Outline

Effects of a price change

**Substitution Effect:**
this stuff is more expensive, so I should buy less

**Income Effect:**
my real income has gone down, because the things I buy cost more
so I should buy less of everything (with some exceptions ...)

**Slutsky Equation:** how big are the two effects?
Slutsky Equation

\[ \frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x^s_1}{\Delta p_1} - \frac{\Delta x^I_1}{\Delta I} x_1 \]

Compensation for a price change (Slutsky version)
Change income so that the old consumption plan is just affordable
Pivot the budget line through the old plan
Slutsky Equation

**Derivation**
If price increases, add just enough income to pay the extra charge:

\[ \Delta I = \Delta p_1 \times x_1 \]

Total Effect

\[ \Delta x_1 = x_1 (p_1 + \Delta p_1, I) - x_1 (p_1, I) \]

Substitution Effect

\[ \Delta x_1^s = x_1 (p_1 + \Delta p_1, I + \Delta I) - x_1 (p_1, I) \]

Income Effect

\[ \Delta x_1^I = x_1 (p_1 + \Delta p_1, I + \Delta I) - x_1 (p_1 + \Delta p_1, I) \]
Example

Several people go to dinner at a restaurant
They have the same preferences: Cobb-Douglas, over food, $f$, and other stuff, $y$
They all have the same amount of money to spend, $I$

Separate Checks:

\[
\max_f f y \\
pf + y = I
\]

Substitute using the budget constraint

\[
\max_x x (I - x)
\]

$x = pf$ (expenditure on food – or just say $p = 1$, so $x$ and $f$ are the same thing)
A quadratic function, zero at $x = 0$ and at $x = I$, positive in between
symmetric in $x$ and $I - x$, maximal at $x = \frac{I}{2}$
Example

Solution:

\[ f^* = \frac{I}{2p} \]

Split the Check:
Suppose the others spend \( x_0 \) (each)
If there are \( n \) people, you pay \( \frac{x + (n-1)x_0}{n} \)
so

\[
\max_x x \left( I - \frac{x + (n-1)x_0}{n} \right)
\]

or

\[
\max_z z (I_0 - z)
\]

where \( z = \frac{x}{n} \) (your share of the cost of your food)
and \( I_0 = I - \frac{(n-1)x_0}{n} \)
(your money after you’ve paid your share of everyone else’s food)
Example

Solution:

\[ \hat{z} = \frac{I_0}{2} \]

so

\[ \hat{x} = n \frac{I_0}{2} \]
\[ = \frac{nI - (n - 1)x_0}{2} \]

But everyone else is making these same calculations
so \( \hat{x} = x_0 \) and

\[ 2\hat{x} = nI - (n - 1) \hat{x} \]

and then

\[ \hat{x} = \frac{n}{n + 1} I \]
Example

So if there are 4 people, each person spends 80% of their money on food, leaving 20% for other stuff. And everyone is worse off (the optimal split is 50-50). The point is that if there are 4 people, ordering a $20 item only costs $5. So food is cheaper, and the substitution effect means it is optimal to buy more
Example (n=4)

\[ x^* = \frac{1}{2} \cdot 40 = 20 \]

\[ \hat{x} = \frac{4}{5} \cdot 40 = 32 \]
Example

**Remark**: This is probably the real reason why restaurants don’t like separate checks (especially for large parties).