Rethinking Bargaining Theory

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Abstract

This paper aims for a new approach to bargaining, quite different than Rubinstein (1982), but using his version of the classic bargaining problem: *Two impatient individuals wish to agreeably divide a unit pie under complete information.* I submit that the players' key motivation is not that offerers enjoy a *temporal monopoly,* so that spurning any proposal is discretely costly. For that inexorably leads to bargaining game forms like Rubinstein's. Rather, I suppose that *players' behaviour is governed solely by their aspiration values* (expected payoffs); hence, I simply must consider a continuous time game form.

In a partial reprise of the pre-1970 agenda of bargaining as a concession game, I explore the resulting stationary complete information bargaining game, with endogenous timing and content of offers. When delay occurs, any offer concedes the entire stopping rent — to wit, players are locked in a war of attrition, awaiting the next concession. Offers are (usually) turned down with positive probability, and aspiration values are a martingale stochastic process.

By tossing out the temporal monopoly, I have done away with any point predictions. Still, there is room here for falsifiable probabilistic implications, as the choice of what and when to offer are stochastically intertwined. An aspired goal of this paper is therefore not the standard deduction that delay occurs or can occur in bargaining, but rather a serious simultaneous consideration of the timing and content of offers. Moreover, I shall investigate how the equilibrium payoff set adjusts with greater patience or risk aversion.

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[‡]This is an extremely early release version with few results, solely seeking feedback. Please email me. The most recent version of this and all my research is available from my web page at mit.edu/lasmith/www.

1. INTRODUCTION

"The whole creation groans and yearns, desiderating a principle of arbitration..."
— Edgeworth (1881), part II, page 46

The classic *bargaining problem* is one the great outstanding puzzles of economic theory, offering lessons of humility and triumph to economists of many generations. What was sparked by the first serious attack on general equilibrium analysis grew into the pre-eminent problem of modern game theory. It presents two individuals with the opportunity to share a dollar and only demands that they first agree on a partition. Here, I take as a primitive a simple non-cooperative strategic form of it that has attracted the greatest attention since 1965, capturing its critical "realistic" elements: (i) two risk neutral players $1, 2$ must divide a unit pie between themselves, by concurring on a feasible split (x_1, x_2) with $x_1 + x_2 \leq 1$ and $x_1, x_2 \geq 0$; (ii) an agreement means a proposal by one player and an immediate acceptance by the other; (iii) the force for timely resolution is the players' impatience.

The modern noncooperative dynamic approach to this problem dates to Ståhl (1972) — the first rigorous unique prediction in a truly dynamic model with credible threats — but blossoms with elegant clarity in the watershed (1982) paper by Rubinstein. In an infinite-horizon, discrete time alternating offer model with payoff discounting, he found a unique perfect equilibrium yielding immediate agreement.

The Paradigm Identi ed. Any (possibly randomly) alternating offer bargaining model is intrinsically a story of *temporal monopoly:* Agreement is seen as the outcome of a process in which individuals in turn are exogenously given the right to ask the other party to accept an offer, or to burn some of the pie by declining (given discrete time). The offerer is then at a strategic advantage, for he can then speak and his counterpart cannot. Yet noneconomists would surely find it strange that in the very best economic theory of bargaining, individuals truly relish the role of offerer, and generally make an "offer that can't be refused" (or at least is not). This just doesn't ring true to many, and runs counter to most casual empiricism — even ignoring the fact that complete information may miss a key part of the story. And recalling the words of wisdom leading off Rubinstein (1991), "game theory is not simply a matter of abstract mathematics but concerns *the real world.*"

"As is well-known, ordinary economic theory is unable to predict the terms on which agreements tend to be reached in cases of ... bilateral monopoly," observed Harsanyi. "Only on the basis of additional *assumptions* does the theory of games furnish a determinate solution." As nature has neither endowed us with the bargaining game form nor the right notion of equilibrium, this task falls to the presumptuous and/or entrepreneurial theorist. The extensive form and solution concept are therefore both assumptions, and cannot be justified within any model. Rubinstein's proposal has really proved such a rare unambiguous theoretical triumph over prior theories, spawning untold embellishments. It may therefore come as surprise to some students of modern game theory that his extensive form is not the bargaining problem. Rather, it is merely a proposed representation of it.

As testimony to its impact, Rubinstein's model has truly shaped how economists *think* of bargaining, per se. For instance, in defiance of observed fact, his outcome entails immediate agreement; this prompted efforts to "explain delay in bargaining" — meaning "... in Rubinstein's model, suitably modified." This has motivated two additional subplots, both adding incomplete information to his simple parable. First, there may be multiple *rational* 'types' of bargainers, with one or both players uncertain about the other's type.[1] The substance of all such models is to embrace *type-signalling.* Second, one or both players may be uncertain about their opponent's sanity;[2] *irrational* types thus adds *bluffing.*

Proving that delay is not just an incomplete information phenomenon has prompted many other twists on Rubinstein's idea.[3] Stahl (1990) considers durable offers and possibly simultaneous moves; Haller and Holden (1990) and Fernandez and Glazer (1991), each model union-firm negotiations, and allow the union to call a strike. Many recent papers explicitly change the disagreement point (see Farusawa and Wen, 1997). All adhere to the temporal monopoly paradigm, but adjust the game form to get their preferred outcome.

A Paradigm Proposed. The sheer genius of temporal monopoly is self-evident: It converts an intractable bilateral monopoly into a succession of pure monopolies. Yet I argue that at its heart, the bargaining problem *is* one of bilateral monopoly, and we surely ought not assume away its very essence. This is "economic warfare", as Zeuthen (1930) aptly labelled his bargaining chapter. We must instead focus this battle through a suitable lens. Like Rubinstein, I assume that time preference is the driving force for settlement, and that bargaining proceeds by a sequence of well-chosen offers and immediate accept/reject decisions. I also maintain his complete information straitjacket: While this sidesteps a key aspect of many bargaining settings, it is surely the cornerstone for any theory. Incomplete information has not really played a critical role in the pre-1970 history of bargaining theory. The focus has been on (incredible) threats, concessions, and relative bargaining power.

[1] Seminal papers here are Rubinstein (1985) and Fudenberg, Levine, and Tirole (1985). The role of the Coase Conjecture (the durable monopolist) also cannot be underestimated in this research program. The emergent refinements literature also used bargaining as a test ground for many new solution concepts.

[2] See §8:8 in Myerson (1991), or Abreu and Gul (1994).

[3] His paper has proved truly democratizing, teaching the fine art of subgame perfect implementation.

If temporal monopoly is the critical factor in bargaining, then parties should literally be tripping over each other for the right to propose, and dialogue and not its absence ought to explain drawn-out negotiations. Rubinstein (1991) has forcefully argued that the "most basic principle in the art of formal modelling" (in game theory) is that the constructed model ought to capture the players' understanding of the game.[4] For instance, citing Aumann (1959), he notes that a finite horizon context ought only be modelled as a finitely repeated game when "the finite period enters explicitly into the players' consideration." Otherwise, an infinitely repeated game is called for. With great aplomb, Binmore (1997) uses this principle (in §1.7) to dispatch of a host of critiques of Rubinstein (1982), such as discrete money. But his defence of the alternating offers model is not so convincing: "Rubinstein's model seems to capture the cut and thrust of bargaining in the real world. The two sides do indeed alternate in making offers, whether they are bargaining over a loaf of bread in an oriental bazarre or over the sale of a skyscraper in Manhattan." Disregarding the validity of this claim, which I do not take for granted, it misses the substance of the "Aumann-Rubinstein principle": Do these real-life players feel critically *motivated* by the proposer's temporal monopoly? I suggest not. Yet realism was not Rubinstein's big goal. Solving Edgeworth's century old open problem was quite an achievement! He found a simple bargaining model with credible threats and a unique, efficient, symmetric outcome.

On the choice of game form and solution concept, I therefore start anew. For my point of departure, I venture that *the single most important* consideration for parties bargaining is *not* a belief that declining an offer burns a boundedly positive fraction of the pie. Rather, I construct a complete information bargaining model without temporal monopoly. Since discrete time forces this paradigm, I have no choice but to shift to continuous time.

Temporal monopoly is a strong, simple, and compelling assumption on the bargaining *game form.* In its stead, I substitute another strong, simple — and hopefully compelling — assumption on the *equilibrium concept.* I submit that the single most important factor in a theory of bargaining ought to be that players' decisions are governed by their *aspiration values* — their expected payoffs in the game. The importance of expectations in bargaining should surprise no one. What is unexpected may be their cutting power as a state variable, for on this pillar is it possible to construct an entire theory of bargaining. Discrete time being a 'subset' of continuous time, temporal monopoly essentially restricts the feasible action space. By way of contrast, I restrict their equilibrium strategy space.[5]

[4] This principle, with which I motivate my contribution, stands in contrast to Friedman's (1953) claim in his "Methodology of Positive Economics" that predictions matter and assumptions do not.

[5] This may sound overly harsh, but as usual, the players may freely choose non-Markovian strategies in response. It just so happens that the best reply to any aspiration value strategy is another such strategy.

Once we abandon the temporal monopoly foundation block, we simply must forsake uniqueness — perhaps Rubinstein's primary (1982) achievement. Interestingly, this is very closely related to a strong version of Zermelo's (1913) Theorem that says that generic finite games of *perfect information* (i.e. no simultaneous moves) have a unique sequential equilibrium.[6] Rubinstein's result suggests that with payoff discounting, this extends to infinite horizon competitive games, like bargaining. In part for this reason, I propose a game of *almost perfect* (yet complete) information — where both are aware of the payoff functions, and the other player's past actions, but not contemporaneous actions.

Fortunately, a falsifiable theory need not deliver a unique prediction. We simply need a consistent set of implications across equilibria, that excludes some possibilities — more exclusive being better. As Zeuthen (1930) recognized, bargaining outcomes may even be probabilistic in nature: "Every solution is not equally probable, and if you take a large enough number of cases, you may expect the economic forces to express themselves." For instance, the choice of what and when to offer are stochastically intertwined.

The strength of this aspiration value approach is first seen in the rational foundation for 'real-life' bargaining that naturally emerges. Consider an equilibrium without immediate agreement. As strategies depend on expected payoffs alone and not on time, any finite delay owes solely to the players' randomization. With strict time preference, there are rents from ending this bargaining hiatus. And since any proposer is indifferent about stepping forth, he must then concede the entire stopping rent to his opponent. In other words, any delay implies that the players are locked in a *war of attrition*: Each strictly prefers that his counterpart and not they stop the clock, and make the concession. *Ipso facto,* the offerer is now at the strategic disadvantage, contrary to temporal monopoly.

Next consider what transpires when some player, say Mr. 1, finally tenders an offer to Mr. 2. The latter might accept it with probability one. If so, game over. Assume not. An agreement brings us to the Pareto frontier, and is therefore efficient, while rejection incurs further delay, and is clearly inefficient. Since Mr. 2 weakly prefers to reject, Mr. 1 must be discouraged by this outcome, suffering a strict payoff loss if Mr. 2 declines.

Altogether, we already have three desirable and realistic features arising: (*i*) wars of attrition explain negotiation lags; (*ii*) all serious offers are concessions; and finally, (*iii*) some offers may be turned down, in which case the proposer is strictly disappointed.[7] To my knowledge, no other complete information bargaining model rationalizes these joint common sense predictions of the actual bargaining process (without assuming them).

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[6] This is Theorem 4.7 in Myerson (1991), one of a surprising number of new results found in that text.
[7] Feature (iii) is also true in Rubinstein's model, where offers are not turned down in equilibrium.

Supporting an Aspiration Class. A modelling innovation necessitated by the paradigm shift is a characterization of an interlaced set of subgames. In my *Dynamic Aspirations Equilibrium* (DAE), the players start with exogenously-given aspiration levels — the initial state of a Markov process. In each war of attrition, players concede at a constant rate by assumption, i.e. the concession time is a Poisson random variable. The moment some player, say Mr. 1 stops and offers x, the responder's aspiration value jumps to this new level x. Afterwards, the concession that is offered may yet be rejected, restarting another war of attrition; however, as already deduced, the pursuant subgame is strictly worse for the player whose offer was rejected, and better for the rejecter — that is, their aspiration values have moved in opposite directions. This really has the flavour of *forward induction* in extensive form games: By declining a given offer, a player signals that in the subgame that follows, he expects no less in the future. The current offer then becomes his new aspiration value. Off-path offers that tempt a player to end the hiatus, and accept a slightly stingy offer, may be punished by moving to a seed aspiration pair that is lower for the deviant, and higher for his opponent.

The fluid evolution of bargaining aspiration pairs obeys a simple regularity condition. The negotiations lead to a Markov and martingale stochastic process on the space of possible pairs of discounted aspiration levels, as in Aumann and Hart's (1993) cheap talk paper. The content of offers and their rejection performs the role of dialogue in their paper. Reflecting the players' indifference at each proposal and acceptance, each party clearly expects that his current aspiration value equals his final discounted payoff.

'Rubinstein-in-the-Small'. That the dynamic aspirations equilibrium does not collapse is in itself worthy of note, and a contribution of this paper. For Perry and Reny (1993), Sákovics (1993), and Stahl (1993) (different than Ståhl) have also endogenized the timing in Rubinstein's model. Among their findings, they show that in a continuous time bargaining game with instantaneous reaction times, so long as players must wait a fixed amount of time between offers, the unique subgame perfect equilibrium is Rubinstein's outcome. This essentially still adheres to the temporal monopoly paradigm, viewing continuous time as 'very fine' discrete time, yielding very small monopoly rents. Since these results surely obtain for random but fixed but boundedly positive mean time between offers, at first blush one might presume that my sought for class of equilibria do not exist. The device that I employ is to insist that to the player with aspiration levels approaching the highest one that will be supported, offers must be made arbitrarily quickly.

Unlike Rubinstein, both the content and the timing of offers are key choice variables in this paper; however, the above literature strain shows that temporal monopoly in no way precludes endogenous timing. Still, the key normative predictions of this model — wars of attrition and strict concessions — clearly cannot arise with a proposer advantage.

Asymmetry and Bargaining Power. One serendipitous reason for the success of Rubinstein's modelling approach was how cleanly it reduces to Nash's bargaining solution.[8] Since the players' bargaining strength owes solely to their relative impatience levels, this is the only source of asymmetry in the Rubinstein's model.[9] But in the aspirations paradigm, a strategic component to bargaining power exists that is a separate source of asymmetry. Parties gain strength here from their *refusal* to bargain, or poor chance of accepting tendered proposals. As is so often true in social bargaining, one may gain advantage by giving the other party the "silent treatment", forcing them to make all overtures.

This paper therefore formalizes the rather vague psychological notion of "bargaining ability" (or at least bargaining strength) found in the literature from Edgeworth (1881) at least through Nash (1950). Soon afterwards, Nash (1953) repudiated this view on p. 138: "With people who are sufficiently intelligent and rational there should not be any question of 'bargaining ability', a term which suggests something like skill in duping the other fellow. The usual haggling process is based on imperfect information," Nash claimed, in a rare incautious moment. "Our assumption of perfect information[10] makes such an attempt meaningless." This paper provides a formal role for players' bargaining ability / strength in a complete information model of haggling. Aspirations may differ because: (a) the aspiration payoff set is skewed in favour of one party, (b) or the players initially differ, or (c) current bargaining history.

To better focus on the initial conditions, consider for a moment a larger social context with random matching — for any repeated bargaining relationship will induce an unwanted folk theorem result. Part (a) bites with distinguished bargaining roles. Social custom may favour firms over workers in wage bargaining, sellers over buyers in price wrangling. To wit, the aspiration set is skewed in favour of that side. In this paper, I restrict attention to nondistinguished roles, as makes sense to overlook this asymmetry.

To see (b), assume that social conventions determine the equilibrium strategies. Some societies may have very little delay in bargaining, some substantial. When two individuals

[8]See the excellent account of the "Nash program" in chapter 4 of Osborne and Rubinstein.

[9]This by itself was a tremendous advance. Harsanyi (1977) writes that "some symmetry postulate ... is common to all possible theories of bargaining as a matter of strict logical necessity."

[10]Based on the context (also: "trying to propagandize each other into misconceptions of the utilities involved"), Nash means complete and incomplete information in the modern terminology, rather than perfect and imperfect information. He is speaking about parametric and not action uncertainty.

are drawn from a large population to bargain, might players may have a reputation for toughness or timidity, with relative strengths determining the aspiration values for that bargaining match? Perhaps.[11] I shall more simply assume that history matters not, and that intelligent and rational players must entertain symmetric aspirations.

The third source of disparate aspirations (c) always remains. An initially disfavored party may gain bargaining momentum, as he finds himself the beneficiary of multiple offers that he so chooses to decline. Harsanyi (1956) saw the random aspects to bargaining strength: "Of course, information on the two parties . . . strength alone may not suffice: the outcome may depend significantly on such 'accidental' factors as . . . bargaining skill."

The Nash Demand Game. The classic folk theorem in bargaining theory is Nash's (1953) *demand game*: Any pie split is a static Nash equilibrium. While by extension, every pie split is a subgame perfect outcome of the continuous time game, this is no longer a worrisome possibility in a DAE. If the proposer almost surely makes some demand $(a, 1-a)$ at time 0, then he must do so continuously, barring time dependence.[12] Since the proposer is now in a position of weakness, some plausible stories can defeat this behaviour. First, since such offers can simply be ignored, with a small offering cost, the outcome is not robust to players' trembling over ε-time intervals. For the proposer may incur an infinite cost repeatedly tendering his offer. Second, and not pursued here, if there is slight uncertainty over the players' initial aspiration values, then this is also not robust. But the simple point is that the Nash demand game is no longer an *interesting* outcome to examine.

The paradox of the Nash demand game is replaced by the reciprocal bargaining refusal conundrum. What if some player simply refuses to make any offers? We can rule this out, for this can only be rationalized if his counterpart plays a Nash demand game, which we have argued can be defeated. In other words, the extent of any asymmetry is limited.

Exploding O ers. As in Rubinstein (1982), I assume that offers imply no future commitment: Bygones are bygones.[13] One may motivate this assumption by simplicity, for otherwise the problem is no longer stationary in aspiration space: There is an additional state variable to the problem, namely one's best outstanding (and last) offer to the other player, which greatly complicates the analysis. But I also consider this model a realistic end in itself, as it captures the inherent risk of declining any offer: One may ultimately offer or accept a strictly worse outcome. Accepting an offer is the irreversible decision to exercise

[11] For instance, in a large population with nonanonymous pairwise matching, and one-shot plays of the Battle of the Sexes, one may construct a simple model where individuals initially stochastically develop a reputation for submission or domination that becomes self-fulfilling.

[12] If he proposes at a finite rate, then he is willing accept slightly less from the other player at time-0.

[13] eg. Fershtman and Seidmann (1993) assume players can't accept worse offers than they have rejected.

an option, and spurning one is very much a risky decision not to sell an asset. If offers imply irrevocable commitments, some sort of third party commitment technology is required to enforce equilibria where the players progressively concede parts of the bargaining pie.

Falsi able Dynamic Implications. An aspirations value approach is rich, and perhaps too rich. How well-behaved is it? Are there are robust properties on the distribution of final offers? Because the ultimate division of the spoils and the bargaining duration are entwined, a joint analysis of the timing and content of offers may be possible.

Rubinstein's bargaining outcome is wholly determined by the players' time preference and risk aversion. On both scores, he delivers the intuitive comparative statics answers: When a player grows either more impatient or more risk averse, his determinate share of the pie shrinks. We can perform similar parametric sensitivity analysis here, but now the comparative statics exercises apply to the entire aspiration class. The big and simple idea is that by declining an offer, an individual exchanges a sure thing for an risky option. Clearly, this option is worth more to someone who is less risk averse or more patient.

This extended introduction is meant both to motivate an important problem that many economists believe is already solved, and to substitute for a dearth of hard results in the current early relase paper. Currently, I only describe the equilirbium notion, and intuitively flesh out a few of the implications in this bargaining paradigm. More to come. . .

2. REPRISING THREE PREVIOUS BARGAINING THEMES

> "It sometimes happens in science and art that the very perfection of a certain style demonstrates its limitations and calls for new approaches."
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> — Morgenstern, writing in 1973 of bargaining theory prior to Ståhl (1972)

I began this project with a proposed bargaining model, and a simplistic understanding of the history of the problem: Edgeworth's open question, then Nash's axiomatic and static solutions, and finally Rubinstein's dynamic one.[14] I have since found a fount of discarded insights that separately appear in my equilibrium that I wish to lay out. Suffice it to say that aside from Nash's island of perspicuity, one can fairly describe pre-1970 bargaining theory as mathematical handwaving at best, and surely not well-founded economics. A psychological description of union-management bargaining was understood, but any formalism was thoroughly frustrated for want of a sensible notion of equilibrium. Zeuthen

[14]Binmore, Osborne, and Rubinstein (1992) is an engaging summary article, half subsumed by Osborne and Rubinstein (1990). Fudenberg and Tirole (1991) is a good textbook summary of the vast 1982–91 bargaining explosion; Binmore (1997) and Cross (1965) address the older literature as well.

and Hicks proposed static bargaining games (masquerading as dynamic ones) two decades before Nash formalized his namesake solution concept.[15] Much later, Cross had exquisitely unfortunate timing: As he ventured the first attempt at a truly dynamic bargaining solution (aside from Nash' (1953) simple two and three stage mechanisms) just as Selten's subgame perfection was rolling of the presses in 1965 (in German, no less).[16] Reflecting this rotten timing, Cross not surprisingly assumed that expectations adjusted arbitrarily, and posited linear strategies that were not even mutual best responses.

The temporal monopoly paradigm first appears in Ståhl (1972). This is much more ad hoc than Rubinstein, analyzing a discrete outcome, finite-horizon bargaining game. With an infinite horizon, Ståhl assumes nonstationary and decreasing payoffs, and derives an order-independent bargaining solution.[17] In the wake of Rubinstein's much simpler (1982) model, the new paradigm has entirely swept away the old, and the insights of earlier bargaining theory have been tossed aside. These insights are next summarized.

A. Bargaining as a Concession Game. Without a doubt, the principle theme of the pre-1970 literature (essentially, everything but Nash's work) is bargaining as a concession game. The fixation on the concession idea was hard-wired into the action space, and likely blinded economists to the simple offer-accept/decline structure. Indeed, in his summary of the state of knowledge on the bargaining problem, Coddington (1968) amazingly formalizes the bargaining problem as "represented quite generally by ... (1) a pair of variables q_1, q_2 representing the demands of the bargainers at any point in time."

Absent any intertemporal structure, the classical stories of Hicks (1932) and Zeuthen (1930) are purely theoretical concession games, with the entire dynamic process played out in one instantaneous mental flash. *Zeuthen's Principle* (see §1.4 of Harsanyi (1977)) was that the next concession always comes from the party less willing to risk a conflict, as measured by the highest chance of conflict they are willing to accept rather than accept the terms proposed by the other party. To his credit, Hicks wanted only credible threats: He assumed that parties choose to strike (i.e. to delay) iff this is more profitable than conceding. With his demand game, Nash (1953) fully indulged himself in incredible threats. In contrast, here I assume that individuals are indifferent between delay and concession.

While this paper reprises this theme of the older literature, I avoid a pure concession game, just as Rubinstein, by assuming exploding offers. No offer remains on the table.

[15]Perhaps not so well-known, in a scoop on Selten's (1975) trembling hand perfection, Nash' (1953) justification of his (1950) axiomatic solution solution concept used payoff-perturbed demand games.

[16]This was the year after Cross' 1964 thesis.

[17]This complexity in part accounts for why, notwithstanding Morgenstern's plaudits, Harsanyi's 300 page (1977) book on bargaining made absolutely no reference to Ståhl's book!

Even thought every offer is a concession, a shift in bargaining momentum may later undo any previous concession: Players may spurn an offer they ultimately wish they had taken.

B. Bargaining as a War of Attrition. The role of time preference in the bargaining problem was first formally recognized in Bishop (1964) and Foldes (1964) — albeit only as a reinterpretation of the static incredible threats models of Hicks and Zeuthen.[18] As Cross wryly observed (p. 72), "If it did not matter when the people agreed, it would not matter whether or not they agreed at all."

Given this force for early settlement, it is natural to investigate the complete information war of attrition. Along these lines, and as a precursor to a more involved model, Osborne (1985) offers a complete information concession game in the spirit of efforts of a generation before. It is a simple timing game: Both players initially demand some fraction of the pie, and then engage in a war of attrition, to see who first caves in. It is important that pure timing games have a much more restricted action space, with players unable to make offers and counteroffers. Transcending this restrictive action space was a key (if under appreciated) contribution of Ståhl (1972).

C. Bargaining as an Aspirations Equilibrium. Most surprisingly, the importance of expectations in determining behaviour is also not new. As long ago as Siegel (1957), the role of payoff aspirations in decision theory was recognized: Siegel defined an aspiration value as the infimum acceptable continuation utility in a dynamic decision problem. In fact, their specific importance in bargaining was already recognized, and experimentally tested in Siegel and Fouraker (1960). A key role played by offers here — to ratchet up the opponent's aspiration level — was observed in a specific experiment (pp. 80–81), as was the intertemporal nonmonotonicity of a player's offer (pp. 77–90). Most recently, payoff aspirations are implicit in the untimely work of Cross (1965).[19] Perhaps because he exogenously postulated the behaviour of expectations, and then deduced dynamics, this plausible line of thought was never subsequently pursued, to my knowledge; in this paper, these expectations adjust within the confines of Bayes-rational game.

[18]For instance, the quite comprehensive survey Bishop (1963) makes no mention of time discounting or impatience. But the role of time per se was implicit at least as early as Hicks (1932).

[19]Confusion must be avoided with the role of aspirations in cooperative game theory, which have a wholly different connotation. See Bennett, Maschler, and Zame (1997).

3. THE MODEL AND EQUILIBRIUM CONCEPT

3.1 The Structure of the Game

Action Space. The game is played in continuous time. Each player's action set in the "stage game" at time $t \in [0, \infty)$ depends on the immediate history: Between offers, an action is a joint choice in $A_N = [0, \infty) \times [0, 1]$ of *when* to offer, and *what*: namely, the stopping time should one offer first, and the *offer,* or the opponent's pie share; with an offer in hand, the action space is $A_O = \{\text{accept, reject}\}$. An agreement consists of a proposal by one of player and an immediate acceptance by the other. In other words, any tendered offer is a momentarily binding verbal or written agreement that must be acted upon at once. Since offers are assumed exploding, a nonimmediate response is tantamount to rejection. We shall call any positive time elapse between offers *delay.*

Strategies. The time-t *history* $h^t = \langle (i, s, x_s), \forall s < t \rangle$ of the game is the list of previous offers, specifying for each, the tenderer $i \in \{1, 2\}$, the time $s < t$, and the proposal $x_s \in [0, 1]$. A *behavior strategy* then consists of a history-dependent mixture over A_N when an offer is not in hand, and an acceptance chance α with an offer in hand.

There are well-known problems with continous time games, since strategies might not uniquely identify an outcome — basically because there is no last moment before any given time (see Bergin and MacLeod (1993)). While merely a tedious technical issue, it must be seriously addressed. Since the stopping rule strategies will later be assumed not to depend on time, I believe that this critique will not bite.

Payo s. The feasible payoff space is $\mathcal{P} = \{(x, 1-x) | 0 \leq x \leq 1\}$. Players are assumed risk neutral (for now), and Mr. $i = 1, 2$ discounts future payoffs at the interest rate $r_i > 0$.

While it is currently not an essential feature of the paper, I may eventually wish to assume that offers cost a nonnegative amount $c \geq 0$ to make. In union-management negotiations, this may be the cost of calling together the 'board' to agree on a proposal. In buyer-seller haggling, this is an insignificant decision cost. This assumption does not affect the nature of the equilibria I focus on, but it does help prune the equilibrium possibility of continuous time Nash demand games, as described earlier. (Details are omitted.)

Adding a reponse cost $c' > 0$, by way of contrast, does not greatly affect the nature of the equilibrium: We still have the war of attrition concession game structure. If the responder cannot choose to be silent, then this is a cost that must be borne anyway, and therefore is formally equivalent to a proposer cost of $c + c'$ in equilibrium.[20] If the responder

[20] Similarly, in public finance, it does not matter whether buyer or seller is taxed in a transaction.

can choose to be silent, then only the final offer will be responded too. In this case, each continuation game is slightly worse for the proposer following a decline, to pay for the equilibrium possibility that the other party accepts and pays his cost c'.

Aspiration Values. Given a set of strategies, each player i can compute his expected present value v_i of the eventual pie split, less his incurred offer costs. Assume that player i makes proposals at random times $\tau_1, \ldots, \tau_n$ in $[t, \infty)$ and finally accepts the random share x of the pie at time τ_n. Then his *aspiration value* at time t is the conditional expected value

$$v_i^t \equiv E\left(e^{-r(\tau_n - t)}x - \sum_{k=1}^{n} e^{-r(\tau_k - t)}c | h^t\right)$$

3.2 The Equilibrium Concept

Rubinstein's bargaining outcome is the unique subgame perfect equilibrium (SPE) of his game. The flexibility of continuous time by itself unfortunately introduces a vast array of new SPE that are impossible to catalogue, and tedious to ponder. There are two types of 'bad' SPE. First, there are those that admit time dependent strategies. For instance, player 1 may decide not to propose until time $\sqrt{e}$, and thereafter to stop and offer at a diminishing Poisson rate. Second, there are those where it is common knowledge that bargaining is not serious, but merely a form of communication. One might, for example, imagine that the first five offers are simply a form of mandatory cheap talk consisting of unreasonable demands; only afterwards does the real bargaining start.

There is obviously no hope in saying anything general about the entire class of SPE. Any outcome at any time can be supported as an SPE. At the very least, I must rule out explicit or implicit strategic time dependence (even from the outset), as well as the latter form of nonserious bargaining. Players must not to possess any clock to calibrate strategies by, and all equilibrium offers must be accepted with positive probability (hopefully ruling out the cheap talk possibility). Call this a nice subgame perfect equilibrium.

To refine the class of SPE, a typical route is to employ Maskin and Tirole's (1992) *Markov perfect equilibrium.* MPE rules out all payoff irrelevant history dependence — meaning everything but an offer that is in hand. Hence, the stopping times and offers are time-invariant within a subgame, and not degenerate at $\tau = 0$; however, it also rules out any expectational dependence that is at the heart of my proposed bargaining process. For one can only be indifferent about accepting or rejecting an offer in hand if it equals one's *new* aspiration value. But MPE precludes a shift in this value.

I obviously must employ a weaker solution concept than MPE, where I allow such shifts. In what I shall call a *Dynamic Aspirations Equilibrium* (DAE), strategies can depend on any payoff-relevant state structure, as well as both players' aspiration values; moreover, the aspiration value can only shift given a payoff-relevant change: a new offer, or an accept-reject decision.[21] I next venture that this new markovian solution concept, though not Markov perfect, conveniently consists of all SPE that are not bad.

Conjecture 1 (Equivalence) *Any nice SPE is an DAE, and conversely.*

A DAE is summarized by a 2-tuple $(\mathcal{A}, \sigma)$ — namely, the *aspiration space* consisting of all pairs $(v_1, v_2) \in \mathcal{A} \subset \{(w_1, w_2) | 0 \leq w_i \leq 1, w_1 + w_2 \leq 1 - c\}$, and a strategy $\sigma = (\sigma_1, \sigma_2)$, which maps $\mathcal{A}$ into pairs $(\mu, \alpha) = (\mu_1, \mu_2, \alpha_1, \alpha_2)$. Here, μ_i is a nonnegative measure over offers in $[0, 1]$, and $\alpha_i(v_1, v_2)$ is the acceptance chance in $[0, 1]$ of offer v_i. (Note that as soon as i tenders an offer x to $-i$, the aspiration value of $-i$ jumps to x, and replaces the v_{-i} argument of $\alpha_i(v_1, v_2)$.) The interpretation is that i makes an offer from any subset $S \subset [0, 1]$ at the stationary Poisson rate $\mu_i(S)$.

Any Markovian game[22] is fully described by specifying the action spaces and payoff functions, as we have done, as well as the state space and transition kernel. In this paper, the space $\mathcal{A}$ of aspirations is the state space for the bargaining Markov process, and σ implies the transition kernel.

Recall that a complete information *war of attrition* is a timing game, with action space {stop, continue}. The game usually ends when one player stops. Both players prefer that the other stop first, and, conditional on a given individual stopping, both players prefer that the game stop sooner rather than earlier.

A DAE consists of a possibly countably infinite number of dual phase subgames: In an initial war of attrition phase, each player chooses a random stopping time τ measured from the outset of that subgame, and will offer first if she stops before the other player. The player having stopped immediately makes a feasible offer $(x, 1-x)$, and his opponent then decides whether to accept or reject given that offer. Given the lack of time dependence, player i's strategy within a subgame consists of a *proposal rate* $m_i = \mu_i([0, 1])$, such that $P(\tau \geq t) = e^{-m_i t}$; a possibly random offer $(x, 1 - x)$ drawn according to the probability measure μ/m to make should i offer first; and a chance $\alpha(x) \in [0, 1]$ with which i will accept the offer x from the other player.

The next result should be pretty standard, since SPE exists.

Conjecture 2 (Existence) *A DAE always exists.*

[21] A shift in the value can only happen in games with almost perfect information on some moves.

[22] This is a degenerate stochastic game where all randomness owes to player randomizations.

4. ROBUST EQUILIBRIUM PROPERTIES

"There is no determinate . . . arrangement towards which the system tends under the operation of . . . a law of Nature, and which would be predictable if we knew beforehand the real requirements of each. . . ; but there are an indefinite number of arrangements à priori possible, towards one of which the system is urged by . . . the *Art of Bargaining* — higgling dodges and designing obstinacy, and other incalculable and often disreputable accidents." — Edgeworth (1881), II, p. 51

4.1 The Nature of Equilibria

Some (a recent instance is Foldes (1964), pp. 117–9) have blamed the above quotation for a half century of inactivity on the bargaining problem. Yet it offers a rhetorical glimpse into the nature of the equilibria in my model. To see how, it helps conceptually to consider the following partitioning of DAE:

- offers are almost surely accepted
- acceptance is probabilistic, and offers deterministic
- offers and acceptance is randomized

Consider first those equilibria where offers are almost surely accepted after a single war of attrition. In this case, there is no incentive to randomize one's offer, since an offer is either accepted or is not.[23] This is essentially Osborne (1985) — a complete information concession game in the spirit of a generation before. It is a simple timing game: First fix what each player will offer, as in a Battle of the Sexes; the players then engage in a war of attrition. It is important that this paper has a much more restricted strategy space, with players unable to choose offers. Indeed, transcending this limiting framework was the very paradigm shift offered by Ståhl (1972).

4.2 The General Structure of Equilibria On the Path

An equilibrium does not specify where one starts out in aspiration space, but takes the initial values (v_1^0, v_2^0) as given, determined outside the system. Still, bargaining negotiations are intrinsically random, so that two individuals may enter negotiations with quite different expectations than are reflected in the final pie split, as the lead Edgeworth quote alluded to. The outcome of the bargaining game is not *a priori* determined by unmodelled initial specification of the aspiration pair.

[23] The most efficient such outcome entails asymmetric strategies: One player immediately offers, and the other accepts, game over.

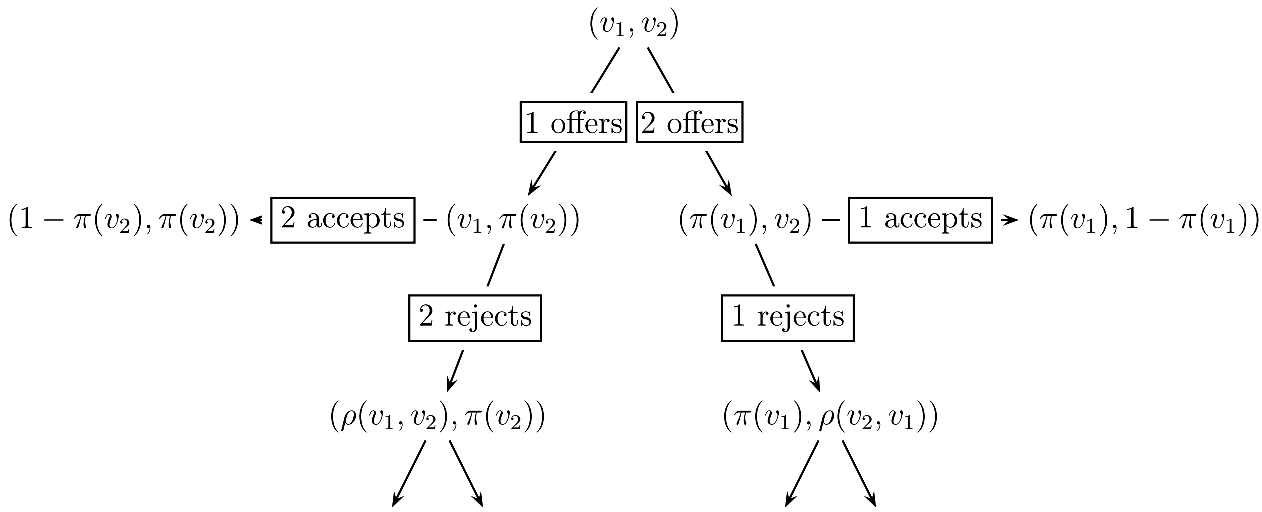

Figure 1: **The Stochastic Process of Aspiration Levels.**

As usual, mixing probabilities must be chosen so as to induce indifference on the part of one's opponent, enabling her to randomize. If one is indifferent about declining an offer, then it clearly must provide one exactly the aspiration value for the continuation game.

Given is a Markov process $\langle v_2^t, v_2^t \rangle$ with support $\mathcal{A} \subset \mathcal{P}$ in a DAE. Let $\pi_i(v) = \int_0^1 x\mu_i(dx)/m_i$ be the *expected offer* that player i makes given his value v. Denote by x_i a typical realized offer in the support of $\mu_i(v)$. In equilibrium, Mr. 1 (with value v_1) plans to suggest to her opponent Mr. 2 (with value v_2) some possibly random pie split $(1 - x_1, x_1)$, while Mr. 2 will suggest to Mr. 1 the split $(x_2, 1 - x_2)$.

Player i with value v will be indifferent about *when* to propose iff

$$v = \int_0^\infty m_i(v) e^{-(m_i(v)+r)t} \pi_i(v) dt = \frac{m_i(v)}{m_i(v) + r} \pi_i(v) \Leftrightarrow \pi_i(v) = [1 + r/m_i(v)]v \qquad (1)$$

Thus, $\pi_i(v) > v$ iff $m_i(v) < \infty$. In other words, all offers are strict concessions given delay: a player expects to have offered to her a fraction of the pie strictly in excess of her aspiration level, should she not offer first. Each subgame is then a war of attrition.

Next, in order that players optimally choose exactly *what* to propose, aspiration levels must be properly calibrated. Hence, their continuation *rejection values* $\rho_i(x_i|v_1, v_2) \leq v_i$, upon having each possible offer x_i declined, must implicitly satisfy

$$\begin{aligned} v_i &= \alpha_{-i}(x_i, v_i)[1 - x_i] + [1 - \alpha_{-i}(x_i, v_i)]\rho_i(x_i|v_1, v_2) - c \\ \Longleftrightarrow \rho_i(x_i|v_1, v_2) &= \frac{v_i + c - \alpha_{-i}(x_i, v_i) + \alpha_{-i}(x_i, v_i)x_i}{1 - \alpha_{-i}(x_i, v_i)} \end{aligned} \qquad (2)$$

Finally, after an offer by Mr. i of x to Mr. $-i$, the latter enjoys the new aspiration value of x, and is therefore willing to mix.

As motivated earlier, the DAE share many intuitive features (rigorous proof missing):

Lemma 1 *In any DAE: (a) any delay arises because of a war of attrition; (b) all serious offers (i.e. on the equilibrium path) are strict concessions; and (c) individuals are strictly disappointed when their offers are declined.*

One question that I wish to resolve is: *Who is more likely to propose, or accept an offer: High or low aspiration players?* Absent a proposer advantage, we intuitively have:

Conjecture 3 *Assume that initial aspiration values are equal. Then the last offer tends to be made by the ex post worse-off player.*

4.3 O -Path Strategies

Delay in bargaining is inefficient, and there are obvious strategic incentives to accelerate the process. Why doesn't the offerer make a slightly stingier than equilibrium proposal that dominates the aspiration values of both players, provided it induces immediate agreement? In models of incomplete information, such offers are deterred simply by interpreting them as evidence that one's type is weak. Here, given the multiplicity of supportable aspiration values, I can simply revert to a feasible equilibrium aspiration pair that is worse for the deviant, and provides an incentive to decline any extra generous offers. Figure 2 illustrates how this may not be possible if there are extremal proposals that are accepted with probability one. For then, such no such punishment aspiration pair exists. To rule this out, $\mathcal{A}$ must touch the Pareto frontier of $\mathcal{P}$ at its horizontal and vertical suprema.

That the equilibrium does not collapse merits some reflection. Indeed, Perry and Reny (1993) and Sákovics (1993) show that in a continuous time bargaining game with instantaneous reaction times, so long as players must leave their offer on the table for some time period $\Delta > 0$, then the range of equilibrium payoffs is bounded by the (very close) different first mover Rubinstein outcomes.[24] That extremal efficiency holds here circumvents this conclusion. To player i with aspiration level approaching the highest level $\bar{v}_i \equiv \sup\{v_i | (v_1, v_2) \in \mathcal{A}\}$ that he can enjoy, offers must be made arbitrarily quickly — giving him a weak incentive to decline offers. By this logic:

Lemma 2 (The Exploding Proposal Rate) *In any DAE,* $\lim_{v \uparrow \bar{v}_i} m_{-i}(v) = \infty$.

As I only support the interior values in $(1 - \bar{v}_2, \bar{v}_1)$, I never need an infinite proposal rate.

[24] Or equivalently, in this context, some random time with mean $\Delta > 0$.

4.4 Intertemporal Dynamics

Ariing quite naturally in this setting is the martingale property. Let $\langle e^{-rt}v_1^t, e^{-rt}v_2^t\rangle$ be the process of discounted aspiration values or the resulting final splits. That is, if an agreement $(x, 1-x)$ obtains at time τ, then $(v_1^\tau, v_2^\tau) = (x, 1-x)$. In equilibrium, players ought not turn down an offer if they expect their value to remaining in the game to worsen; conversely, if their value is expected to improve, they'll accept no offer.

Lemma 3 *The stochastic process $\langle e^{-rt}v_1^t, e^{-rt}v_2^t\rangle$ is a martingale until an agreement.*[25]

4.5 Cross Sectional Dynamics

A major hope of this project is to relate the duration of bargaining to the distribution of ultimate pie splits. Recall that equilibrium demanded an exploding proposal rate for more unequal pie splits. Thus, in a symmetric equilibrium, one cannot decouple the content of a proposal from its timing. I hope to establish the next cross-sectional intertemporal link:

Conjecture 4 (Correlation of Delay and O ers) *For the same initial aspiration pair, longer bargaining sessions are associated with more disparate settlements.*

Here's a partial intuitive explanation. Suppose that Mr. 1 is indifferent about making one of several proposals to Mr. 2. Then the only reason for him to tender a more generous proposal is if it is either more likely to be accepted, or if leads to a higher rejection value (or both). If we ignore the latter possibility, then Conjecture 4 holds: Indeed, the final proposer tends to gets shafted (Conjecture 3), and accelerating the agreement leads him to an even more disadvantageous split for Mr. 1. Next suppose that the rejection aspiration value pair (v_1', v_2') arising from a more generous offer Pareto dominates the corresponding pair (v_1, v_2) stemming from a less generous offer, or $(v_1', v_2') \gg (v_1, v_2)$. Let the associated stochastic agreement times be τ' and τ, respectively. Since $v_1' + v_2' \equiv E[e^{-r\tau^0}] > E[e^{-r\tau}] \equiv v_1 + v_2$, the pair (v_1', v_2') is generally associated with a quicker agreement than is (v_1, v_2). This suggests that quick settlement is associated with more disparate offers along the path. But the final offer is still not nailed.

It may help to link this notion with Harsanyi's quick (1956) derivation of the Nash bargaining solution from Zeuthen's (1930) psychological concession game. He appealed to four axioms: symmetric strategies, common knowledge of strategies; expected utility maximization; and lastly "monotonicity". This last axiom asserts that the chance that

[25] One useful implication of this martingale structure is that any convex function of discounted aspiration values trends up, and any concave function trends down. This may deliver some useful result.

Mr. 1 accepts Mr. 2's offer $(x, 1-x)$ in favour of his own tabled proposal $(x', 1-x')$ is weakly increasing in $x - x'$. But this does not follow from optimality considerations alone, for in the usual reversal with mixed strategies, Mr. 1's mixture must be incentive compatible for Mr. 2 and not himself.

Still, this seems a basic property. What if a player tries to sweeten the pot? Let Mr. 1 be indifferent about offering more to Mr. 2, and therefore be willing to randomize across several offers. As noted, indifference demands that higher offers lead to agreement more often, or induce a greater rejection aspiration value. A lower acceptance chance therefore cannot be rejected out-of-hand, but it perversely leads to Pareto rankable outcomes: Since Mr. 2 prefers the higher offer, both players are clearly better off when Mr. 2 rejects a high offer than when he rejects a low offer. If this cannot occur, then a player can accelerate the pace of negotiations with a higher offer. Higher offers are riskier for the proposer, but lead to a quicker settlement. I might then assume Zeuthen's intuitive operative principle:

A1 (Monotonicity) *Given an aspiration value, the acceptance chance rises in the offer.*

4.6 Sensitivity Analysis

The two main comparative statics derivable in Rubinstein's model should also obtain here, but will be true of the entire class, and thus will only be probabilistically true ex ante. I have yet to properly formulate this notion, so this section is totally heuristic.

Suppose that player i's interest rate rises. If both parties maintain their old strategies, then Mr. i's value from waiting for an offer falls. To maintain an equilibrium, something must give on Mr. $-i$'s behalf. Either he must increase his offer rate, or his pie proposal. But if Mr. $-i$ is proposing more, then either his aspiration value is lower, or Mr. i must be accepting with higher chance, or some combination. Either way we have:

Conjecture 5 (Impatience) *When a player grows more impatient, he tends to accept more often, and be proposed to more frequently. Thus, the game ends stochastically sooner.*

When risk aversion appears, the waiting option is not worth as much, unless players are accepted more often.

Conjecture 6 (Risk Aversion) *When a player grows more risk averse, his offers tend to be accepted more often.*

5. SYMMETRIC PURE STRATEGY OFFERS

Now assume that players are equally patient ($r_1 = r_2 = r$), and both risk neutral. Assume the offers are turned down with positive probability, but that in any subgame a player only contemplates a single offer.

A2 (Pure Proposals) *Players employ a pure strategy proposal function.*

Assume that player $i = 1, 2$ offers $\pi_i(v_1, v_2)$ with flow chance $m_i(v_1, v_2)$ and accepts any offer weakly above $\pi_{-i}(v_1, v_2)$ with chance $\alpha_i(v_1, v_2)$. Call π_i player i's *proposal function,* since it describes the offer that i proposes.

Next, posit that players have undistinguished bargaining roles, so that:

A3 (Symmetry) *Players use identical acceptance functions and proposal functions.*

Thus, under A3, let $\pi(x) = [1 + r/m(x)]x$ be the common proposal function, and $m(x)$ the proposal rate (deduceable from π), and $\alpha(x)$ the acceptance rate function.

Finally, let's assume a basic regularity property:

A4 (Continuity) *Strategies π and α are continuous in aspiration values.*

5.1 The Aspiration Set and Frontier

Given symmetry, the supremum and infimum of aspiration values of $\mathcal{A}$ are $0 \leq \underline{v} < \bar{v} \leq 1$. It helps to pay special attention to the *aspiration frontier* $\langle (x, f(x)), \underline{v} < x < \bar{v} \rangle$, where $f(x) = \sup\langle y | (x, y) \in \mathcal{A} \rangle$ — namely, the maximal attainable continuation aspiration level for player 2 in any continuation game whenever 1 has aspiration level at most x. The next assumption is not generally justifiable, but helps to procure a tractable class of examples. It asserts that the sequence of offers and rejections over time can only achieve a higher aspiration level for one player at the expense of the other: No pair of aspiration values may Pareto dominate another.

A5 (Global Stationarity) *Aspiration pairs always lie on the aspiration frontier.*

I suspect that assumption A5 is true in any pure offering-strategy equilibrium, given A4.

Here, $f : [\underline{v}, \bar{v}] \to [0, 1]$ is a strictly decreasing function, satisfying *interior inefficiency*: $f(x) < 1 - x$ on $(\underline{v}, \bar{v})$. Delay (*viz.* a finite proposal rate $m(x) < \infty$) implies that in present value terms, the aspiration frontier must lie inside the actual payoff frontier. So if $v_2 = f(v_1)$, then $v_1 + v_2 < 1$ and $v_1, v_2 \geq 0$. Since the aspiration frontier arises endogenously, it remains to verify that interior inefficiency is in fact a legitimate restriction.

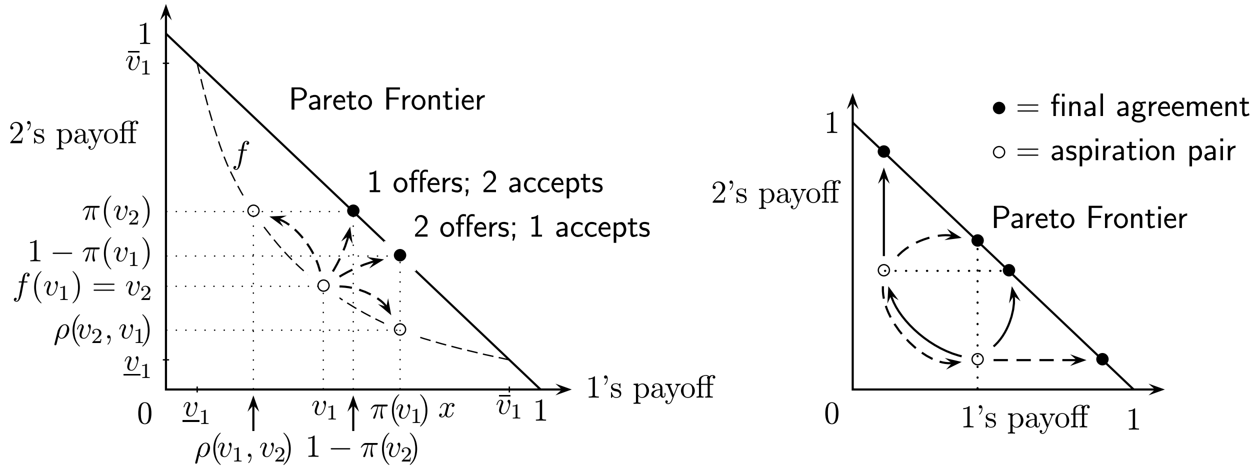

Figure 2: **Pure Strategy Payo Frontier and Transitions.** Discounted aspiration values are a martingale (Lemma 3) with eventual absorption on the Pareto frontier with agreement. The left figure shows how efficiency may prevail at either extreme with a continuum of possible aspiration levels. The right figure depicts the simpler model with just two aspiration value pairs ○, and four possible final agreements •. Solid (dashed) lines are continuations following proposals or concessions by Mr. 1 (Mr. 2). At his lower aspiration value, a player accepts the equilibrium proposal, but his proposals are sometimes turned down by his opponent who is indifferent; see the dotted lines). A two-point aspiration set offers no way of credibly punishing a slightly frugal deviant offer by the player with the worse aspiration value.

Under A4, we may assume for simplicity that strategies are solely functions of one of the pair of aspiration values. Since one's offer rate and proposal function must satisfy one's opponent's incentive constraints, I shall make each a function of one's opponent's aspiration value. Consequently, in equilibrium, v_1 always plans to suggest to her opponent $v_2 = f(v_1)$ the pie split $(1-\pi(v_2), \pi(v_2)) = (1-\pi(f(v_1)), \pi(f(v_1)))$, while player v_2 will suggest to v_1 the pie split $(\pi(v_1), 1-\pi(v_1))$. Here π is the *proposal function,* namely an increasing function $\pi : [\underline{v}, \bar{v}] \to [0,1]$ satisfying $\pi(x) > x$ on $(\underline{v}, \bar{v})$.

5.2 Global Stationarity

I now describe the nature of these DAE, and comment on their cross-sectional structure. This turns out to be linked to intertemporal dynamics.

Lemma 4 *Any stationary players' strategies π and α induce a unique strictly decreasing payoff frontier function f; moreover, any such function f is induced by a unique pair of stationary players' strategies m and α.*

Proof: The first point is quite obvious, while the basic argument behind the second is as follows. If an agreement is not reached at the aspiration pair $(x, f(x))$, then we transit (see figure 2) either to the splits $(\rho(x, f(x)), \pi(f(x)))$ or $(\pi(x), \rho(f(x), x))$. Since these pairs must lie on the frontier, it suffices to show that there are unique and well-defined functions $\alpha(\cdot) \in (0,1)$ and $\pi(\cdot) \in (0,1)$ such that $f(\pi(x)) = \rho(f(x), x)$ and $\pi(f(x)) = f(\rho(x, f(x)))$, i.e.

$$\pi(f(x)) = f\left(\frac{x + c - \alpha(x) + \alpha(x)\pi(f(x))}{1 - \alpha(x)}\right) \tag{3a}$$

$$f(\pi(x)) = \frac{f(x) + c - \alpha(f(x)) + \alpha(f(x))\pi(x)}{1 - \alpha(f(x))} \tag{3b}$$

Since f is decreasing and continuous by assumption A5, $\phi = f^{-1}$ exists. Apply ϕ to both sides of (3a), and evaluate at $\phi(x)$, and similarly rewrite (3b), to get the equivalent equations

$$[1 - \alpha(\phi(x))]\phi(\pi(x)) = \phi(x) + c - \alpha(\phi(x)))[1 - \pi(x)] \tag{4a}$$

$$[1 - \alpha(f(x))]f(\pi(x)) = f(x) + c - \alpha(f(x))[1 - \pi(x)] \tag{4b}$$

The detailed demonstration that (4a) and (4b) uniquely nail down α and π is omitted. □

Since the proposal rate $m(x)$ is embedded in the proposal function $\pi(x)$, there are just two degrees of freedom when thinking about the equilibrium, namely the acceptance and proposal functions. To get a feel for what's going on, it helps to simply focus on one of them. Since Lemma 2 shows that $m(x)$ cannot be constant, and $\pi(x)$ is clearly dependent on x, the most natural class of equilibria to consider entail a well-behaved acceptance rate. We shall see that skewness of the aspiration set derives from the acceptance rate.

Proposition 1 (Symmetry) *Assume* A2–A4*, as well as a constant acceptance function satisfying* $\alpha(x) \equiv \alpha > 0$ *for all* $x \in [\underline{v}, \bar{v}]$*, then*

(a) *The payoff frontier function f is self-inverse, i.e.* $f(f(x)) = x$*;*

(b) *The frontier exhibits interior inefficiency iff delay occurs:* $f(x) < 1 - x \Leftrightarrow m(x) < \infty$*;*

(c) *The proposal function π is increasing.*

For instance, the acceptance function may be constant: $\alpha(x) \equiv \alpha \in (0,1)$.

Proof of part (a): Subtracting (4b) from (4a) yields $(1 - \alpha(f(x)))[f(\pi(x)) - \phi(\pi(x))] = f(x) - \phi(x)$. Define $Q(x) = f(x) - \phi(x)$. Then $Q(x) = [1 - \alpha]Q(\pi(x))$. Iteration then yields

$$|Q(x)| \le (1 - \alpha)^k Q(\pi^k(x)) \tag{5}$$

where π^k denotes compositional power. Now clearly $|Q(x)| \le 1$ for $x \in [\underline{v}, \bar{v}]$. So (5) implies that $|Q(x)| \le (1 - \alpha)^k$ for all $x \in [\underline{v}, \bar{v}]$, and all $k = 1, 2, 3 \ldots$. Since $1 - \alpha < 1$, we have that $(1 - \alpha)^k \to 0$ as $k \to \infty$. Hence $Q(x) = 0$, or $f(x) = \phi(x)$ for all $x \in [\underline{v}, \bar{v}]$, so that f is self-inverse.

Proof of part (b): It suffices to argue $f(x) < 1 - x \Leftrightarrow \pi(x) > 0$ for $x \in (\underline{v}, \bar{v})$. Since f is decreasing, and $f(y) < 1 - y$:

$$f(y) < (1-\alpha)f(y) + \alpha(1-y) \tag{6}$$

for all $y \in (\underline{v}, \bar{v})$ and any $\alpha < 1$. Then

$$\pi(x) = f(f(\pi(x))) > f[(1-\alpha(\pi(x)))f(\pi(x)) + \alpha(\pi(x))(1-\pi(x))] = f[f(x)] = x \tag{7}$$

follows in order from the facts that (i) f is self-inverse; (ii) inequality (6) holds and f is decreasing; (iii) equation (3b) implies that $f(x) = (1-\alpha)f(\pi(x)) + \alpha(1-\pi(x))$; and (iv) f is self-inverse.

Proof of part (c): Recall that $f(x) = (1-\alpha)f(\pi(x)) + \alpha(1-\pi(x)) - c$. Hence, f decreasing implies that if $x > y$ then

$$[1-\alpha]f(\pi(x)) + \alpha[1-\pi(x)] = f(x) > f(y) = [1-\alpha]f(\pi(y)) + \alpha[1-\pi(y)]$$

so that $\pi(x) < \pi(y)$, again because f is decreasing. □

Symmetry of $\mathcal{A}$ ought to hold more generally from A3. But the montonicity of π certainly appeals to the constancy of α.

A Simple Example. There exists a nice one-parameter family of solutions for the case $\underline{v} = 0$, $\bar{v} = 1 - c$. For constants $\psi > 0$ or $\psi < -1$, it's easy to verify that any f of the form $f(x) = \psi(1-c-x)/(x+\psi)$ is self-inverse, decreasing, and satisfies $f(0) = 1 - c$, $f(1-c) = 0$.[26] As it turns out, to force $\pi(x) > x$, we'll only be interested in $\psi > 0$.

Note that for f self-inverse, the two relations (4a) and (4b) are equivalent, so let's just work with (4b), assuming as in Theorem 1, that $\alpha(\cdot) = \alpha$. Temporarily replace $\pi(x)$ by y to get

$$(1-\alpha)\psi(1-c-y)/(y+\psi) = \psi(x-1+c)/(x+\psi) - \alpha + \alpha y \tag{8}$$

This yields a quadratic equation for y in terms of x, α, c, ψ. For instance, with $\psi = 1$, we have the following proposal function

$$\pi(x) = \left(\alpha(1-c+x) - 2 + \sqrt{[\alpha(1-c+x)-2]^2 + 8\alpha x}\right)/2\alpha(1-c+x)$$

[26] The idea, which I owe to Dan Cass, is to work with an easy class of self-inverse functions, namely the conics. The function in question is a rescaling and coordinate change of the hyperbola $xy = k$. Another choice of 'near' conic would be generalized ellipses $x^p + y^p = 1 - c$. For $p = 1$ this gives $f(x) = 1 - c - x$, while for $0 < p < 1$ it bows below this line. While having the advantage of being more symmetric, the drawback of this family is that there are no closed form solutions.

6. CONCLUSION

Summary. Correctly understanding bargaining is clearly an important task for economists and social scientists. For as Schelling realized, "most conflict situations are essentially bargaining situations." While static bargaining payoffs are literally a constant sum game, Schelling's point was rather that pure competition is quite rare. Instead, the *dynamic* bargaining game is a pure coordination in time (sooner is better) adjoined to a pure competition game in the pie split; the overly trite 'coopetition' unfortunately well describes the bargaining blend. With pure payoff discounting, this is a multiplicative mixture, while with explicitly costly bargaining it is additive. Temporal monopoly renders the coordination aspect purely discrete, while maintaining continuous the competitive element. As a result, coordination is perfectly achieved. In the model of van Damme, Selten, and Winter (1990) with discrete money that Binmore so well critiqued, both elements are discrete, and with such a balance anything can happen. This paper takes another balanced tack, assuming continuous competitive and coordination aspects.

Until Rubinstein's exploitation of the temporal monopoly paradigm to deduce his unique outcome on rationality considerations alone, equilibrium-based theories — arguing how players should behave — of bargaining were standard fare (even though a rational strategic notion of equilibrium did not exist). Hicks' (1932) and Zeuthen's (1930) outcomes can be viewed as equilibrium approaches; Schelling himself in 1957 offered his own overarching informal theory of bargaining based on focal points. I offer a simple new equilibrium-based approach to complete information bargaining theory that offers a richer dynamic story of the bargaining process than Rubinstein. This theory is motivated by observation, and normative considerations. I find that traditional informational screening explanations of many extended labour strikes are sometimes especially stretched. Here, delay is entirely a complete information phenomenon.

A dynamic theory of bargaining depends on players being able to make credible threats. Ståhl's bargaining model was among the earliest applications of subgame perfection — the first credible dynamic solution concept. Yet the temporal monopoly assumption is so powerful as to swamp all other considerations in bargaining. It assigns extreme importance to a factor that intuitively ought not play a key role in players' considerations: Namely, that a slight time elapse occurs before their opponent can speak. I have seen no serious justification of this principle aside from its self-evident technical appeal. I have instead tried to motivate an alternative approach by quoting from the words of giants.

In all but the final logical step, this paper follows closely in the footsteps of Rubinstein

(1982). He explores the bargaining model with offer-accept/reject structure and time preference in discrete time. This paper does so in continuous time, and where players' strategies are governed by the expected payoffs alone. This framework clearly cannot be justified on the theoretical grounds of uniqueness or efficiency, as can Rubinstein's game, but then again neither can observed fact. Neither is surely a litmus test, since repeated games is deemed an interesting and important paradigm despite the folk theorem. And theoretical considerations aside, the model offers parameters such as the acceptance and proposal rates that can be calibrated to actual bargaining situations; it should yield many testable predictions about the joint distribution of the length of the bargaining process and the ultimate distribution, as well as the short run bargaining dynamics.

Extensions and Di erent Approaches. In this paper, I explored pure time impatience. I could just as well assume that time passage is explicitly costly. In Rubinstein (1982), this alternative assumption wrecked havoc with uniqueness and symmetry, but here the change is not so dramatic. It will still induce a war of attrition along the lines of the game of chicken. I suspect that many of the properties of the equilibria will persist.

A richer theory of bargaining will incorporate uncertain discount rates or valuation parameters — the next most important consideration in conflicts. In that case, *incomplete information* wars of attrition will no doubt emerge[27] — for much the same reason that, for instance, the 'all-pay' auction does. With informational signalling and bluffing, the mixing assumed in this paper may no longer be needed, as players will earn informational rents, as in traditional incomplete information bargaining models. A different theory may also adjust the strategy space I allow to admit the possibility of binding commitments, or nonexploding offers, that has been the focus of the historical literature, may be properly addressed. In such a setting, since the amount of the pie that remains to be bargained over is falling, this introduces a nonstationary element to the problem that intuitively ought to hasten the final agreement. All such extensions require a solid understanding of this stationary complete information paradigm. Finally, we might consider the often observed possibility that negotiations may sometimes temporarily reach an impasse, as either party may credibly "break off negotiations" and "walk away from the table" for a set time.

[27] See the discussion in Fudenberg and Tirole (1991), page 219, for definitions, and Abreu and Gul (1994) or Ordover and Rubinstein (1985) for applications of this genre.

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