1. Suppose that a preference ordering on $\mathbb{R}_+^2$ can be represented by the utility function

$$u(x) = \frac{1}{2}x_1^2 + \ln(x_2)$$

(a) Suppose the prices are $p_1 = 2$, $p_2 = 1$. If wealth is $w = 4$, what is the optimal consumption plan?

(b) Suppose that wealth increases to 5. Now what is the optimal consumption plan?

(c) Can you find a price vector and a wealth level such that $x_1(p, w) = 1$?

2. (a) Prove Hotelling’s Lemma (if $\pi(p)$ is the profit function for the production set $Y$, and $y(p)$ is the supply function, then $\nabla \pi(p) = y(p)$, where $\nabla \pi(p)$ is the gradient of $\pi$ - the vector of partial derivatives with respect to the prices).

(b) Show that a factor of production is inferior if and only if an increase in the price of that factor reduces marginal cost.

3. Prove that a risk averse person buys full insurance if the price is actuarially fair, and buys less than full insurance if the price is not actuarially fair.

4. Pick TWO of the following assertions. State whether they are true, false or ambiguous, and explain why.

(a) A firm uses 10 units of labor and 20 units of capital to produce 10 units of output. The marginal product of labor is 0.5. If there are constant returns to scale, the marginal product of capital must be 0.25.

(b) The utility function $u(x_1, x_2) = x_1^2 + \sqrt{x_2}$ is not quasi-concave, so the expenditure function derived from this utility function is not concave in prices.

(c) Constant relative risk aversion implies that the demand for insurance is a decreasing function of wealth.