categories. One such alternative is bounded rationality, where agents are “bounded” by model misspecification yet are “rational” in their use of least squares (Sargent 1999). Gabaix (2014) proposes and analyzes a “sparse max” operator in which agents build a simplified model of the world, paying attention to only some of the relevant variables as attention bears a positive cost in the model. This approach is motivated by the limited capacity of agents to follow and relate macroeconomic variables. Gabaix continues by analyzing the results of consumer demand and competitive equilibrium when agents use a sparse max operator. Of particular relevance is his analysis of a Phillips curve in the Edgeworth Box. He shows that under a sparse max operator, each equilibrium price level corresponds to a different real equilibrium, similar to a Phillips curve. Natural expectations, in Fuster, Laibson, and Mendel (2010), is a related concept in the sense that it is a middle ground between rational expectations and naïve intuitive expectations. Agents with natural expectations use simple, misspecified models to forecast a complex reality.

A similar approach where agents have misspecified models is that of diagnostic expectations. This type of expectations is motivated by Kahneman and Tversky’s (1972) representativeness heuristic that characterizes our non-Bayesian tendency to overestimate the probability of a trait in a group when that trait is representative or diagnostic to that group (e.g., red hair among the Irish). Gennaioli and Shleifer (2010) and Bordalo, Gennaioli, and Shleifer (2016) take this behavioral intuition and formalize it into diagnostic expectations. Agents with diagnostic expectations overweight future outcomes that become more likely with incoming data. Bordalo, Gennaioli, and Shleifer (2016) show that diagnostic expectations in a model of credit cycles can account for stylized facts on credit spreads such as their excess volatility, predictable reversals, and an overreaction to news.

The second approach is to use models of learning. The most common formulation of learning is adaptive learning where the agent acts as an econometrician in each period and uses observed outcomes to estimate a perceived law of motion. From the perceived law of motion, which is not necessarily the actual law of motion, the agent forms expectations and maximizes subject to those expectations. Evans and Honkapohja (1999, 2012) provide an extensive review of the adaptive learning literature and discuss other learning approaches. In these models, agents often observe shocks (and so in this dimension, their information is full) and have full rationality but do not know parameters governing dynamics in the economy.

Models of learning have been used to study inflation and inflation expectations. On the theoretical side, Orphanides and Williams (2005) demonstrate that in a model where agents use a model of finite memory, least squares, perpetual learning to form inflation expectations, significant and persistent deviations of inflation expectations from those implied by rational expectations may arise. On the empirical front, models of learning have done well matching observed inflation persistence and inflation expectation survey data (e.g., Milani 2007 and Branch and Evans 2006a). These empirical successes suggest learning models may capture important deviations from FIRE.

A closely related body of work emphasizes the possibility that agents switch across
different forecasting processes over time. Branch and Evans (2006b), for example, model agents as choosing from a list of misspecified models (parameters, however, are formed optimally) based on prior forecast performance. This formulation is able to give rise to intrinsic heterogeneous expectations under suitable conditions. In particular, with high intensities of choice (intensity of choice parameterizes the agent’s bounded rationality, and as it approaches infinity, the agents approach full optimization), expectations must influence actual outcomes. Pfajfar and Žakelj (2014) use an inflation forecasting experiment to assess the process of forecast formation and the extent to which agents switch forecasting rules. They find that expectations are heterogeneous, with some subjects behaving in line with rational expectations while others appear to adhere to methods of adaptive learning or trend extrapolation with frequent switching between forecasting models. Forecast-switching behavior has also been modeled as a result of social dynamics as in Hachem and Wu (2017). Agents have pairwise meetings where they compare their recent forecast errors, and the agent with the larger error earns a “strike.” After a threshold level of strikes, an agent will switch their forecasting rule.

Recap: There are a variety of alternatives to FIRE that can explain why we observe pronounced and persistent deviations from FIRE in survey data. Options include sticky information, noisy information, rational inattention, bounded rationality, diagnostic expectations, and learning. Identifying which approach can best characterize the expectations formation process of different agents should be a key area of future research.

5. Application: The Phillips Curve

As discussed above, FIRE often appears at odds with real-time, survey-based expectations. This section demonstrates that in a prominent and important application—the Phillips curve—incorporating real-time expectations into the analysis can address a number of otherwise puzzling shortcomings of the NKPC that arise under the FIRE assumption.

Of course, the NKPC was derived under the assumption of FIRE, and including subjective inflation expectations is a deviation from this assumption. However, Adam and Padula (2011) show that one can use survey expectations in the Phillips curve as long as economic agents respect the law of iterated expectations (LIE), a weaker assumption than FIRE. This constraint is satisfied, e.g., when agents are rational but not sufficiently informed. Coibion and Gorodnichenko (2012, 2015a) and others report evidence consistent with this condition being satisfied.

We begin with a discussion of the early successes of the NKPC and then move to the failures and puzzles generated by the formulation. Then, survey-based expectation data availability is explored and the literature using survey-based expectations is reviewed and shown to have solved some of the puzzles associated with the NKPC. Our own empirical analysis confirms the importance of the inclusion of survey-based inflation expectations in the estimation of the Phillips curve.

5.1 Successes of the Full-Information Rational Expectations Phillips Curve

The expectations-augmented Phillips curve combined with the assumption that expectations are rational and fully informed experienced early theoretical and empirical successes.

Theoretical Success: The workhorse framework with rational expectations codified in Clarida, Galf, and Gertler (1999) and Woodford (2003) has been instrumental in guiding empirical analyses to link the nominal and real sides of the economy. For example, the framework can help answer such
questions as what measure of slack (e.g., output gap, unit labor costs, unemployment) and expected inflation (e.g., one-year- or one-quarter-ahead inflation, lagged or future) should be used in the Phillips curve. The framework can also provide a benchmark to evaluate empirical estimates of Phillips curve parameters. The baseline formulation of the curve takes the following form:

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\pi_t = \beta E_t \pi_{t+1} + \kappa X_t + \text{shock}_t
\]

where \(\pi_t\) is the rate of inflation at time \(t\), \(E_t \pi_{t+1}\) is the mathematical (FIRE) expectation of inflation at time \(t+1\) given information available at time \(t\), \(\beta\) is the discount factor, \(X_t\) is the output gap (more generally, a measure of slack in the economy), \(\kappa\) measures the slope of the Phillips curve and is a function of structural parameters, and \(\text{shock}_t\) is a “cost-push” shock. \(^{13}\) Note that this formulation nests the expectations-augmented Phillips curve advocated by Friedman (1968) and Phelps (1967).

**Empirical Success:** Galí and Gertler (1999) and others estimate the NKPC after imposing FIRE. Using labor’s share of income as the forcing variable in the Phillips curve, Galí and Gertler find estimated coefficients that conform closely to those predicted by the theory. Galí and Gertler also present a model where some firms are forward looking (set prices as in Calvo pricing) and some firms are backward looking (set prices equal to the average price set in the previous period with a correction for inflation). With these assumptions, a hybrid Phillips curve is developed in terms of structural parameters. An estimation of the curve is then conducted using real unit labor costs as the slack variable. The backward-looking component is statistically significant, but smaller than the forward-looking component. The authors conclude that the NKPC is a reasonable approximation of inflation dynamics.

### 5.2 Limitations of the Full-Information Rational Expectations Phillips Curve

Next, we review the empirical limitations of the Phillips curve when estimated under the assumption of FIRE. The literature has documented the following shortcomings.

**Ad-hoc lags, instability, and structural breaks.**—The micro-foundation of the NKPC suggests a purely forward-looking inflation dynamics model. However, to incorporate the persistence observed in inflation data, authors have relied on ad hoc, backward-looking terms and estimated a “hybrid” NKPC. Fuhrer and Moore (1995), Fuhrer (1997), Lindé (2005), and Rudd and Whelan (2005) argue that these backward-looking components can be very important. A potential reason why this may be occurring is structural breaks. That is, the traditional NKPC is a linearization around a zero steady state inflation rate, and it ignores the possibility of changes in the steady state inflation rate. By doing so, the lagged terms of inflation will spuriously capture these changes (e.g., Kozicki and Tinsley 2002; Cogley and Sbordone 2008). Mavroeidis, Plagborg-Møller, and Stock (2014), in an earlier survey of the literature, extensively review the empirics of the NKPC with special attention to weak identification. They conclude that estimation of the NKPC is fraught with uncertainty, as small changes

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\(^{13}\)The most common formulation of the NKPC relates inflation to expected inflation and marginal costs. However, because the latter are not directly measurable, there has been considerable debate about what the relevant forcing term should be in the NKPC in empirical applications, as we discuss in more detail in sections 5.2 and 5.3.

\(^{14}\)Sheedy (2010) provides another justification for the presence of backward-looking terms. If prices that have not changed for longer periods are more likely to be changed than those set recently, the Phillips curve will have backward-looking terms even though all pricing decisions are entirely forward looking.
in specifications can lead to large variation in point estimates due to weak identification.

**Missing (dis)inflation.**—Several researchers have argued that inflation should have fallen much more in the United States and other advanced economies during the Great Recession, given the amount of slack in the economy (e.g., Hall 2013, IMF 2013). Similarly, a missing inflation puzzle arose in real time during the late 1990s. Amid a booming economy with unemployment falling below estimates of the non-accelerating inflation rate of unemployment (NAIRU), policy makers disagreed about why inflation had yet to be triggered (Meyer 2004).

**Low out-of-sample predictive power.**—Atkeson and Ohanian (2001) contend that Phillips-curve-based inflation forecasts have been no more accurate than those of a naïve model where inflation next year equals inflation in the prior year. The Phillips-curve-based methods shown to be inferior are: the textbook NAIRU Phillips curve, the Stock and Watson NAIRU model (lagged values of inflation and the slack variable are included in this specification), and the Greenbook forecasts. Stock and Watson (2007) also find that gap-based backward-looking Phillips curves are less successful in forecasting inflation, relative to simple univariate models, after 1984.

**Sensitivity to the slack variable employed.**—Since traditional measures of economic slack such as unemployment and output gap yield the puzzles just described, authors have proposed the use of other slack variables, such as the labor share of income. Overall, the results reveal sensitivity in the slope of the Phillips curve to which slack variable is used.

Galí and Gertler (1999) argue that real unit labor cost (ULC), commonly measured by labor's share of income, is the superior forcing variable because of a strong contemporaneous correlation with inflation, unlike the output gap, which leads inflation. Furthermore, when estimating the Phillips curve with the output gap as a measure of real economic conditions, the coefficient obtained is counterfactually negative and significant, while labor's share of income yields a positive and significant coefficient. Similarly, Woodford (2001) and Sbordone (2002) contend unit labor cost is the best proxy for marginal costs using different approaches. However, Rudd and Whelan (2005) argue that neither detrended real GDP nor real unit labor costs allow the NKPC to fit the data well. In addition, King and Watson (2012) find that since 1999, the behavior of real unit labor costs should have implied a decline in inflation of fifteen percentage points. In reality, actual inflation stayed relatively unchanged, allowing King and Watson to conclude, “conventional unit labor cost measure is no longer a useful construction for inflation dynamics and has not been at least since the early 2000s.” Elsby, Hobijn, and Sahin (2013) further document that labor's share of income is subject to a number of measurement issues...
that can make it a poor measure of marginal costs.

Another measure proposed was the unemployment recession gap, the difference between the current unemployment rate and the minimum unemployment rate over the current and previous eleven quarters. Stock and Watson (2010) show that the empirical regularity of US recessions being accompanied by declines in inflation can be explained by a model where the unemployment recession gap explains the deviation of core inflation from its trend.

Recap.—The Phillips curve, as derived under FIRE, has encountered several empirical shortcomings: the lack of persistence has led to ad hoc backwards-looking terms; periods of missing (dis)inflation are puzzling; Phillips-curve-based forecasts have low out-of-sample predictive power in comparison to naïve forecasts; and there is sensitivity to the slack variable used.

5.3 Real-Time Expectations and the Phillips Curve

Bernanke (2007) summarizes the strengths and weaknesses of FIRE in the context of the NKPC with, “The traditional rational-expectations model of inflation and inflation expectations has been a useful workhorse for thinking about issues of credibility and institutional design, but, to my mind, it is less helpful for thinking about economies in which (1) the structure of the economy is constantly evolving in ways that are imperfectly understood by both the public and policymakers and (2) the policymakers’ objective function is not fully known by private agents.” In light of this assessment, survey expectations may provide an appealing alternative to FIRE in the estimation of the Phillips curve and their use does appear to help solve many of the aforementioned puzzles and limitations associated with the Phillips curve in recent years.

Ad hoc lags, instability, and structural breaks.—As emphasized by Bernanke (2007) and many others, using a traditional rational-expectations model of inflation and inflation expectations may be problematic when the structure of the economy is constantly evolving. Survey expectations can adapt and thus lead to a more robust Phillips curve. Roberts (1995), for example, finds that when using survey measures of inflation and either detrended output or the unemployment rate as the slack variable, the NKPC is stable over the two subsamples tested. In contrast, “McCallum’s approach,” which utilizes the actual future inflation rate and instrumental variables (i.e., this approach builds on FIRE), yields qualitatively unstable coefficients.

Brisсимis and Magginas (2008) utilize SPF inflation forecasts, Greenbook inflation forecasts, and final data on future inflation to estimate both a forward-looking and a hybrid Phillips curve. Their findings suggest that once one allows for deviations from rationality (i.e., by using surveys), the pure NKPC provides a reasonable description of inflation dynamics in the United States during the 1968–2006 period. In particular, notice from table 1 (their table 2) that the use of inflation expectations from the Greenbook and the SPF moves more weight to the expectation term, rather than the lagged term in a hybrid Phillips curve specification ($\pi_t = \beta_1 E_t \pi_{t+1} + \beta_2 \pi_{t-1} + \beta_3 ulc_t + \epsilon$). The lagged term is no longer significant with the inclusion of the surveys. Furthermore, the dominance of the forward-looking component remains in subsamples as shown in table 2 (their table 5).

Others have assessed that changes in the inflation trend and target are well captured by survey measures. Cecchetti et al. (2007) provide evidence that survey inflation expectations from the Fed, the SPF, and the MSC are correlated with future inflation. They conclude that “(1) Signals from several survey measures of US inflation expectations
### TABLE 1

<table>
<thead>
<tr>
<th>Specification with Greenbook forecast</th>
<th>$E_t \pi_{t+1}$</th>
<th>$\pi_{t-1}$</th>
<th>$ulc_t$</th>
<th>$R^2$</th>
<th>$J$-test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.84</td>
<td>0.18</td>
<td>0.04</td>
<td>0.85</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>(4.30)</td>
<td>(1.03)</td>
<td>(5.06)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Specification with SPF forecast</td>
<td>0.86</td>
<td>0.21</td>
<td>0.05</td>
<td>0.86</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>(5.03)</td>
<td>(1.51)</td>
<td>(5.25)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Specification with final data on future inflation</td>
<td>0.61</td>
<td>0.38</td>
<td>0.01</td>
<td>0.83</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>(6.19)</td>
<td>(3.98)</td>
<td>(0.28)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: This table reproduces table 2 in Brissimis and Magginas (2008). ©2008 The Association of the International Journal of Central Banking. Reprinted with permission. All rights reserved.

Notes: $E_t \pi_{t+1}$ denotes inflation expected by the private sector for period $t+1$, expressed in terms of the annualized rate of change in the GDP deflator; $\pi_{t-1}$ is the lagged value of the annualized rate of change of the GDP deflator; and $ulc_t$ is real unit labor cost. Numbers in parentheses are $t$-statistics, and the last column shows the $p$-values associated with a test of the model’s overidentifying restrictions (Hansen’s $J$-test). The instrument set includes two lags of real unit labor cost, the output gap, nominal wage growth and three lags of inflation.

### TABLE 2

<table>
<thead>
<tr>
<th></th>
<th>$E_t \pi_{t+1}$</th>
<th>$\pi_{t-1}$</th>
<th>$ulc_t$</th>
<th>$R^2$</th>
<th>$J$-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1968:IV–1979:II (Pre-Volcker period)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Greenbook-based specification</td>
<td>0.86</td>
<td>0.17</td>
<td>0.05</td>
<td>0.59</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>(4.16)</td>
<td>(1.08)</td>
<td>(4.17)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1979:III–2000:IV (Volcker-Greenspan period)</td>
<td>0.78</td>
<td>0.29</td>
<td>0.02</td>
<td>0.67</td>
<td>0.69</td>
</tr>
<tr>
<td>Greenbook-based specification</td>
<td>(3.91)</td>
<td>(1.77)</td>
<td>(2.35)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1968:IV–1979:II (Pre-Volcker period)</td>
<td>1.09</td>
<td>-0.001</td>
<td>0.03</td>
<td>0.57</td>
<td>0.55</td>
</tr>
<tr>
<td>SPF-based specification</td>
<td>(4.88)</td>
<td>(-0.006)</td>
<td>(2.13)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1979:III–2000:IV (Volcker-Greenspan period)</td>
<td>0.75</td>
<td>0.27</td>
<td>0.02</td>
<td>0.89</td>
<td>0.67</td>
</tr>
<tr>
<td>SPF-based specification</td>
<td>(5.33)</td>
<td>(1.96)</td>
<td>(3.30)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: The table reproduces Table 5 in Brissimis and Magginas (2008). ©2008 The Association of the International Journal of Central Banking. Reprinted with permission. All rights reserved.

Notes: $E_t \pi_{t+1}$ denotes inflation expected by the private sector for period $t + 1$, expressed in terms of the annualized rate of change in the GDP deflator; $\pi_{t-1}$ is the lagged value of the annualized rate of change of the GDP deflator; and $ulc_t$ is real unit labor cost. Numbers in parentheses are $t$-statistics, and the last column shows the $p$-values associated with a test of the model’s overidentifying restrictions (Hansen’s $J$-test). The instrument set includes two lags of real unit labor cost, the output gap, nominal wage growth and inflation.
anticipate future movements in the US inflation trend; and (2) when the inflation trend changes, survey measures of expectations are likely to follow.” Del Negro and Eusepi (2011) find that time variation in the inflation target is important in explaining inflation expectations. These results based on survey measures of expectations are consistent with other studies (e.g., Cogley and Sbordone 2008, Kim and Kim 2008, Zhang, Osborn, and Kim 2008) that emphasize structural breaks in explaining away the importance of backward-looking components of the Phillips curve.

Survey measures have also been shown to generate the persistence that ad hoc lags have otherwise frequently been used to capture. Fuhrer (2015b) analyzes the implications of using survey data in three key building blocks of standard DSGE models: a price-setting Euler equation, an investment/savings (IS) curve, and a forward-looking policy rule. Fuhrer finds that using survey expectations eliminates the need for ad hoc lags. What formerly appeared to be a need for ad hoc lags of endogenous variables is better represented as inertia in inflation expectations. In addition, he finds that in a horse-race test, survey expectations dominate rational expectations in DSGE models. This finding leads to the question: why are inflation expectations persistent? Fuhrer (2015a) empirically demonstrates that individual forecasters in the SPF and the MSC tend to revise their forecasts toward the lagged central tendency of expectations. If agents had FIRE, one would not expect this behavior. Rather, it is suggestive of not fully informed agents using lagged central tendencies as a way to average out some of the agent’s own idiosyncratic error, and thus building in persistence in inflation expectations.

Furthermore, agents may change how they forecast inflation over time, for example, due to information costs. Using survey expectations will help capture any such possible changes. Evans and Ramey (1992, 1998) and Brock and Hommes (1997, 1998) demonstrate theoretically that agents facing information costs may rationally choose not to use rational expectations as their expectation formation process. Branch (2004) presents evidence that dynamic switching appears to occur in survey data. With MSC data, he finds evidence of heterogeneous expectations in which agents dynamically switch predictors based on relative mean squared errors of the predictor functions and the costs associated with each.

**Missing disinflation.**—Coibion and Gorodnichenko (2015b) suggest that the missing deflation during the Great Recession, documented in figure 1 (their figures 5 and 6), can be explained by the rise of household inflation expectations (assuming firm expectations match those of households) from 2009 to 2011. Panel C of figure 1 shows the increase in inflation expectations of consumers during the Great Recession and demonstrates that the missing disinflation is alleviated with the use of consumer expectations. The increase in expectations was attributed to rising oil prices, which consumers appear to perceive as salient indicators of inflation.\(^\text{16}\) In a related work, Friedrich (2014) investigates the “twin puzzle” across advanced countries of higher-than-expected inflation despite economic slack from 2009 to 2012 and weakening inflation despite economic recovery post 2012. He estimates a global Phillips curve for 1995 to 2013 using survey-based inflation expectations and finds that these measures of inflation expectations account for the “twin puzzle.”

\(^{16}\)Del Negro, Giannoni, and Schorfheide (2015) provide an alternative explanation for the missing disinflation during the Great Recession. They demonstrate that a DSGE model with FIRE for short-term inflation and survey-based ten-year inflation expectations can predict a decline in output without a decline in inflation. The insight behind this finding offered by the authors is that inflation is more dependent on expected future marginal costs than on current macroeconomic activity.
Low out-of-sample predictive power.— Stock and Watson (2007) and others document that it has been increasingly difficult to nowcast or forecast inflation in recent periods. As a result, (semi)structural approaches based on a Phillips curve have also become less successful in accounting for observed inflation. At the same time, Ang, Bekaert, and Wei (2007), Croushore (2010), and others find that survey-based forecasts of inflation continue to have better root mean squared forecast error (RMSFE) than autoregressive moving average (ARIMA) models and other popular alternatives. Furthermore, as we show below, Phillips curves using survey measures of inflation expectations tend to have better in-sample fit in the post-1978 period and better out-of-sample fit during the Great Recession and its aftermath. Thus, although Phillips curves do not yield consistently superior forecasts, employing survey expectations of inflation in a Phillips curve can still provide useful insights into future inflation dynamics.

Figure 1. Time Variation in the Slope of the Phillips Curve, Coibion and Gorodnichenko (2015b)

Source: The figure reproduces figures 5 and 6 in Coibion and Gorodnichenko (2015b).
Notes: Panels A through C show changes in the slope of the Phillips curve over time. Panels A and C use CPI inflation rate. Panel B uses GDP deflator inflation rate.
curve tends to improve our ability to rationalize and forecast inflation dynamics.

**Sensitivity to the slack variable employed.**—The issue of sensitivity to the slack variable arose after traditional measures failed to deliver anticipated results and a search for alternative measures ensued. If using surveys allows traditional measures to deliver anticipated and significant coefficients and stability, then the use of alternative slack measures would be unnecessary. Adam and Padula (2011) demonstrate that using either the output gap or unit labor costs as a proxy for marginal costs yields the expected signs in the slope of the Phillips curve when survey measures are used for inflation expectations. In a similar spirit, Roberts (1995) considers two approaches to the treatment of expectations in the Phillips curve. The first approach is to use surveys to construct a measure of expectations. The second approach is to impose FIRE as in McCallum (1976). These two approaches amount to running the following regressions:

**Expectation approach:**

\[
\Delta p_t - E_t \Delta p_{t+1} = c_0 + \gamma y_t + c_1 \Delta rpoi_t + c_2 \Delta rpoi_{t-1} + \epsilon_t;
\]

**McCallum approach:**

\[
\Delta p_t - \Delta p_{t+1} = c_0 + \gamma y_t + c_1 \Delta rpoi_t + c_2 \Delta rpoi_{t-1} + \epsilon_t + (E_t \Delta p_{t+1} - \Delta p_{t+1}),
\]

\[
\Delta p_t - \Delta p_{t+1} = c_0 + \gamma y_t + c_1 \Delta rpoi_t + c_2 \Delta rpoi_{t-1} + \epsilon_t + v_t,
\]

where \(\Delta p_t\) is the inflation rate, \(E_t \Delta p_{t+1}\) is a survey-based measure of inflation expectations, \(y_t\) is a measure of slack, \(\Delta rpoi_t\) is the percent change in the real price of oil, and \(\epsilon_t\) and \(v_t\) are the error terms. Roberts finds that regardless of the slack proxy (detrended output or unemployment rate), the coefficient on the slack measure is in the correct direction and statistically significant when the expectation approach is used. The McCallum approach, on the other hand, yields insignificant slack coefficients and a poor \(R^2\). See his specifications and estimation results in Table 3 (his 1) below.

**Survey measures are empirically preferred to the rational-expectations assumption in Phillips curves.**—In addition to addressing most of the weaknesses of the FIRE-based NKPC, using survey measures of expectations often empirically dominates rational-expectations Phillips curves. For example, Roberts (1995) estimates the Phillips curve using both survey expectations (Livingston Survey and MSC) and rational expectations. Similar coefficients are found on the slack variable with both approaches, but only with survey expectations is the coefficient statistically significant. He suggests, “One explanation for the larger standard error is that actual future inflation is a worse proxy for inflation expectations than are the surveys.” Subsequent work has largely confirmed this finding.

Fuhrer and Olivei (2010) document that, over the preceding three decades, rational expectations have had little effect on inflation, whereas survey measures have played a considerable role. The influence of survey measures in some models was found to have even increased in recent years. Fuhrer, Olivei, and Tootell (2012) similarly find that US inflation from 1990 to 2010 is not well modeled by a forward-looking, rational-expectations Phillips curve but rather is well described by a model that uses a survey-based, one-year-ahead inflation expectation term and lagged inflation terms; see Table 4 (their table 5). Panel A shows the importance of survey expectations...
in the inflation process and panel B demonstrates that a rational-expectations term has no impact and a lagged inflation term has little effect on inflation, once a one-year-ahead survey expectation term is included. Fuhrer (2012) estimates a Phillips curve with both a rational-expectations term and a survey-expectations term using maximum likelihood (with three variants on trend inflation) and generalized method of moments (GMM) (with two variants on the weight matrix: “standard” and “optimal” weights). He finds, in all specifications but one, survey expectations play a dominant role and rational expectations are insignificant.17

Recap.—Incorporating real-time survey data in the estimation of the Phillips curve addresses many of the puzzles that arose under FIRE. Various studies suggest that survey-based inflation expectations tend to yield a stable, forward-looking Phillips curve.

5.4 LIE (Law of Iterated Expectations) Detector for the Phillips Curve

Studies using survey-based expectations conventionally replace FIRE expectations in specification (1) with the average (or median) expectations. One may be concerned that this mechanical approach leads to a mis-specification as an alternative expectations formation process can yield a different Phillips curve. Indeed, table 5 demonstrates that the specific formulation of the Phillips curve depends on assumptions about the information structure and other elements of the employed models: there is variation in how expectations should be defined, what should be used as a measure of slack,

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Detrended output</td>
<td>Proxy for inflation expectations</td>
</tr>
<tr>
<td></td>
<td>Livingston</td>
</tr>
<tr>
<td>Constant</td>
<td>0.765</td>
</tr>
<tr>
<td></td>
<td>(0.255)</td>
</tr>
<tr>
<td>( \gamma ) (slack)</td>
<td>0.337</td>
</tr>
<tr>
<td></td>
<td>(0.081)</td>
</tr>
<tr>
<td>( c_1 ) (current oil price change)</td>
<td>0.050</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
</tr>
<tr>
<td>( c_2 ) (lagged oil price change)</td>
<td>0.027</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Source: This table reproduces table 1 in Roberts (1995). Reprinted with permission from Ohio State University Press.

Notes: Columns 1, 2, 4, and 5 use specification (2). Columns 3 and 6 use specification (3)–(4).

\[17\] The one specification that yields a dominant role for rational expectations is GMM with standard weights. This is the same specification as that used in Nunes (2010), the sole paper arguing that rational expectations in the NKPC outperform survey measures. Fuhrer (2012) argues that this simple GMM approach likely suffers from weak instruments that are unable to identify the effects of both lagged inflation and inflation expectations on current inflation.
### TABLE 4
PHILLIPS CURVE ESTIMATES, FUHRER, OLIVEI, AND TOOTELL (2012)

#### Panel A. The US Phillips Curve with survey expectations

\[
\pi_t = a \pi_{t-1}^y + (1-a) \pi_t + c_s + c_0 \quad \text{and} \quad \pi_{t}^y = \sum_{i=0}^{4} d_i \pi_{t-i} + \sum_{j=0}^{2} e_j y_{t-j} + e_0
\]

<table>
<thead>
<tr>
<th>Estimate</th>
<th>SE</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Survey expectation (a)</td>
<td>0.70</td>
<td>0.12</td>
</tr>
<tr>
<td>Lagged inflation (1 - a)</td>
<td>0.30</td>
<td>0.12</td>
</tr>
<tr>
<td>Marginal costs (c)</td>
<td>0.053</td>
<td>0.028</td>
</tr>
<tr>
<td>Intercept (c_0)</td>
<td>-0.22</td>
<td>0.093</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.69</td>
<td></td>
</tr>
<tr>
<td>Standard error of the regression: 0.58</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Current and lagged inflation ($\sum d$)
Current and lagged output gap ($\sum e$)
Intercept ($e_0$)
$R^2 = 0.84$
Standard error of the regression: 0.26

#### Panel B. A hybrid Phillips Curve model for the United States

\[
\pi_t = a E_t \pi_{t+1} + b \pi_{t-1}^y + (1-a-b) \pi_{t-1} + c_s + c_0 \quad \text{and} \quad \pi_{t}^y = \sum_{i=0}^{2} d_i \pi_{t-i} + \sum_{j=0}^{2} e_j y_{t-j} + e_0
\]

\[
\begin{align*}
\text{and } & s_t = \sum_{i=1}^{2} B_i X_{t-i} \quad \text{and} \quad y_t = \sum_{i=1}^{2} \Gamma_i X_{t-i}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Estimate</th>
<th>SE</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Rational expectation (a)</td>
<td>0.00</td>
<td>0.018</td>
</tr>
<tr>
<td>Survey expectation (b)</td>
<td>0.74</td>
<td>0.13</td>
</tr>
<tr>
<td>Lagged inflation (1 - a - b)</td>
<td>0.26</td>
<td>0.034</td>
</tr>
<tr>
<td>Marginal costs (c)</td>
<td>0.048</td>
<td>0.039</td>
</tr>
<tr>
<td>Intercept (c_0)</td>
<td>-0.19</td>
<td>0.060</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td>log-likelihood: 254.94</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Current and lagged inflation ($\sum d$)
Current and lagged output gap ($\sum e$)
Intercept ($e_0$)

Source: This table reproduces table 5 in Fuhrer, Olivei, and Tootell (2012). Reproduced with permission from Wiley.

Notes: The dependent variable is core CPI inflation. The data frequency is quarterly, and the sample period is 1990:I to 2010:II. The parameters in the top panel are estimated via ordinary least squares, and standard errors are corrected for heteroskedasticity and autocorrelation. The parameters in the bottom panel are estimated via full information maximum likelihood, and the standard errors are computed via a BHHH algorithm. Panel B jointly estimates the VAR coefficients $B$ and $\Gamma$ with the other parameters. $X$ is the vector of variables in the two-lag VAR (inflation, output gap, marginal costs).

- $a$ Indicates $t$-statistic for the sum of coefficients.
- $b$ Indicates $p$-value for joint significance of contemporaneous and lagged values.
and whether a lagged inflation or the nominal interest rate should be included. At the same time, the listed specifications have a number of common elements: expected inflation, a forcing term like output gap, etc. Furthermore, using average values of inflation expectations reported in a survey can be appropriate under certain conditions.

Specifically, Adam and Padula (2011), whose derivation is outlined in the appendix, show that the Phillips curve given in equation (1) can be derived with expectations other than FIRE. Let firm \( i \in [0, 1] \) have subjective expectations \( F_{i,t} [x] \) for variable \( x \) in another standard New Keynesian model with Calvo pricing. Optimal price setting requires that the reset price \( p_{i,t}^* \) for firm \( i \) obeys

\[
(5) \quad p_{i,t}^* = (1 - \theta \beta) \sum_{j=0}^{\infty} (\theta \beta)^j F_{i,t} [mc_{i,t+j}],
\]

where \( \theta \) is the probability that a firm is unable to adjust its price in a given period, \( \beta \) is a time discount factor, and \( mc_n \) is the nominal marginal cost. Let \( \bar{F}_t [x] \equiv \int_0^1 F_{i,t} [x] \, di \) be the average expectation for variable \( x \) in the economy. Then, if agents are unable to predict revisions in their own or other agents’ forecasts (condition 1 in Adam and Padula 2011), one has

\[
F_{i,t} [mc_{i,t+s}] - F_{j,t} [mc_{i,t+s}] = 0
\]

\( \forall i, j, \) and \( s > 0, \)

and, after standard steps in the derivation of the New Keynesian FIRE Phillips curve (see Galí 2008), one obtains

\[
(6) \quad \pi_t = \beta \bar{F}_t [\pi_{t+1}] + \frac{(1 - \theta)(1 - \theta \beta)}{\theta} mc_t.
\]

Following the standard derivation, one can replace the marginal cost with output gap

<table>
<thead>
<tr>
<th>Information structure</th>
<th>Phillips curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full-information rational expectations with time-dependent pricing (Calvo 1983)</td>
<td>( \pi_t = \beta E_t[\pi_{t+1}] + b_1 X_t )</td>
</tr>
<tr>
<td>Full-information rational expectations with time-dependent pricing (Gertler and Leahy 2008)</td>
<td>( \pi_t = \beta E_t[\pi_{t+1}] + b_2 X_t )</td>
</tr>
<tr>
<td>Sticky prices and backwards rule-of-thumb firms (Galí and Gertler 1999)</td>
<td>( \pi_t = (1 - b_4) \pi_{t-1} + b_4 E_t[\pi_{t+1}] + b_3 X_t )</td>
</tr>
<tr>
<td>Sticky information (Mankiw and Reis 2002)</td>
<td>( \pi_t = \bar{E}<em>{t-1}[\pi_t] + b_5 \bar{E}</em>{t-1}[\Delta y_t] + b_6 y_t )</td>
</tr>
<tr>
<td>Adaptive learning (Milani 2005)</td>
<td>( \pi_t = \hat{E}<em>{t-1}\pi</em>{t-1} + b_7 X_t )</td>
</tr>
<tr>
<td>Rational inattention (Afrouzi and Yang 2016)</td>
<td>( \pi_t = \bar{E}<em>{t-1}[\pi_t] + \bar{E}</em>{t-1}[\Delta y_t] + b_8 y_t + b_9(\bar{E}<em>t[\pi</em>{t+1} + \Delta y_{t+1}] - \pi_t) )</td>
</tr>
</tbody>
</table>

Notes: This table shows Phillips curves derived under various assumptions about how economic agents form expectations. Coefficients “\( b \)” vary across specifications and depend on structural parameters and details of information structure. \( \pi_t \) is inflation, \( X_t \) is output gap, \( y_t \) is output, \( i_t \) is the nominal interest rate. \( E_t \) denotes full-information rational expectations given information available at time \( t \). \( \bar{E}_t \) denotes average expectations (not necessarily full information, but rationality is preserved) across agents. \( \hat{E}_t \) denotes expectations when agents use information up to period \( t \) to learn about structural parameters in the economy.

18 There are also a number of hybrid models which generate similar specifications. For example, Dupor, Kitamura, and Tsuruga (2010) combine sticky prices and sticky information and derive the associated Phillips curve.
X_t and, thus, arrive at a specification that resembles specification (1). “Condition 1” is essential for applying LIE to equation (5).

To the extent that real-time measures of expectations may not satisfy this requirement, the estimation of the Phillips curve with the cross-sectional average of survey-based inflation expectations may no longer be micro-founded. Despite a large literature using subjective expectations in the estimation of the Phillips curve, to our knowledge, the current literature has not evaluated whether the cross-sectional average operator satisfies LIE or if condition 1 holds.

There is, however, a straightforward test of whether expectations in surveys satisfy the LIE, at least when applied to the NKPC. Recall that before applying the LIE, one operates with the following expectation:

$$\pi_t = (1 - \theta)(1 - \theta\beta) \sum_{j=0}^{\infty} (\theta\beta)^j F_t X_{t+j}$$

$$+ (1 - \theta) \sum_{j=0}^{\infty} (\theta\beta)^j F_t \pi_{t+j},$$

where $F_t Y_{t+j}$ denotes date-$t$ forecast for variable Y at time $t+j$. The LIE allows us to collapse equation (7) to the Phillips curve:

$$\pi_t = \beta F_t \pi_{t+1} + \lambda X_t.$$

If the law of iterated expectations holds, then inflation forecast $F_t \pi_{t+1}$ is a sufficient statistic for forecasts of macroeconomic variables in periods $t+1, t+2, \ldots$. As a result, adding future output gaps or forecasts of longer-horizon inflation expectations to equation (8) should not be significant in estimation.

We use this insight for two surveys where forecasts are available for multiple horizons: MSC and SPF. Table 6 demonstrates that the additional future output gap and inflation terms are not statistically significant and generally only marginally increase the fit relative to specification (8). The results fail to detect deviations from LIE and allow us to use mean survey expectations in the estimation of the NKPC.

Recap.—Prior work has often replaced the expectations term in the NKPC with non-FIRE, survey-based expectations; however, one may be concerned that non-FIRE expectations may lead to an entirely different specification. Table 5 shows how different assumptions change the specification of the Phillips curve, but there are many commonalities. Furthermore, Adam and Padula (2011) demonstrate that if agents are unable to predict revisions in their own or other agents’ forecasts, the LIE can be applied, and a Phillips curve can be derived resembling specification (1). We present evidence that in the context of the NKPC, surveys appear to satisfy LIE.

5.5 Challenges in Using Market and Survey Measures of Expectations

What measures of inflation expectations are available in the United States, which should we use, and what are the potential challenges with each? There are several surveys of US inflation expectations, varying in composition and construction, including Blue Chip Economic Indicators, Business Inflation Expectations (Atlanta Fed), the Federal Reserve’s Greenbook forecasts, FOMC member forecasts, the Livingston Survey, the MSC, Survey of Consumer Expectations (NY Fed), Consensus Economics forecasts, and the SPF. Table 7 provides a summary of key characteristics of the surveys. Financial markets can also provide real-time forecasts as an alternative to surveys.

A key limitation of the currently available market and survey measures is that none provides a direct historical measure of firms’ inflation expectations, which are the relevant ones from the perspective of estimating
Phillips curves.\textsuperscript{19} Indeed, the NKPC is derived from the firm’s optimization problem, and the expectation term in the canonical relationship is therefore that of the firm.

\textsuperscript{19}The Federal Reserve of Atlanta does conduct a survey of firm expectations. Unfortunately, it only surveys businesses in the sixth district and only began in 2011. Coibion, Gorodnichenko, and Kumar (2015) conduct a firm expectation survey; however, it was taken in New Zealand and is a single cross-section. The New Zealand survey cannot be used to estimate the Phillips curve, a time series object.

\textsuperscript{20}Research on topics that require firm expectations of variables other than inflation have been able to utilize surveys. For example, Gennaioli, Ma, and Shleifer (2015) use Duke University’s quarterly survey of chief financial officers’ expectations of earning growth to document their effect on firm investment plans.

Market-based measures.—There are two primary approaches for deducing inflation expectations from financial markets. The first uses the difference in yields between Treasury inflation-protected securities (TIPS) and nominal Treasuries of the same maturity, and the second uses inflation swap data. A notable strength of using these market-based measures is their high-frequency nature that cannot be matched by surveys. However,
### TABLE 7
**US Surveys of Inflation Expectations**

<table>
<thead>
<tr>
<th>Survey respondents</th>
<th>Survey of</th>
<th>Survey of</th>
<th>Livingston</th>
<th>Business</th>
<th>Survey of</th>
<th>Blue chip</th>
<th>Federal Reserve</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>consumers</td>
<td>professional forecasters</td>
<td>survey of consumers</td>
<td>inflation expectations</td>
<td>consumer expectations</td>
<td>economic indicators</td>
<td>greenbook</td>
</tr>
<tr>
<td>Number of surveys collected</td>
<td>500</td>
<td>Varies by survey, ≈ 40 in recent surveys.</td>
<td>Varies by survey, ≈ 30 in recent surveys.</td>
<td>300</td>
<td>1,300</td>
<td>50</td>
<td>1</td>
</tr>
<tr>
<td>Price inflation measure(s)</td>
<td>CPI, core CPI, GDP price index, PCE, core PCE</td>
<td>CPI, PPI</td>
<td>Each firm’s unit costs</td>
<td>Inflation</td>
<td>CPI, GDP price index, PPI, PCE</td>
<td>GDP deflator, CPI, core CPI, PCE</td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>Monthly</td>
<td>Quarterly</td>
<td>Semi-annually</td>
<td>Monthly</td>
<td>Monthly</td>
<td>Monthly</td>
<td>8–12 per year</td>
</tr>
<tr>
<td>Survey conducted by</td>
<td>Survey Research Center at the University of Michigan</td>
<td>Federal Reserve of Philadelphia</td>
<td>Federal Reserve of Atlanta</td>
<td>Federal Reserve of New York</td>
<td>Aspen publishers</td>
<td>Federal Reserve Board of Governors</td>
<td></td>
</tr>
<tr>
<td>Short term forecast</td>
<td>Next 1 year (annual)</td>
<td>Current and next 4 quarters (Q/Q annualized).</td>
<td>Current month (value). Next 6 and 12 months (value). Next 3 years (value, annually).</td>
<td>Next 1 year (annual)</td>
<td>Next 1 year (annual)</td>
<td>Current year and next year (annually)</td>
<td>Current and next 8 quarters (Q/Q annualized)</td>
</tr>
<tr>
<td>Long term forecast</td>
<td>Next 5 to 10 years (annual average)</td>
<td>Next 5 and 10 years (annual average)</td>
<td>Next 10 years (annual average)</td>
<td>Next 5 to 10 years (annual average)</td>
<td>Rate 3 years from now (rate from now +2 years to now +3 years)</td>
<td>Next 7 years (annually). Next 5 years (annual average)</td>
<td>N</td>
</tr>
<tr>
<td>Point prediction</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Probability distribution</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>N</td>
</tr>
</tbody>
</table>

Notes: The monthly MSC began in 1978; however, earlier data is available at lower frequencies. The SPF has changed their questionnaire over the years; the status of the survey as of 2014.1 is reported in the table. The Sixth District in column 4 includes Alabama, Florida, Georgia, and parts of Louisiana, Mississippi, and Tennessee. Greenbook forecasts for CPI became reported in 1979. Other inflation measures were introduced after that.
there are shortcomings of both bond-market and swap-market inflation expectation measures, taken in turn below.

First, the break-even inflation rate, or the difference between yields on nominal and real US debt of similar maturities, is often quoted as a measure of inflation expectations. However, the break-even inflation rate is a measure of inflation expectations confounded with the inflation risk premium and the difference in liquidity premiums between TIPS and nominal debt (e.g., Christensen, Lopez, and Rudebusch 2010; D’Amico, Kim, and Wei 2014; Gürkaynak, Sack, and Wright 2010), thus posing a dilemma for researchers who want to use a TIPS-based measure of inflation expectations.21 Furthermore, TIPS began trading in 1997. Analyses where longer horizons are needed will be constrained using TIPS-based measures. Many surveys, in contrast, provide a longer history of inflation expectations (see table 7).

Second, inflation swap data has been used to determine expected inflation rates. In an inflation swap, two counter-parties exchange payments. One party pays fixed payments, while the other pays variable payments that depend on the realized inflation rate. For both parties to be willing to engage in the inflation swap, the fixed payment must be roughly the amount of expected inflation. Like when using bond-market data, one must account for inflation risk premium, posing some difficulty. Furthermore the inflation swap market is relatively new with meaningful trading volumes beginning only in 2003.

Professional forecasts.—While expectations of professional forecasters have been the primary type of survey data employed, these surveys may suffer from respondents not revealing their true beliefs for a variety of reasons. Papers have demonstrated this potential both theoretically and empirically. Ottaviani and Sørensen (2006), for example, propose and analyze a cheap talk game in the context of forecasters. The primary finding is that truth telling could be an unlikely equilibrium. Laster, Bennett, and Geoum (1999) suggest a model where forecasters are fully knowledgeable about the true probability distribution of outcomes and the forecaster who makes the best forecast in a given period gains publicity for his firm and is rewarded. Forecasters in this model are willing to compromise accuracy to gain publicity, thus the distribution of the forecasts will reflect the true probability distribution function as well as this trade-off. Empirically, forecaster deviations from consensus, in the Blue Chip survey, are correlated with the type of firm the forecaster works for (i.e., nonfinancial corporations may value accuracy for planning, but advisory firms may value publicity to attract clients). Ehrbeck and Waldmann (1996) show that when making forecasts is a repeated game, the pattern of forecasts can reveal private information about the forecaster, so that rational forecasters will choose to compromise between minimizing errors and imitating the patterns of more able forecasters.

Consumer expectations.—Another approach to deal with the absence of direct measures of firm expectations is to use consumer expectations in their place. There are several reasons why one might think consumer expectations are likely to be a better proxy for firm expectations than professional forecasts.

First, incentives to provide untruthful forecasts for profit-based reasons are smaller for consumers. Armantier et al. (2015) conduct an experiment to assess if consumer surveys suffer from cheap talk and if consumers act on their inflation beliefs. They find that a respondent’s inflation expectation

21 Another issue in the use TIPS inflation expectations, often ignored, arises from TIPS payments being tied to the CPI three months prior to the payment date. TIPS are thus not fully protected from inflation.
gathered in a survey strongly correlates with the respondent’s response in a financially incentivized experiment. Arnold, Dräger, and Fritsche (2014) similarly find, using a German survey of households, that differences in households’ beliefs about inflation expectations parlay into their portfolio decisions.

Second, consumer forecasts appear to fit the Phillips curve better than professional forecasts, both before the Great Recession and after. Coibion and Gorodnichenko (2015b) document this feature of the data. Hence, consumers’ inflation expectations may be a better historical proxy for firms’ expectations than professional forecasts.

Third, surveys of firms’ inflation forecasts from New Zealand indicate that first and second moments of firms’ forecasts are much more aligned with those of households than professionals. For example, Coibion, Gorodnichenko and Kumar (2015) find that the average forecast of firms in the fourth quarter of 2013 was over 5 percent, much closer to the average forecast of around 4 percent for consumers than the average forecast of 1.5 percent from professionals. Similarly, there was tremendous disagreement among firms about future inflation, a well-noted characteristic of consumer forecasts that stands in sharp contrast to the very limited disagreement observed among professionals. Thus, along both metrics, firm forecasts do seem to resemble household forecasts much more closely than those of professional forecasters. Kumar et al. (2015) further document that most firm managers rely primarily on their personal shopping experience to inform them about price changes and use their inflation expectations primarily for their personal decisions, providing an additional justification for why the forecasts of firm managers so closely resemble those of households.

Finally, some have suggested household expectations may inform firm price-setting behavior, thus household expectations may play an even more direct role in the Phillips curve. The seminal behavioral findings in Kahneman, Knetsch, and Thaler (1986) have motivated this literature. They find in a survey that consumers regard it as unfair for firms to raise prices in response to shifts in demand, but acceptable to raise prices in response to increasing costs. Additionally, consumers were willing to punish firms for unfairness (e.g., drive five extra minutes to another drugstore if the closest one had increased prices when its competitor was temporarily forced to close). Building on these behavioral findings, Rotemberg (2005, 2010, 2011) and Eyster, Madarasz, and Michaillat (2015) develop theoretical models where consumer perceptions of firm fairness and feelings of regret arise from paying more than expected or in excess of marginal cost. Firms thus engage in price-setting behavior so as to not upset the firm’s consumers and, as a result, consumer expectations may be used in price setting.

Despite the aforementioned reasons why consumer expectations may be a good proxy for firm expectations, they are not firm expectations and possible shortcomings remain. Sensitivity to survey language appears to differ between households and firm managers. Households have been shown to have higher and more dispersed expectations when asked about “overall price changes” rather than “inflation rates” (e.g., Bruine de Bruin et al. 2012 and Dräger and Fritsche 2013). This sensitivity to language is not observed among managers (Coibion, Gorodnichenko, and Kumar 2015). Furthermore, a firm may be more incentivized to track economic developments and have informed inflation expectations.

22Alternatively, one can follow Carroll (2003). In his model, a consumer (or a firm) has a constant probability of updating his inflation forecast each period toward the views of professional forecasters.
Additional limitations of survey data.—
There are several other concerns that arise with the use of survey data that will need to be addressed in the literature going forward. One is how to reliably aggregate expectations, if at all. As Figlewski and Wachtel (1981) highlight, aggregation of expectations rather than using the full set of individual expectations may lead to biased results in tests of rationality in survey data. However, when testing models, most models imply that mean expectations are the relevant metric when focusing on inflation or other macroeconomic dynamics. But it is common to rely on median measures of expectations in survey data (e.g., Fuhrer and Olivei 2010, Malmendier and Nagel 2016, and Trehan 2015). The latter can introduce more stability when the composition of respondents is changing over time, but can also mask significant variation over time when respondents frequently provide integer responses (such as in the MSC).

A second consideration is whether to treat survey expectations as given (predetermined) or not. Zhang, Osborn, and Kim (2009) suggest that survey expectations may contain information correlated with the contemporaneous error term if forecasts are collected in the middle of the current period as done, for example, with SPF forecasts. If survey measures are believed to be endogenous with respect to contemporaneous economic conditions, an instrumental variables approach could be taken. Finding valid and strong instruments for survey expectations without imposing ad hoc assumptions on dynamics is a challenge, as using instrumental variables requires assumptions about how inflation expectations are formed, an area of active research and heated debates. Mavroeidis, Plagborg-Møller, and Stock (2014) contend that, when using survey measures, endogeneity should be considered. Others have assumed exogenous survey expectations and utilized ordinary least squares (OLS) in estimation of the Phillips curve (e.g., Roberts 1995, Rudebusch 2002, and Adam and Padula 2011).

A third issue is whether we should use point predictions and/or subjective probability distributions. Historically, surveys have tended to collect point predictions; however, subjective probability distributions are increasingly being elicited (Armantier et al. 2013). Point predictions of an agent are often in line with the central tendencies of his or her subjective probability distribution (e.g., Engelberg, Manski, and Williams 2009 and Coibion, Gorodnichenko, and Kumar 2015). However, when deviations arise they appear systematic. For example, point predictions may be rounded to the nearest “five” (Binder 2015) or be more optimistic than the subjective probability distribution would imply (Engelberg, Manski, and Williams 2009).

Recap.—The United States and many other countries currently lack a long historical measure of firm inflation expectations. As a result, researchers who wish to measure expectations and estimate the Phillips curve must rely on other measures of expectations. There are two types of expectation measures available: market based and survey based. Market measures of expectations offer high-frequency data, but suffer from confounding factors and a relatively short history. Survey measures with long time series are available for professional forecasters and consumers. Professional forecasts offer long time series, but professionals may not reveal their true beliefs. Consumer expectations do not suffer from cheap talk, and are empirically similar to firm expectations in New Zealand; however, they suffer from sensitivity to survey language and consumers may not have a strong incentive to track economic developments. These market-based or survey-based measures can be used as a direct measure of firm expectations, or one can attempt to infer firm expectations by assuming that either (i) firms have a constant
probability of updating their beliefs toward professionals, or (ii) consumer expectations influence firm price-setting behavior.

5.6 Illustration

In this section, we attempt to synthesize previous studies to highlight the differences arising from using FIRE- and survey-based inflation expectations in the Phillips curve. Specifically, we investigate the stability of the Phillips curve across various measures of inflation expectations, run a series of horse-race regressions to identify inflation forecast measures with the best predictive power, and explore how using various measures of inflation expectations translates in matching the dynamics of inflation during the Great Recession and its aftermath. These exercises are not meant to go over an exhaustive list of specifications considered in the literature. Instead, the objective is to illustrate the effect of using non-FIRE expectations in the standard framework given by equation (1).

In the first exercise, we run the standard forward-looking, expectations-augmented Phillips curve:

$$\pi_t = a_0 + a_1 E_t \pi_{t+1} + b_1 (U_E t - U_E t^N) + \text{error},$$

where $\pi_t$ is the actual quarter-on-quarter inflation rate (CPI, annualized), $E_t \pi_{t+1}$ is one-year-ahead inflation forecast (CPI), $U_E t$ is the unemployment rate, and $U_E t^N$ is the natural rate of unemployment (CBO’s NAIRU). Table 8 presents OLS estimates of specification (9) for various measures of inflation expectations over different periods. When we use the mean inflation forecast from the MSC (panel A), we observe a stable, strong relationship between inflation and unemployment. A one percentage point deviation of the unemployment rate above NAIRU is associated with a 0.2 percentage point decline in inflation. While $R^2$ is high for the full sample and early part of the sample (1978–89), the magnitude of $R^2$ gradually declines with time. This reduced predictability of inflation has been documented in previous studies (e.g., Stock and Watson 2007) for a variety of models and, hence, it should not be interpreted as increasing obsolescence of the Phillips curve relative to other forecasting methods. When we use the Binder (2015) approach to identify households who are more confident in their beliefs about inflation and are more likely to represent beliefs of firm managers (panel B), the sensitivity of inflation to the unemployment gap varies somewhat but remains significant for the 2000–2014 period.

Panel C presents results for the case of naive expectations as in Atkeson and Ohanian (2001), i.e., treating expectations of future inflation as equal to average inflation over the previous four quarters. Apart from large variation in the estimated slope of the Phillips curve from $−0.747$ for 1978–89 to $−0.216$ for the full sample, we observe that the estimated coefficient on expected inflation turns negative for 2000–2014. Furthermore, the Phillips curve with naive expectations has little predictive power during this recent period ($R^2 = 0.075$). In general, this version of the Phillips curve has an $R^2$ consistently below that in panel A. These results suggest that modeling inflation expectations as backward-looking can lead to “puzzles” and inferior forecasting properties of the Phillips curve.

In panels D, E, and F, we use inflation projections from the SPF, Greenbooks (reports prepared by the Fed staff for FOMC meetings), and financial markets (measured according to the method of Haubrich, Pennacchi, and Ritchken 2012). Arguably, these projections are closer to FIRE than expectations of households. In a pattern common across these projections, the estimated slope of the Phillips curve gets smaller
### Table 8: Stability of the Phillips Curve

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
</tbody>
</table>


$UE gap_t$: -0.229  -0.233  -0.210  -0.249  0.989  
(0.087)  (0.128)  (0.278)  (0.110)  
$E_t \pi_{t+1}$: 1.432  1.500  1.388  0.828  
(0.072)  (0.090)  (0.387)  (0.184)  
Observations: 146  48  40  58  
$R^2$: 0.773  0.834  0.516  0.218  


$UE gap_t$: -0.333  -0.568  -0.145  -0.237  0.105  
(0.102)  (0.130)  (0.217)  (0.112)  
$E_t \pi_{t+1}$: 1.592  1.519  2.053  1.495  
(0.068)  (0.093)  (0.474)  (0.276)  
Observations: 145  48  40  57  
$R^2$: 0.764  0.814  0.534  0.234  


$UE gap_t$: -0.216  -0.747  -0.217  -0.370  0.348  
(0.158)  (0.264)  (0.238)  (0.155)  
$E_t \pi_{t+1}$: 0.818  0.856  0.706  -0.187  
(0.111)  (0.097)  (0.217)  (0.198)  
Observations: 146  48  40  58  
$R^2$: 0.598  0.709  0.328  0.075  


$UE gap_t$: -0.177  -0.374  -0.462  -0.157  0.520  
(0.103)  (0.122)  (0.226)  (0.192)  
$E_t \pi_{t+1}$: 0.729  1.182  1.731  0.732  
(0.116)  (0.222)  (0.336)  (1.188)  
Observations: 132  34  40  39  
$R^2$: 0.265  0.348  0.520  0.070  


$UE gap_t$: -0.348  -0.495  -0.365  -0.174  0.185  
(0.164)  (0.149)  (0.267)  (0.098)  
$E_t \pi_{t+1}$: 0.954  1.265  1.459  0.371  
(0.128)  (0.125)  (0.338)  (0.506)  
Observations: 120  41  40  39  
$R^2$: 0.592  0.752  0.498  0.038  


$UE gap_t$: -0.141  -0.492  -0.063  -0.041  0.036  
(0.100)  (0.135)  (0.205)  (0.184)  
$E_t \pi_{t+1}$: 0.613  1.105  1.626  1.068  
(0.105)  (0.312)  (0.310)  (0.379)  
Observations: 130  32  40  58  
$R^2$: 0.205  0.216  0.460  0.125  

Notes: The dependent variable in specification (9) is the quarterly inflation rate (CPI, annualized). In all panels (except panel C), $E_t \pi_{t+1}$ is the one-year ahead inflation forecast (mean). In panel C, $E_t \pi_{t+1}$ is the average inflation rate over the previous four quarters. The top row indicates estimation samples. $UE gap_t$ is the difference between the actual unemployment rate and the CBO’s NAIRU. All data are final vintage. Newey–West robust standard errors (five lags) are in parentheses. The last column reports p-values for the null hypothesis that the slopes are equal over the time periods listed in columns 2–4. The estimation sample excludes 2008:IV, which is an outlier in the data.
over time and becomes insignificantly different from zero for the 2000–2014 period. This pattern is consistent with previous studies documenting the flattening of the Phillips curve with FIRE. Interestingly, using MSC inflation expectations tends to yield a higher $R^2$ than what one could obtain with these presumably more rational expectations.

For the second exercise, we augment specification (9) to include expectations from multiple sources:

$$\pi_t = a_0 + a_1 E^X_t \pi_{t+1} + a_2 E^Y_t \pi_{t+1} + b_1 (UE_t - UE^N_t) + \text{error},$$

where $E^X_t \pi_{t+1}$ and $E^Y_t \pi_{t+1}$ are inflation expectations from sources X and Y. This exercise can help identify the type of agents whose expectations have the most predictive power in the Phillips curve framework. Consistent with results in table 8, we find that household expectations dominate expectations from other sources (see table 9). For example, column 2 in table 9 illustrates that adding naïve expectations does not change the estimate of the slope of the Phillips curve or the sensitivity of inflation to inflation expectations. When we control for expectations of SPF, Greenbooks, or financial markets (columns 3 through 5), the coefficient on household expectations is reduced, but it remains large and the slope of the Phillips curve is stable. Similarly, controlling for long-term inflation expectations (column 6) does not alter results materially. Finally, household expectations continue to be a strong predictor of inflation even when we include all sources of expectations in specification (10). In short, household expectations appear to play a special role in making the Phillips curve stable and inflation predictable.23

In the third exercise, we estimate Phillips curves with various sources of expectations on the pre–Great Recession period and then use the estimated relationships to predict inflation for the 2009–11 period, i.e., the “missing disinflation” period. Table 10 summarizes our findings. Although households expected inflation to be higher than other agents did, the Phillips curve based on household expectations has the second smallest absolute mean forecast error. The smallest absolute mean forecast error was found using Binder’s (2015) household expectation measure. In contrast, professional forecasters, financial markets, and the staff of the Federal Reserve had larger absolute mean forecast errors and under-predicted inflation.24

These results suggest that using household expectations appears to yield a stable relationship between nominal and real variables, so that the Phillips curve is useful even in times of crisis. Why household expectations work better in the context of the Phillips curve than other sources of expectations is a fruitful area for future research. Preliminary evidence (e.g., Kumar et al. 2015) suggests that ordinary consumers and firm managers are remarkably similar in how they form and use their inflation expectations. Consumers and firms may similarly depart from employing FIRE, whereas professional forecasters, economists, and financial markets may be closer to FIRE. To be clear, not all departures from FIRE will be helpful, but rather departures from FIRE that are consistent with firms’ expectation formation.

23The dominance of household survey measures remains after accounting for the direct effect of oil prices on inflation. Higher oil prices could have a direct effect on inflation due to higher input costs and an indirect effect through inflation expectations. Estimates suggest the direct, short-run effect of oil prices on inflation is approximately 1–2 percent, while the direct long-run effect is close to 4 percent. Controlling for the direct effect of the price of oil, household survey measures remain dominant over other survey measures.

24Because Greenbook projections are available with a five-year delay, we have only four observations to evaluate the Phillips curve based on Greenbook expectations.
<table>
<thead>
<tr>
<th>Dep. var.: $\pi_t$</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{UEGap}_t$</td>
<td>-0.229</td>
<td>-0.227</td>
<td>-0.222</td>
<td>-0.180</td>
<td>-0.207</td>
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<td>0.653</td>
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Notes: The dependent variable in specification (10) is the quarterly inflation rate (CPI, annualized). For all measures of expected inflation (except the naïve expectations and the MSC five-year-ahead case), $E_t \pi_{t+1}$ is the one-year-ahead inflation forecast (mean). For the naïve expectations case, $E_t \pi_{t+1}$ is the average inflation rate over the previous four quarters. All regressions are estimated on the largest possible sample that covers 1978–2014. The number of observations varies across columns because some measures of expectations are available for fewer periods. $\text{UEGap}_t$ is the difference between the actual unemployment rate and the CBO’s NAIRU. All data are final vintage. The estimation sample excludes 2008:IV, which is an outlier in the data. Newey–West robust standard errors (five lags) are in parentheses.
Additional results presented in the appendix assess the robustness of our findings. Using output gap, constrained specification, or lags of employment gap does not affect results materially. Likewise, using current and lagged changes of oil prices as additional controls does not influence results. The greatest sensitivity is to which measure of inflation one uses as the dependent variable in the Phillips curve. With core CPI, personal consumption expenditures, or GDP deflator, we fail to find a stable, downward-sloping Phillips curve. This finding suggests that consumers may form their expectations based on CPI, rather than other price indexes, and therefore MSC can provide an inappropriate measure of expected inflation for Phillips curves based on price indexes other than CPI. This observation underscores the need for further research on the matter as well as better measurement of inflation expectations.

Recap: Relative to a number of popular alternative measures of inflation expectations (lagged inflation, professional surveys, Greenbook expectations, and the Cleveland Fed expectations), consumer expectations yield the most stable Phillips curve (CPI-based) and provide the best fit during recent years. In a horse race of inflation expectations, consumer expectations remain a strong predictor of inflation. Furthermore, a Phillips curve based on consumer expectations has a lower absolute forecast error than NKPCs based on other expectation measures. Overall, the evidence suggests that survey-based consumer expectations can play an important role in the Phillips curve.

6. Conclusion

FIRE is a useful theoretical framework that has allowed economists across fields to incorporate expectations into models in a meaningful manner while maintaining tractability. In a prominent application of FIRE, the NKPC connecting real and nominal variables has emerged as a cornerstone of mainstream macroeconomic models used in policy and research.
However, pronounced deviations from FIRE in the short run have now been well-documented in empirical work. These findings should give one pause before relying on the commonly used FIRE assumption and highlight the need for using alternative frameworks (e.g., sticky information, noisy information, bounded rationality, models of learning) in describing how expectations are formed, since these alternatives can significantly improve the ability of macroeconomic models to fit the data and change policy prescriptions.

To illustrate the potential of this approach, we reviewed recent work that incorporates the real-time expectations of economic agents as observed in surveys into expectations-augmented Phillips curves. This approach can address a number of otherwise puzzling features of FIRE Phillips curves (e.g., the need for ad hoc lags, instability, missing disinflation during the Great Recession, sensitivity to the slack variable used). This supports the notion that agents may not be rational or face pervasive information rigidities, and accounting for these can help reconcile theory and data.

But the use of survey data does face a number of shortcomings. First and most practically, we lack direct empirical evidence on the real-time beliefs of firms, those agents whose expectations play a central role in price-setting, hiring, and investment decisions. Second, there are many possible explanations for the observed deviations from FIRE. Distinguishing between these, and identifying a tractable model for the expectations formation process, should therefore be a prominent area of future research.

While our empirical work focused exclusively on the role of real-time expectations data in the Phillips curve, the issue is much broader. For example, controlling for the real-time expectations of monetary policy makers plays an important role in the identification of monetary policy shocks (Romer and Romer 2004) and the response function of central banks (Coibion and Gorodnichenko 2011). Crump et al. (2015) find that controlling for the real-time expectations of households helps in the estimation of consumption Euler decisions. However, little attention has yet been devoted to how real-time expectations might matter along other dimensions, such as wage bargaining between employees and firms, or the employment and investment decisions of firms.
### APPENDIX TABLE 1
**Horserace Regressions, Output Gap**

<table>
<thead>
<tr>
<th>Dep. var.: $\pi_t$</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Output\ Gap_t$</td>
<td>0.140</td>
<td>0.139</td>
<td>0.130</td>
<td>0.098</td>
<td>0.119</td>
<td>0.157</td>
<td>0.126</td>
<td>0.086</td>
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<tr>
<td></td>
<td>(0.051)</td>
<td>(0.050)</td>
<td>(0.052)</td>
<td>(0.076)</td>
<td>(0.049)</td>
<td>(0.062)</td>
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<td>(0.049)</td>
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<tr>
<td>$Expected\ inflation, E_t \pi_{t+1}$</td>
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<td>(0.208)</td>
<td>(0.167)</td>
<td>(0.254)</td>
<td>(0.240)</td>
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<td>−0.474</td>
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<td>−0.166</td>
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<td></td>
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<td>−0.153</td>
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<td>(0.151)</td>
<td>(0.314)</td>
<td>(0.325)</td>
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</table>

| Observations       | 146  | 146  | 132  | 120  | 130  | 118  | 113  | 94   |
| $R^2$              | 0.775| 0.775| 0.387| 0.721| 0.339| 0.658| 0.372| 0.403|

**Notes:** The dependent variable in specification (10) is the quarterly inflation rate (CPI, annualized). For all measures of expected inflation (except the naïve expectations and the MSC five-year-ahead case), $E_t \pi_{t+1}$ is the one-year-ahead inflation forecast (mean). For the naïve expectations case, $E_t \pi_{t+1}$ is the average inflation rate over the previous four quarters. All regressions are estimated on the largest possible sample that covers 1978–2014. The number of observations varies across columns because some measures of expectations are available for fewer periods. $UEGa_t$ is the difference between the actual output rate and the CBO’s potential output. All data are final vintage. The estimation sample excludes 2008:IV, which is an outlier in the data. Newey–West robust standard errors (five lags) are in parentheses.
### APPENDIX TABLE 2

**HORSE RACE REGRESSIONS, TREND-INFLATION SPECIFICATION**

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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
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<td>( \text{UEGap}_t )</td>
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<td>-0.344</td>
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<td>-0.077</td>
<td>-0.177</td>
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<td>(0.190)</td>
<td>(0.094)</td>
<td>(0.245)</td>
<td>(0.302)</td>
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<td>1.246</td>
<td>1.568</td>
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<td>(0.194)</td>
<td>(0.139)</td>
<td>(0.209)</td>
<td>(0.187)</td>
<td>(0.332)</td>
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<td>0.216</td>
<td>0.221</td>
<td>0.362</td>
<td>0.436</td>
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**Notes:** The dependent variable in specification (10) is the quarterly inflation rate (CPI, annualized) where inflation and inflation expectations are detrended with \( \pi_t^{\text{trend}} \), the ten-year-ahead inflation forecast from the SPF. For all measures of expected inflation (except the naïve expectations and the MSC five-year ahead case), \( E_t \pi_{t+1} \) is the one-year-ahead inflation forecast (mean). For the naïve expectations case, \( E_t \pi_{t+1} \) is the average inflation rate over the previous four quarters. All regressions are estimated on the largest possible sample that covers 1978–2014. The number of observations varies across columns because some measures of expectations are available for fewer periods. \( \text{UEGap}_t \) is the difference between the actual unemployment rate and the CBO’s NAIRU. All data are final vintage. The estimation sample excludes 2008:IV, which is an outlier in the data. Newey–West robust standard errors (five lags) are in parentheses.
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</tbody>
</table>

Notes: The dependent variable in specification (10) is the quarterly inflation rate (CPI, annualized). The constraint is that the sum of coefficients on expected inflation is equal to one. For all measures of expected inflation (except the naïve expectations and the MSC five-year-ahead case), $E_t \pi_{t+1}$ is the one-year-ahead inflation forecast (mean). For the naïve expectations case, $E_t \pi_{t+1}$ is the average inflation rate over the previous four quarters. All regressions are estimated on the largest possible sample that covers 1978–2014. The number of observations varies across columns because some measures of expectations are available for fewer periods. $UEGap_t$ is the difference between the actual output rate and the CBO’s potential output. All data are final vintage. The estimation sample excludes 2008:IV, which is an outlier in the data. Newey–West robust standard errors (five lags) are in parentheses.
### APPENDIX TABLE 4

**Horserace Regressions, Distributed Lag for Unemployment Gap**

<table>
<thead>
<tr>
<th>Dep. var.: $\pi_t$</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>sum $UEGap_t$</td>
<td>$-0.242$</td>
<td>$-0.238$</td>
<td>$-0.231$</td>
<td>$-0.166$</td>
<td>$-0.214$</td>
<td>$-0.254$</td>
<td>$-0.270$</td>
<td>$-0.194$</td>
</tr>
<tr>
<td></td>
<td>$(0.083)$</td>
<td>$(0.081)$</td>
<td>$(0.087)$</td>
<td>$(0.124)$</td>
<td>$(0.082)$</td>
<td>$(0.101)$</td>
<td>$(0.091)$</td>
<td>$(0.139)$</td>
</tr>
<tr>
<td>Expected inflation, $E_t \pi_{t+1}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSC</td>
<td>$1.426$</td>
<td>$1.395$</td>
<td>$1.014$</td>
<td>$1.324$</td>
<td>$0.967$</td>
<td>$1.276$</td>
<td>$1.376$</td>
<td>$1.531$</td>
</tr>
<tr>
<td></td>
<td>$(0.074)$</td>
<td>$(0.166)$</td>
<td>$(0.189)$</td>
<td>$(0.149)$</td>
<td>$(0.199)$</td>
<td>$(0.156)$</td>
<td>$(0.248)$</td>
<td>$(0.233)$</td>
</tr>
<tr>
<td>Naïve</td>
<td>$0.024$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(0.145)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPF</td>
<td></td>
<td>$0.229$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$-0.073$</td>
<td>$0.439$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(0.147)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$(0.547)$</td>
<td>$(0.641)$</td>
</tr>
<tr>
<td>Greenbook</td>
<td></td>
<td>$0.114$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$-0.199$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(0.114)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$(0.314)$</td>
</tr>
<tr>
<td>Financial markets</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$0.192$</td>
<td>$0.618$</td>
<td>$0.093$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$(0.143)$</td>
<td>$(0.397)$</td>
<td>$(0.461)$</td>
<td></td>
</tr>
<tr>
<td>MSC (5 year ahead)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$0.163$</td>
<td>$-0.085$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$(0.151)$</td>
<td>$(0.293)$</td>
<td>$(0.315)$</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>144</td>
<td>144</td>
<td>132</td>
<td>120</td>
<td>130</td>
<td>118</td>
<td>113</td>
<td>94</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.766</td>
<td>0.766</td>
<td>0.389</td>
<td>0.722</td>
<td>0.340</td>
<td>0.653</td>
<td>0.387</td>
<td>0.409</td>
</tr>
</tbody>
</table>

**Notes:** This table reports results for specification (10) modified as follows: $\pi_t = a_0 + a_1 E_t \pi_{t+1} + a_2 E_t^Y \pi_{t+1} + \sum_{s=0}^2 b_s (UE_{t-s} - UE_{t-s}^N) + \text{error}$. The table reports results for $\sum_{s=0}^2 b_s$ in the first row. The dependent variable in specification (6) is the quarterly inflation rate (CPI, annualized). For all measures of expected inflation (except the naïve expectations and the MSC five-year-ahead case), $E_t \pi_{t+1}$ is the one-year-ahead inflation forecast (mean). For the naïve expectations case, $E_t \pi_{t+1}$ is the average inflation rate over the previous four quarters. All regressions are estimated on the largest possible sample that covers 1978–2014. The number of observations varies across columns because some measures of expectations are available for fewer periods. $UEGap_t$ is the difference between the actual output rate and the CBO’s potential output. All data are final vintage. The estimation sample excludes 2008:IV, which is an outlier in the data. Newey–West robust standard errors (five lags) are in parentheses.
### APPENDIX TABLE 5

**Horserace Regressions, Control for Oil Price Shocks**

<table>
<thead>
<tr>
<th>Dep. var.: $\pi_t$</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$UEGap_t$</td>
<td>-0.201</td>
<td>-0.180</td>
<td>-0.161</td>
<td>-0.165</td>
<td>-0.145</td>
<td>-0.214</td>
<td>-0.188</td>
<td>-0.126</td>
</tr>
<tr>
<td></td>
<td>(0.067)</td>
<td>(0.071)</td>
<td>(0.070)</td>
<td>(0.105)</td>
<td>(0.064)</td>
<td>(0.070)</td>
<td>(0.062)</td>
<td>(0.052)</td>
</tr>
<tr>
<td>Expected inflation, $E\pi_{t+1}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSC</td>
<td>1.357</td>
<td>1.035</td>
<td>0.530</td>
<td>0.794</td>
<td>0.585</td>
<td>0.875</td>
<td>0.566</td>
<td>0.595</td>
</tr>
<tr>
<td></td>
<td>(0.089)</td>
<td>(0.107)</td>
<td>(0.155)</td>
<td>(0.170)</td>
<td>(0.150)</td>
<td>(0.161)</td>
<td>(0.203)</td>
<td>(0.172)</td>
</tr>
<tr>
<td>Naïve</td>
<td>0.231</td>
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<td></td>
<td></td>
<td>0.130</td>
<td>-0.050</td>
</tr>
<tr>
<td></td>
<td>(0.089)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.104)</td>
<td>(0.093)</td>
</tr>
<tr>
<td>SPF</td>
<td></td>
<td>0.533</td>
<td></td>
<td></td>
<td>-0.077</td>
<td>-0.050</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.136)</td>
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<td></td>
<td>(0.359)</td>
<td>(0.429)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Greenbook</td>
<td></td>
<td>0.445</td>
<td></td>
<td></td>
<td>0.406</td>
<td>0.182</td>
<td>-0.219</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(0.103)</td>
<td></td>
<td></td>
<td>(0.108)</td>
<td>(0.323)</td>
<td>(0.380)</td>
<td>(0.222)</td>
</tr>
<tr>
<td>Financial markets</td>
<td></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.508</td>
<td></td>
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<td>0.438</td>
<td>0.457</td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(0.118)</td>
<td></td>
<td></td>
<td>(0.198)</td>
<td>(0.191)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSC (5 year ahead)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sum $d\log(Oil\ price_t)$</td>
<td>0.039</td>
<td>0.046</td>
<td>0.064</td>
<td>0.067</td>
<td>0.058</td>
<td>0.062</td>
<td>0.068</td>
<td>0.073</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.010)</td>
<td>(0.011)</td>
<td>(0.012)</td>
<td>(0.011)</td>
<td>(0.013)</td>
<td>(0.013)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>Observations</td>
<td>144</td>
<td>144</td>
<td>132</td>
<td>120</td>
<td>130</td>
<td>118</td>
<td>113</td>
<td>94</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.815</td>
<td>0.825</td>
<td>0.587</td>
<td>0.815</td>
<td>0.571</td>
<td>0.765</td>
<td>0.591</td>
<td>0.644</td>
</tr>
</tbody>
</table>

**Notes:** This table reports results for specification (10) modified as follows: $\pi_t = a_0 + a_1 E_t \pi_{t+1} + a_2 E_t Y_{t+1} + b(UE_t - UE_t^N) + \sum_{s=0}^2 \gamma_s d\log(Oil\ price_{t-s}) + \text{error.}$ The table shows reports results for $\sum_{s=0}^2 \gamma_s$ in the last row. The dependent variable in specification (6) is the quarterly inflation rate (CPI, annualized). For all measures of expected inflation (except the naïve expectations and the MSC five-year-ahead case), $E_t \pi_{t+1}$ is the one-year-ahead inflation forecast (mean). For the naïve expectations case, $E_t \pi_{t+1}$ is the average inflation rate over the previous four quarters. All regressions are estimated on the largest possible sample that covers 1978–2014. The number of observations varies across columns because some measures of expectations are available for fewer periods. $UEGap_t$ is the difference between the actual output rate and the CBO’s potential output. All data are final vintage. The estimation sample excludes 2008:IV, which is an outlier in the data. Newey–West robust standard errors (five lags) are in parentheses.
APPENDIX B. DERIVATIONS OF THE NEW KEYNESIAN PHILLIPS CURVE AND ADAM AND PADULA (2011)

The derivation of the Phillips curve is briefly outlined below to highlight where the FIRE assumption is used. For a complete derivation see Galí (2008). Facing Calvo pricing, firms will be allowed to adjust prices in a given period with constant probability \(1 - \theta\). If firm \(i\) resets its price in period \(t\), it will choose the following optimal price:

\[
\begin{align*}
  p_{i,t}^* &= (1 - \theta) \sum_{j=0}^{\infty} (\theta \beta)^j E_{i,t} \text{mc}^n_{i,t+j,t}.
\end{align*}
\]  

(B1)

Assuming constant returns to scale and perfect factor mobility, the nominal marginal cost faced by firm \(i\) in period \(t + j\) that last updated prices in \(t\) is equal to the aggregate nominal marginal cost \(\text{mc}^n_{i,t+j,t} = \text{mc}^n_{t+j}\)\(^2\). Hence:

\[
\begin{align*}
  p_{i,t}^* &= (1 - \theta) \sum_{j=0}^{\infty} (\theta \beta)^j E_{i,t} \text{mc}^n_{i,t+j}.
\end{align*}
\]  

(B2)

\(^2\)In the typical NKPC derivation, the production function is assumed to be \(Y_{i,t} = Z_{i,t} L_{i,t}^{1-\alpha}\) resulting in \(\text{mc}^n_{i,t+j,t} = \text{mc}^n_{i,t+j} + \frac{\alpha}{1-\alpha} (p_{i,t} - p_{i+j})\). We have used the simplification of constant returns to scale and perfect factor mobility for easier analogy to the Adam and Padula (2011) derivation that follows. The importance of FIRE in the NKPC derivation remains apparent, and the NKPC only changes by losing a constant multiplicative factor on marginal cost.

---

**APPENDIX TABLE 6**

Sensitivity of the Phillips Curve Estimates to Using Alternative Measure of Inflation

<table>
<thead>
<tr>
<th>Dep. var.: (\pi_t)</th>
<th>Inflation measure</th>
<th>CPI (1)</th>
<th>Core CPI (2)</th>
<th>PCE (3)</th>
<th>Core PCE (4)</th>
<th>GDP deflator (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(UE\ Gap_t)</td>
<td></td>
<td>-0.229</td>
<td>-0.089</td>
<td>-0.006</td>
<td>0.129</td>
<td>-0.029</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.087)</td>
<td>(0.111)</td>
<td>(0.076)</td>
<td>(0.141)</td>
<td>(0.097)</td>
</tr>
<tr>
<td>(E_t \pi_{t+1})</td>
<td></td>
<td>1.432</td>
<td>1.281</td>
<td>1.212</td>
<td>1.010</td>
<td>1.050</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.072)</td>
<td>(0.122)</td>
<td>(0.080)</td>
<td>(0.105)</td>
<td>(0.113)</td>
</tr>
<tr>
<td>Observations</td>
<td></td>
<td>146</td>
<td>146</td>
<td>146</td>
<td>146</td>
<td>146</td>
</tr>
<tr>
<td>(R^2)</td>
<td></td>
<td>0.773</td>
<td>0.737</td>
<td>0.820</td>
<td>0.734</td>
<td>0.760</td>
</tr>
</tbody>
</table>

Notes: The dependent variable in specification (9) is the quarterly inflation rate (annualized). The definition of inflation is indicated in the top row. \(E_t \pi_{t+1}\) is the one-year-ahead inflation forecast (mean) in the MSC. The estimation sample is 1978:1–2014:III. \(UE\ Gap_t\) is the difference between the actual unemployment rate and the CBO’s NAIRU. All data are final vintage. PCE is the chain-type price index for PCE. Core CPI and PCE exclude food and energy. The estimation sample excludes 2008:IV, which is an outlier in the data. Newey–West robust standard errors (five lags) are in parentheses.
Recalling $mc_t^n = mc_t + p_t$, and $\pi_t = p_t - p_{t-1} = (1 - \theta)(p_t^* - p_{t-1})$ and defining the cross-sectional average expectation as $E_t = \int_0^1 E_{t,i} \, di$:

(B3) \[ p_t^* - p_{t-1} = (1 - \theta \beta) \sum_{j=0}^{\infty} (\theta \beta)^j E_t [mc_{t+j}^n + p_{t+j} - p_{t-1}], \]

(B4) \[ \pi_t = (1 - \theta)(1 - \theta \beta) \sum_{j=0}^{\infty} (\theta \beta)^j E_t mc_{t+j} + (1 - \theta) \sum_{j=0}^{\infty} (\theta \beta)^j E_t \pi_{t+j}. \]

Shifting the equation forward one period and applying the law of iterated expectations:

(B5) \[ E_t \pi_{t+1} = (1 - \theta)(1 - \theta \beta) \sum_{j=0}^{\infty} (\theta \beta)^j E_t mc_{t+j+1} + (1 - \theta) \sum_{j=0}^{\infty} (\theta \beta)^j E_t \pi_{t+j+1}. \]

Combining the previous two lines results in the NKPC:

(B6) \[ \pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \theta)(1 - \theta \beta)}{\theta} mc_t. \]

Adam and Padula (2011), whose derivation is outlined next, show that the Phillips curve can be derived under a more general information structure if a certain condition is satisfied. They call the needed condition “condition 1” and it requires that agents are unable to predict revisions in their own or other agent’s forecasts. As in the NKPC case, assume monopolistic competition and Calvo pricing with reset probability $1 - \theta$. As in the previous derivation, assume constant returns to scale and perfect factor mobility resulting in equal marginal costs across firms. Suppose there are $I$ professional forecasters that each advise $1/I$ firms. Forecaster $i$ has subjective expectation $F_{i,t}$. When allowed to update prices, a firm advised by forecaster $i$ will reset to its optimal price:

(B8) \[ p_t^* = (1 - \theta \beta) \sum_{j=0}^{\infty} (\theta \beta)^j F_{i,t} mc_{t+j}^n. \]

The new price level is:

(B9) \[ p_t = (1 - \theta) \frac{1}{I} \sum_{i=1}^{I} p_t^* + \theta p_{t-1}. \]

Implying inflation is:

(B10) \[ \pi_{t+1} = (1 - \theta) \pi_{t+1}^* + \theta \pi_t, \quad \text{where} \quad \pi_{t+1}^* = \frac{1}{I} \sum_{i=1}^{I} \pi_{i,t+1}^* \equiv \frac{1}{I} \sum_{i=1}^{I} p_{i,t+1} - p_{i,t}. \]

Let $F_t[\cdot] \equiv \frac{1}{I} \sum_{i=1}^{I} F_i[\cdot]$ and apply it to the previous:

(B11) \[ F_t[\pi_{t+1}] = (1 - \theta) F_t[\pi_{t+1}^*] + \theta \pi_t. \]
Next, apply the subjective expectation operator to the definition of $\pi_{t+1}^*$:

\begin{equation}
F_{i,t}[\pi_{t+1}^*] = \frac{1}{I} F_{i,t} \left[ \sum_{i=1}^{I} p_{i,t+1}^* - p_{i,t}^* \right].
\end{equation}

Condition 1 requires that agents are unable to predict revisions in their own or other agents' forecasts and is as follows:

\begin{equation}
F_{i,t}[F_{h,t+1}[mc_n^{i,s}] - F_{h,t}[mc_n^{i,s}]] = 0 \quad \forall i, h, \text{ and } s > 0.
\end{equation}

Plugging in the optimal prices, using condition 1, rearranging, and switching from nominal to real marginal cost:

\begin{equation}
F_{i,t}[\pi_{t+1}^*] = \frac{1 - \theta}{I} F_{i,t} \left\{ \sum_{i=1}^{I} \left[ \sum_{j=0}^{\infty} (\theta \beta)^j F_{h,t+1}[mc_h^{n,t+1+j}] + \sum_{j=0}^{\infty} (\theta \beta)^j F_{h,t}[mc_h^{n}] \right] - \sum_{j=0}^{\infty} (\theta \beta)^j F_{h,t}[mc_h^{n}] \right\},
\end{equation}

\begin{equation}
F_{i,t}[\pi_{t+1}^*] = (1 - \theta) F_{i,t} \left[ \pi_{t+1}^* + \frac{mc_t}{1 - \theta} - \pi_t \right].
\end{equation}

Arriving at the Phillips curve:

\begin{equation}
\pi_t = \beta F_{t} [\pi_{t+1}^*] + \frac{(1 - \theta)(1 - \theta \beta)}{\theta} mc_t.
\end{equation}

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Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer.


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