Income and Substitution Effects: The Slutsky Equation

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Outline

Effects of a price increase

**Substitution Effect:**
this stuff is more expensive, so I should buy less

**Income Effect:**
my real income has gone down, because the things I buy cost more
so I should buy less of everything (with some exceptions ...)

**Slutsky Equation:** how big are the two effects?
Slutsky Equation

\[ \frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^I}{\Delta I} x_1 \]

Compensation for a price change (Slutsky version)
Change income so that the old consumption plan is just affordable
Pivot the budget line through the old plan
Slutsky Equation

Derivation
If price increases, add just enough income to pay the extra charge:

$$\Delta I = \Delta p_1 \times x_1$$

Total Effect

$$\Delta x_1 = x_1 (p_1 + \Delta p_1, I) - x_1 (p_1, I)$$

Substitution Effect

$$\Delta x_1^s = x_1 (p_1 + \Delta p_1, I + \Delta I) - x_1 (p_1, I)$$

Income Effect

$$\Delta x_1^I = x_1 (p_1 + \Delta p_1, I + \Delta I) - x_1 (p_1 + \Delta p_1, I)$$
Example

Several people go to dinner at a restaurant. They have the same preferences: Cobb-Douglas, over food, $f$, and other stuff, $y$. They all have the same amount of money to spend, $I$.

**Separate Checks:**

$$\max_f fy$$

$$pf + y = I$$

Substitute using the budget constraint

$$\max_x x(I - x)$$

$x = pf$ (expenditure on food — or just say $p = 1$, so $x$ and $f$ are the same thing).

A quadratic function, zero at $x = 0$ and at $x = I$, positive in between symmetric in $x$ and $I - x$, maximal at $x = \frac{I}{2}$.
Example

Solution:

\[ f^* = \frac{I}{2p} \]

Split the Check:
Suppose the others spend \( x_0 \) (each)
If there are \( n \) people, you pay \( \frac{x + (n-1)x_0}{n} \)
so

\[
\max_x x \left( I - \frac{x + (n-1)x_0}{n} \right)
\]

or

\[
\max_z z \left( I_0 - z \right)
\]

where \( z = \frac{x}{n} \) (your share of the cost of your food)
and \( I_0 = I - \frac{(n-1)x_0}{n} \)
(your money after you’ve paid your share of everyone else’s food)
Example

Solution:

\[ \hat{z} = \frac{I_0}{2} \]

so

\[ \hat{x} = n \frac{I_0}{2} = \frac{nI - (n - 1)x_0}{2} \]

But everyone else is making these same calculations so \( \hat{x} = x_0 \) and

\[ 2\hat{x} = nI - (n - 1)\hat{x} \]

and then

\[ \hat{x} = \frac{n}{n + 1} I \]
Example

So if there are 4 people, each person spends 80% of their money on food, leaving 20% for other stuff.

And everyone is worse off (the optimal split is 50-50).

The point is that if there are 4 people, ordering a $20 item only costs $5. So food is cheaper, and the substitution effect means it is optimal to buy more
Example (n=4)

\[ x^* = \frac{1}{2} \times 40 = 20 \]
\[ \hat{x} = \frac{4}{5} \times 40 = 32 \]
Remark: This might be the real reason why restaurants don’t like separate checks (especially for large parties).