Amplification of Productivity Shocks: Why Vacancies Don’t Like to Hire the Unemployed?¹

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PRELIMINARY — COMMENTS WELCOME

Abstract

In this paper I study a new amplification mechanism in search models that arises when workers can choose to search on the job and, despite the fact that all workers are ex-ante identical, employers prefer to hire already employed workers for endogenous reasons. The motivation for on-the-job search in the model is job-shopping, where workers look for jobs they find appealing, and the appeal of a job to the worker is not observed by the firm. In equilibrium, workers arriving from unemployment are more likely to leave a job for a more appealing job, and, knowing this, firms prefer to hire already employed, as opposed to unemployed, workers.

Employers’ preference for hiring already employed workers introduces a new amplification mechanism into search models. This is because vacancies in the model with such preference respond more to aggregate shocks than in the standard search model due to the fact that employed workers reduce their search intensity in a recession, thereby making it less attractive for firms to post vacancies. Using simulations of the proposed model, I explore the extent that the presence of job-to-job transitions can help in explaining the volatility of unemployment and vacancies over the business cycle through this new amplification mechanism. The simulation results show that, for standard parameter values, this new mechanism can generate five times more amplification compared to the baseline model.

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1 Introduction

Close to half of all labor market transitions are job-to-job transitions: moves of workers from one employer to another without any intervening unemployment (Nagypál (2004), Fallick and Fleischman (2004)). Despite their magnitude, such job changes have often been ignored by macroeconomists in the formal modeling of aggregate labor market dynamics over the business cycle. In this paper, I consider a model of on-the-job search with the novel feature that firms prefer to hire already employed workers, expecting this to yield higher profits. While most recruitment professionals are acutely aware of this preference of firms, to date macroeconomists have not considered ways to incorporate such a feature into models of the aggregate labor market. In fact, none of the existing models of on-the-job search have this feature, since they all implicitly feature a preference by firms for hiring the unemployed. I show that the novel feature I introduce gives rise to a new amplification mechanism in search models. Despite the fact that all workers are ex-ante identical, employers’ prefer to hire already employed workers. This “preference” arises endogenously in the model because the expected profits from hiring an unemployed worker are lower than those from hiring an employed worker. The reason for this is that workers hired from unemployment have higher expected turnover as they are willing to accept even low quality matches and then to continue to search on the job. Therefore, it is less profitable for firms to undertake the necessary investment needed to create employment relationships when the composition of searchers shifts towards unemployed workers during a recession, thereby stifling vacancy creation during these bad times.

The model I consider builds on the search and matching framework. In the last two decades, search and matching models have gained wide popularity in the analysis of aggregate labor markets, due to their ability to explain several labor-market phenomena that the standard neo-classical growth model cannot tackle, such as the existence of equilibrium unemployment. Several authors have asserted that these models can also explain quantitatively the cyclical variation in key labor-market variables. Recently, however, this view has been challenged,
and the search and matching approach has been criticized for its lack of amplification (Shimer (2004)). Shimer argues that earlier works wrongly declared success because they did not consider either the magnitude of the exogenous driving forces (Blanchard and Diamond (1989), Mortensen and Pissarides (1994), Cole and Rogerson (1999)) or the highly negative correlation between unemployment and vacancies (Andolfatto (1996), Merz (1995), Ramey and Watson (1997), Gomes, Greenwood, and Rebelo (2001)). He shows that, once both of these facts are taken into account, the textbook search model fails to match quantitatively the cyclical properties of key labor market variables, such as the unemployment and vacancy rate. In response to shocks to the productivity of employment relationships, the standard search model results in an elasticity of unemployment and of vacancies that is approximately a tenth of the elasticities observed in the data. In response to shocks to the separation rate, on the other hand, the standard search model results in a positive correlation between unemployment and vacancies, while the data show a strong negative correlation.

A commonly adopted simplification in models of search is that there are no job-to-job transitions (this holds, for example, in all the works mentioned above), despite the fact that this has been recognized very early on (Tobin (1972)) as a serious shortcoming. There are several models in the literature that explicitly model on-the-job search and the resulting job-to-job transitions, and these provide many important insights into the ways in which on-the-job search alters labor market outcomes. These models, however, either do not study cyclical fluctuations, since that is not the focus of their analysis (Burdett and Mortensen (1998), Burdett, Imai, and Wright (2004)), or they do not consider the extent of amplification generated by the model (Mortensen (1994), Pissarides (1994), Barlevy (2002)). In contrast, in this paper I construct a tractable search model with job-to-job transitions that is capable of generating more amplification than the baseline model and that can quantitatively match salient features of job-to-job transitions both in terms of their magnitude and in terms of their cyclicality.

The reason that existing frameworks with on-the-job search (Pissarides (1994), Mortensen (1994), Burdett and Mortensen (1998), Barlevy (2002)) do not help in resolving the amplification puzzle is that they all have the common feature that the expected payoff to employers
is higher when hiring unemployed workers than when hiring employed workers.\(^4\) This exacerbates the lack of amplification, since, during recessions, the extent of on-the-job search declines and the composition of the searching pool shifts towards the unemployed, meaning that the pool of searchers changes \textit{in favor} of vacancy creation. For example, while Barlevy (2002) does not report the elasticity of unemployment to productivity shocks in his model, it can be calculated from the numbers that he reports to be below a hundredth of the elasticity observed in the data.

I argue that a feature that is both more plausible and provides scope for amplification is one where employers benefit more from hiring employed workers than from hiring unemployed ones. Given this feature, there is a complementarity between vacancies and employed workers in the matching market. Increased search activity from either of them during a boom encourages search activity from the other party, leading to increased amplification. I also argue, based on the work of Eriksson and Lagerstrom (2004), that there is compelling evidence that employers indeed expect to reap a higher payoff from hiring employed workers.

How could a higher expected payoff to firms from hiring employed workers arise? There are essentially two types of mechanisms that can generate higher payoff from the hiring of employed workers. One of them is based on the idea that unemployed workers are “desperate” and will readily accept all matching opportunities, even the ones that lead to a disadvantageous outcome for the firm. The other is based on the idea that unemployment is a sign of being a less desirable worker because, for example, less reliable workers are more likely to become unemployed.

Let us consider a general setup, and denote by the vector \(\mathbf{x}\) the type of match that is being created by the firm and the worker, where \(\mathbf{x}\) contains any variable that is relevant in determining the payoff to the firm from creating the match, such as a measure of productivity net of wages or match quality. Let us normalize \(\mathbf{x}\) so that the payoff to the firm is increasing in \(\mathbf{x}\).

\(^4\) It should be noted, however, that the emphasis of these authors have not been the role of on-the-job search in amplifying productivity shocks. A notable exception is Shimer (2003), who studies amplification with on-the-job search. The mechanism in his model is very different from the one I will study in this project, since he departs in several ways from the standard search model. In his model, a higher expected benefit from hiring \textit{unemployed} workers can actually help in resolving the amplification puzzle.
Let the distribution of $x$ be $F_u(x)$ among unemployed searchers, and $F_e(x)$ among employed searchers. When contemplating vacancy creation, conditional on meeting an unemployed worker, the profits a firm can expect to make are

$$
\Pi_u = \int \pi(x) A_u(x) dF_u(x),
$$

(1)

where $A_u(x)$ is the probability that an unemployed worker will accept a type $x$ job and $\pi(x)$ is the profit from creating a job of type $x$ (this can include the probability that the firm will choose to create the match conditional on possibly observing some elements of $x$). Correspondingly, the expected profits from meeting an employed worker are

$$
\Pi_e = \int \pi(x) A_e(x) dF_e(x).
$$

(2)

A general feature of models with on-the-job search in which an employed worker can choose to quit to unemployment is that the acceptance rate of employed workers is lower than that of unemployed workers for a given type of match. The reason for this is that the outside option of an employed worker (their current employment relationship) is better than the outside option of an unemployed worker; if this was not the case, the employed worker would quit to unemployment. Hence $A_u(x) \geq A_e(x)$ for all possible values of $x$.\(^5\)

Then, if $\pi(x) \geq 0$ for all $x$, which is the case in all models where the firm has full information about $x$ and can reject some matches, and $F_u$ weakly first-order stochastically dominates $F_e$,\(^6\) then it is necessarily the case that the expected profits from meeting an unemployed worker are at least as large as the expected profits from meeting an employed worker; i.e., $\Pi_u \geq \Pi_e$. Indeed, this is the case in the above mentioned models with on-the-job search.

To generate $\Pi_u < \Pi_e$, it is necessary to have at least one of two features, 1) $\pi(x) < 0$ for

\(^5\)Notice that the above formulation encompasses setups in which there is asymmetric information between the worker and the firm at the time they meet, since in that case the law of iterated expectations can be used to arrive to the above formulation.

\(^6\)This can happen even without any unobserved heterogeneity favoring the unemployed searchers, if the nature of wage setting is such that the fact that employed searchers have better outside options leads them to command higher wages than unemployed searchers. This could be the case if the existing and the potential new employer of the employed searcher engage in a Bertrand competition.
some \( x \) or 2) a distribution \( F_e \) that strictly first-order stochastically dominates \( F_u \). I call explanations where \( \Pi_u < \Pi_e \) results from \( \pi(x) < 0 \) for some \( x \) acceptance-rate-differences-based explanations, since in these a higher acceptance rate by unemployed workers does not necessarily increase the expected payoff of the firm. The explanations where \( \Pi_u < \Pi_e \) results from \( F_e \) strictly first-order stochastically dominating \( F_u \), on the other hand, I call unobservable-heterogeneity-based explanations. In this paper I explore an acceptance-rate-differences-based explanation. Exploring the unobservable-heterogeneity-based explanations is left for future work.

Notice that a negative payoff at the time of the creation of the employment relationship, i.e. a negative \( \pi(x) \), is not a feature of the standard search model (in the tradition of Pissarides (2000) or Mortensen and Pissarides (1994)); since in that model, all costs of creating a vacancy are born prior to meeting a worker, through vacancy-creation costs. It is a natural extension to consider, however, that the firm has to expend some additional resources (on training, relocation, or other match-specific investments) at the time the match is formed. In addition, in order for firms to enter matches that lead to a negative payoff (i.e., \( \pi(x) < 0 \) for some \( x \)), it is necessary that some component of \( x \) is not observable to the firm. I introduce asymmetric information in the model by assuming that the firm has less information about the quality of the match that is being formed than the worker. This is naturally the case when the quality of the match is observed only by the worker and enters directly only into the utility function of the worker.

Given this asymmetric-information problem, the basic mechanism of the model is simple. Workers can undertake job shopping at a cost both while unemployed and while employed, where job shopping simply means searching for a match with a higher idiosyncratic value to the worker. Unemployed workers are “desperate:” they are willing to accept any idiosyncratic value above some minimum threshold. Employed workers, on the other hand, are more selective, and only accept matches that have a value above the value of their current match. Turnover declines with the idiosyncratic value of the match for two reasons. First, the probability of finding a better match declines. Second, as a consequence, the incentives to search for a better job also decline, thereby leading to lower endogenous search effort.
This means that the expected turnover of previously unemployed workers is higher than that of previously employed workers, making them less attractive candidates for firms to hire. Of course, this higher turnover has to be weighed against the higher acceptance rate of unemployed workers. One can show that the turnover effect can outweigh the acceptance-rate effect when there is a possibility that the firm will make negative profits in a match. Amplification then is a direct result of this mechanism. Vacancies that reap higher expected payoffs from hiring employed workers respond more to aggregate shocks than in the standard search model, since employed workers reduce their search intensity in a recession, thereby making it less attractive for firms to post vacancies during these bad times.

One crucial issue that needs to be addressed is that of wage setting: the way in which the surplus that arises in the model with search frictions is shared by the worker and the firm. There is no obvious way to solve this issue in a model with on-the-job search and asymmetric information. I adopt the assumption that the parties in a match split the expected surplus according to some sharing rule, since this is the closest equivalent in this model to the standard assumption of Nash bargaining in the textbook search model, thereby making the quantitative results the most comparable. In future versions of this paper I will study the robustness of my results to alternative wage setting mechanisms and I will also study the solution to the appropriately defined social planner’s problem, since this has the advantage of sidestepping the issue of wage setting.

The qualitative and quantitative results of the numerical exercise are quite promising. They show that on-the-job search and job-to-job transitions vary positively with aggregate productivity shocks when comparing stationary equilibria for different aggregate productivity levels. Quantitatively, though, the extent of job-to-job transitions is only half as large as in the data. My results also show that the amplification mechanism embedded in the model shows up clearly when considering the response of the unemployment and the vacancy rate to changes in aggregate productivity. In the standard model, as Shimer (2004) has shown, the elasticity of the vacancy-unemployment ratio with respect to labor productivity (which in the standard model is equal to market tightness) is below 2 for reasonable parameter values. In this model the elasticity of the vacancy-unemployment ratio is 9.07 for the same
parameter values, due to both the decline in unemployment (elasticity of \(-5.44\)) and the increase in vacancy rate (elasticity of 3.63).

Moreover, when considering shocks to the separation rate, my results show that the vacancy unemployment ratio varies negatively with job-destruction shocks and hence with the unemployment rate. Shimer (2004) argues that destruction shocks induce a positive correlation between vacancies and unemployment rate in the standard model, thereby causing the vacancy unemployment ratio to have a positive elasticity with respect to adverse destruction shocks. In my model, because a higher destruction rate discourages vacancy creation, it shifts the composition of searchers towards the unemployed both through its direct effect and its indirect negative effect on the search intensity of employed workers. This brings destruction shocks back into the picture as a plausible source of business cycle variation in the vacancy-unemployment ratio.
2 Environment

Time is continuous and goes on forever. There is a unit measure of infinitely-lived workers, who are ex-ante identical. Workers can be either employed or unemployed, and the objective of workers is to maximize

\[
\int_{t=0}^{\infty} e^{-rt} y_t, \tag{3}
\]

where

\[
y_t = \begin{cases} 
    w_t + \mu_t - c(s_t) & \text{if employed,} \\
    b - c(s_t) & \text{if unemployed.} 
\end{cases} \tag{4}
\]

Here \(w_t\) is the wage received when employed at time \(t\), \(\mu_t\) denotes the attractiveness or appeal to employed workers of their current employment match, in other words, workers derive utility from being on a job they “like”, and \(s_t\) denotes the search effort of the worker at time \(t\). The appeal of a job to the worker, \(\mu\) (which I will also call match quality below), is determined upon meeting a potential employer, and is drawn from the distribution \(F(\cdot)\), where \(F : [\underline{\mu}, \bar{\mu}] \to [0, 1]\) is a continuous, twice differentiable, strictly increasing distribution function, and \(\underline{\mu} \in \mathbb{R} \cup \{-\infty\}\) and \(\bar{\mu} \in \mathbb{R} \cup \{\infty\}\). In addition, workers can choose to engage in search at a flow cost of \(c(s_t)\), where

\[
c(s_t) = \begin{cases} 
    c_0 + \hat{c}(s_t) & \text{if } s_t > 0 \\
    0 & \text{if } s_t = 0 
\end{cases}, \tag{5}
\]

where \(\hat{c}(\cdot)\) is a strictly increasing, strictly convex, twice continuously differentiable function. This means that there is both a fixed cost and a variable cost of searching, so that when the incentives become sufficiently low, the worker stops searching all together. Finally, \(b\) denotes the constant utility flow (derived from leisure and/or from unemployment insurance benefits) that a worker receives while unemployed.
There is a large measure of ex-ante identical firms. Firms’ objective is to maximize

\[
\int_{t=0}^{\infty} e^{-rt}(\pi_t - K\xi_t),
\]

where

\[
\pi_t = \begin{cases} 
0 & \text{if firm is inactive} \\
-c_v & \text{if firm is active with a vacant job} \\
p - w_t & \text{if firm is active with a filled job.} 
\end{cases}
\]

\[
\xi_t = \begin{cases} 
1 & \text{if a new match is created at time } t \\
0 & \text{otherwise} 
\end{cases}
\]

This means that any firm can enter the market and become active by posting a vacancy at flow cost \(c_v\). If a firm posts a vacancy, then it participates in the matching market for creating new matches. When a firm creates a match, it needs to pay a one-time match-specific start-up cost of \(K\), and then it receives a flow profit of \(p - w_t\) until the match dissolves. Here \(p\) is the output of a match, which is assumed to be the same for all matches. Matches dissolve for exogenous reasons at rate \(\delta\) and endogenously when the worker decides to form a new employment relationship as a consequence of on-the-job search.

There is a single matching market with a meeting function determining the number of meetings \((m_t)\) as a function of the total amount of search effort of workers \((s_t)\) and the number of vacancies posted \((v_t)\):

\[
m_t = m(s_t, v_t),
\]

such that \(m_s(s, v) > 0, m(0, v) = 0\) for any \(v\), \(m_v(s, v) > 0, m(s, 0) = 0\) for any \(s\). I assume that \(m(s, v)\) has constant returns to scale, so that the meeting rate per unit of search effort for workers can be written as

\[
\lambda_t = \lambda(\theta_t) = \frac{m(s_t, v_t)}{s_t} = m\left(1, \frac{v_t}{s_t}\right) = m\left(1, \theta_t\right),
\]
where $\theta_t = \frac{v_t}{s_t}$ is market tightness at time $t$. Similarly, the meeting rate for firms can be written as

$$\eta_t = \eta(\theta_t) = \frac{m(s_t, v_t)}{v_t} = \frac{m(s_t, v_t)}{\frac{s_t}{s_t}} = \frac{\lambda(\theta_t)}{\theta_t}. \quad (11)$$

The timing of match formation is as follows. If a worker and a firm meet, the worker observes the appeal of the potential match and can decide whether or not to form the match. In order to form the match, an already employed worker needs to end his current relationship. The relevance of this assumption is that it implies that the outside option of all workers is unemployment. The firm does not observe neither the appeal of the match to the worker nor whether the worker was previously unemployed or employed. If the worker agrees to forming a match, then wages are determined upon the formation of the match by splitting the expected surplus such that the worker receives $\beta$ fraction of it. Wages are subsequently renegotiated only if otherwise the participation constraint of the parties would be violated conditional on the worker not being able to credibly communicate the existence of a possibility to form a new match. Firms rationally form and update their beliefs about the appeal of a job to the worker based on the information available to them.

It is worth commenting on the particular choice of the wage setting mechanism. While there is no micro-foundation or axiomatic basis for the chosen wage-determination mechanism, it is chosen to keep the model as close as possible to the standard search model that is studied in Shimer (2004). Without asymmetric information and on-the-job search, Nash bargaining implies the sharing of the surplus between the worker and the firm. Therefore, assuming surplus sharing in the environment studied provides the most direct comparison with the standard model. It would be worthwhile to study, and it is an issue I will consider in future work, how departing from surplus sharing affects the results reported below.

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7 An alternative way to justify unemployment being the outside option of all workers is to assume that matches cannot be “recalled” and that there is scope for renegotiation between the worker and the firm immediately after the worker has moved from the old employer to their current employer.
3 Equilibrium

3.1 Definition of stationary equilibrium

For the sake of simplicity, and to keep the analysis tractable, I consider what happens in
the above described economy in a stationary equilibrium. Clearly, given the assumptions
on wage-setting above, in a stationary equilibrium, the only new information that arrives
to a worker-firm pair while in a match is whether the match is still in existence or not,
therefore the only state variable that enters the asset values of workers and firms, besides
the wage and the quality of the match, is the length of the relationship. Let the value of
unemployment be $U$, the value of a worker employed in a match of quality $\mu$, tenure $\tau$, and
wage $w$ be $W(w, \mu, \tau)$, the value of a vacancy be $V$, and the value of employment for the
firm in a match of tenure $\tau$ and wage $w$ be $J(w, \tau)$.

Definition 1. A recursive stationary search equilibrium is unemployment rate $u$, vacancy
rate $v$, asset values $\{U, V, W(w, \mu, \tau), J(w, \tau)\}$, wage function $w(\tau)$, workers’ search decisions $s(w, \mu, \tau)$, and distribution of employed workers $G(w, \mu, \tau)$ such that

- $U$ and $W(\cdot)$ are the value of unemployment and of working for workers making optimal
matching, searching, and acceptance decisions given $u, v, w(\cdot)$, and $G(\cdot)$, and $s(\cdot)$ is
the corresponding optimal search policy.

- $V$ and $J(\cdot)$ are the value of a vacancy and of a filled job for firms making optimal
vacancy creation and matching decisions given $u, v, w(\cdot)$, and $G(\cdot)$.

- Agents update their beliefs rationally.

- There is free entry of vacancies.

- Wages are determined by sharing of the expected surplus upon meeting and are subse-
quently renegotiated only if otherwise the participation constraint of the parties would
be violated.

- The distribution $G(\cdot)$ is consistent with the decisions of the agents in the economy.
3.2 Characterization of equilibrium

3.2.1 Wage determination

Let the expected value of a worker upon forming a match be

\[ W_0(w) = \int_0^{\bar{\mu}} W(w, \mu, 0) dH_0(\mu), \]  

(12)

where \( H_0(\mu) \) is the distribution of match quality at the formation of a match conditional on the worker accepting the match, to be derived below. Similarly,

\[ J_0(w) = \int_0^{\bar{\mu}} J(w, 0 | \mu) dH_0(\mu), \]  

(13)

is the expected value of a firm upon forming a match, where \( J(w, 0 | \mu) \) is the value to the firm from matching with a worker with match quality \( \mu \).

The wage is set such that

\[ (1 - \beta) (W_0(w) - U) = \beta (J_0(w) - V). \]  

(14)

Since the information that the wage is conditioned on is the same in all initial matches, and all workers have the same outside option, there is a single initial wage in equilibrium. Clearly, conditional on the worker having accepted the match, this initial wage satisfies the participation constraint of both parties. Moreover, it is straightforward to show that wages will never be renegotiated, since the initial wage will satisfy the participation constraint of the agents at all future tenure conditional on the match having continued. This follows from the fact that match continuation in the model is always favorable information to a firm, meaning that the firm’s posterior belief improves as the match lasts longer and longer. Formally, the lack of renegotiation follows from the lack of firm-initiated separations, as stated in the proposition below, which is proved in the Appendix.

**Proposition 2.** If workers’ search decision has the reservation property, there are no firm-initiated separations in a stationary equilibrium.
3.2.2 Worker side

Given that there is a unique wage in the stationary equilibrium as argued above, the wage $w$ and tenure $\tau$ can be dropped as state variables from the value of an employed worker. The Bellman equation characterizing the value of being a worker with quality $\mu$ is then

$$rW(\mu) = \max_{s \geq 0} \left\{ \mu + w - c(s) + \lambda(\theta)s \int_{\mu}^{\hat{\mu}} \max[W(\mu') - W(\mu), 0]dF(\mu') + \delta(U - W(\mu)) \right\}. \quad (15)$$

The flow payoff from working is the utility derived from being in a match of quality $\mu$ and from the wage $w$. An employed worker needs to choose her search effort, and if a new firm is encountered, she needs to decide whether to form the new match given its quality $\mu'$ that is drawn from the distribution $F$, or to stay with her current employer. Moreover, at rate $\delta$ the worker suffers a loss of asset value due to exogenous separation.

The Bellman equation characterizing the value of being an unemployed worker is

$$rU = \max_{s \geq 0} \left\{ b - c(s) + \lambda(\theta)s \int_{\mu}^{\hat{\mu}} \max[W(\mu') - U, 0]dF(\mu') \right\}. \quad (16)$$

An unemployed worker needs to also choose her search effort, and if a firm is encountered, she needs to decide whether to form the new match given its quality $\mu'$ that is drawn from the distribution $F$, or to remain unemployed.

Equation (15) defines a contraction, and therefore the Contraction Mapping Theorem implies that $W(\mu)$ is increasing in $\mu$ and, given the assumptions on $F(\cdot)$ and $c(\cdot)$, differentiable except at the points where the search decision changes discontinuously. This in turn implies that acceptance decisions have the reservation property with the quality of the current match being the reservation match quality.

Let us next turn to studying the worker’s search decision. Given the structure of the search cost and using the reservation property of acceptance decisions, the worker’s decision problem
can be rewritten as follows:

\[
    rW(\mu) = \max \left\{ \max_{s > 0} \left\{ \mu + w - c(s) + \lambda(\theta) s \int_{\mu}^{\bar{\mu}} W(\mu') - W(\mu) dF(\mu') + \delta(U - W(\mu)) \right\} ; \right. \\
    \mu + w + \delta(U - W(\mu)) \right\} ,
\]

(17)

where the search decision has been broken down into two steps: a decision whether to search at all, and a decision of how much to search if searching. I assume that the worker chooses to search if she is indifferent between searching and not searching. The first-order condition characterizing the second of these maximization problems is given by

\[
    c'(s(\mu)) = \lambda(\theta) \int_{\mu}^{\bar{\mu}} W(\mu') - W(\mu) dF(\mu') = \lambda(\theta) \int_{\mu}^{\bar{\mu}} W'(\mu') \bar{F}(\mu') d\mu',
\]

(18)

where the second equality follows from integration by parts and \( \bar{F} = 1 - F \) is the survival function of the distribution \( F \). With regards to the first maximization problem, clearly, the payoff from search is declining with \( \mu \), hence the optimal policy with respect to whether to search at all has the reservation property. This means that there exists a \( \mu_s \) above which the worker will choose not to search at all and below which she will choose to search. At \( \mu_s \), the condition of optimality states that

\[
    c(s(\mu_s)) = \lambda(\theta)s(\mu_s) \int_{\mu_s}^{\bar{\mu}} W'(\mu') \bar{F}(\mu') d\mu'.
\]

(19)

From these two optimality conditions, and given the properties of \( c(\cdot) \), the following Lemma follows.

**Lemma 3.** There exists a \( \mu_s \), such that for all \( \mu > \mu_s \), \( s(\mu) = 0 \). Moreover, the optimal search effort of the worker, \( s(\mu) \), is continuous and strictly declining in \( \mu \) for all \( \mu < \mu_s \).

In what follows, I assume that the variable part of the search cost function takes on the form \( \hat{c}(s) = c_1 s^{1+\rho} \), for some \( \rho > 0 \). Substituting in this functional form of \( c \), Equations (18)
evaluated at \( \mu = \mu_s \) and (19) imply that

\[
s(\mu_s) = \left( \frac{c_0}{c_1 \rho} \right)^\frac{1}{1 + \rho},
\]

which in turn together with Equation (18) evaluated at \( \mu = \mu_s \) implies that

\[
c_1(1 + \rho) \left[ \frac{c_0}{c_1 \rho} \right]^\frac{1}{1 + \rho} = \frac{\lambda(\theta)}{r + \delta} \int_{\mu_s}^{\bar{\mu}} \tilde{F}(\mu')d\mu',
\]

an equilibrium condition that determines \( \mu_s \) as a function of \( \lambda(\theta) \) and of exogenous parameters.

Taking derivatives with respect to \( \mu \) on both sides of the worker’s asset equation and rearranging gives

\[
\frac{dW(\mu)}{d\mu} = \begin{cases} 
\frac{1}{r + \delta + \lambda(\theta)s(\mu)(1 - F(\mu))} & \text{if } \mu < \mu_s \\
\frac{1}{r + \delta} & \text{if } \mu > \mu_s 
\end{cases}
\]

Substituting into the optimality condition for search for \( \mu < \mu_s \), taking derivatives on both sides with respect to \( \mu \), and using the functional form for \( \hat{c}(s) \) gives the differential equation

\[
s'(\mu) = -\frac{\lambda(\theta)F(\mu)s(\mu)^{1-\rho}}{(1 + \rho)\rho (r + \delta + \lambda(\theta)s(\mu)F(\mu))}.
\]

This differential equation together with the boundary condition in (20) fully characterizes the search decision of workers as a function of the quality of their match, and can be solved numerically for any value of \( \rho > 0 \).

Notice that while \( W(\mu) \) is continuous everywhere, there is a kink in the function \( W(\mu) \) at \( \mu^s \), since there is a positive difference between its derivative from the left (the first expression evaluated at \( \mu^s \)) and the derivative from the right (the second expression evaluated at \( \mu^s \)).

Finally, given that \( W(\mu) \) is increasing as argued above, an unemployed worker will clearly
adopt a reservation match quality policy when searching for a job, hence

\[ rU = \max_{s \geq 0} \left\{ b - c(s) + \lambda(\theta)s \int_{\mu_m}^{\mu} [W(\mu') - U]dF(\mu') \right\}, \tag{24} \]

where \( \mu_m \) is an unemployed worker’s reservation match quality implicitly defined by

\[ W(\mu_m) = U. \tag{25} \]

Comparing the asset equation of a worker at match quality \( \mu_m \) and that of an unemployed worker, it is clear that

\[ s_u = s(\mu_m) \tag{26} \]
\[ \mu_m = b - w. \tag{27} \]

### 3.2.3 Firm side

Again, given that there is a unique wage in the stationary equilibrium as argued above, the wage \( w \) can be dropped as a state variable from the value of employment for the firm. The value of being a firm with a match of tenure \( \tau \) is then

\[ J(\tau) = \int_{\mu}^{\mu} J(\mu)dH_\tau(\mu) \tag{28} \]

where \( H_\tau(\mu) \) is the distribution of match quality for a match of tenure \( \tau \). \( J(\mu) \) in turn satisfies the Bellman equation

\[ rJ(\mu) = \begin{cases} 
    p - w + \lambda(\theta)s(\mu)\bar{F}(\mu)(V - J(\mu)) + \delta(V - J(\mu)) & \text{if } \mu \leq \mu_s \\
    p - w + \delta(V - J(\mu)) & \text{if } \mu > \mu_s . 
\end{cases} \tag{29} \]
The flow payoff of a match to the firm is $p - w$. In addition, the firm needs to take into account that the match might end for exogenous reasons at rate $\delta$ and endogenously if the worker decides to move to another job, where the latter happens at rate $\lambda(\theta)s(\mu)\bar{F}(\mu)$. Since endogenous turnover is decreasing with $\mu$ (and becomes zero once $\mu > \mu_s$), the value of a match to the firm increases in $\mu$.

The Bellman equation characterizing the value of a vacancy can be expressed as

$$rV = -c_f + \eta(\theta)P_a \int_{\bar{\mu}}^{\mu} (J(\mu) - K - V) \, dH_0(\mu),$$

(30)

where $P_a$ is the probability that a match is accepted by the worker. Given the free-entry condition, $V = 0$, and the acceptance policy of the worker, we can write the above as

$$J(\mu) = \begin{cases} \frac{p-w}{r+\delta+\lambda(\theta)s(\mu)\bar{F}(\mu)} & \text{if } \mu \leq \mu_s, \\ \frac{p-w}{r+\delta} & \text{if } \mu > \mu_s, \end{cases}$$

(31)

and

$$\frac{c_f}{\eta(\theta)} = P_a \int_{\mu_m}^{\bar{\mu}} (J(\mu) - K) \, dH_0(\mu).$$

(32)

### 3.2.4 Equilibrium distribution of workers

Next, I turn to the derivation of $G(\mu)$, which denotes the stationary measure of employed workers below match quality $\mu$, and $u$, which is the stationary unemployment rate. Clearly, the support of $G$ is $[\mu_m, \bar{\mu}]$ and $G(\bar{\mu}) = 1 - u$.

In the model, the stationary measure of unemployment can be derived from equating the flow into and out of unemployment

$$u\lambda(\theta)s(\mu_m)\bar{F}(\mu_m) = \delta (1 - u),$$

(33)
so that
\[ u = \frac{\delta}{\delta + \lambda(\theta)s(\mu_m)F(\mu_m)}. \]  

(34)

To determine the distribution \( G(\mu) \), one can equate the flow into and out of \( G(\mu) \) (just as in the Burdett and Mortensen (1998) model). The flow into the pool of employed workers with match quality \( \mu \) or lower is
\[ u\lambda(\theta)s(\mu_m)(\bar{F}(\mu_m) - \bar{F}(\mu)), \]

(35)

while flow out of the pool of employed workers with match quality \( \mu \) or lower is
\[ \delta G(\mu) + \int_{\mu_m}^{\min(\mu, \mu^*)} \lambda(\theta)s(\mu')\bar{F}(\mu)dG(\mu'). \]

(36)

The inflow clearly consists only of unemployed workers, while the outflow consists of workers that separate exogenously, and workers that find a match that is better than \( \mu \), where one has to take into account that only workers below match quality \( \mu_s \) are searching.

Equating these two flows when \( \mu \leq \mu_s \) means
\[ \frac{u\lambda(\theta)s(\mu_m)(\bar{F}(\mu_m) - \bar{F}(\mu))}{\bar{F}(\mu)} = \frac{\delta G(\mu)}{\bar{F}(\mu)} + \lambda(\theta)\int_{\mu_m}^{\mu} s(\mu')dG(\mu'). \]

(37)

Differentiating both sides with respect to \( \mu \) and rearranging gives
\[ G'(\mu) = \frac{u\lambda(\theta)s(\mu_m)\bar{F}(\mu_m) - \delta G(\mu)}{\delta + \lambda(\theta)s(\mu)\bar{F}(\mu)} \cdot \frac{f(\mu)}{\bar{F}(\mu)}. \]

(38)

For \( \mu > \mu_s \) the same steps give
\[ G'(\mu) = \frac{u\lambda(\theta)s(\mu_m)\bar{F}(\mu_m) - \delta G(\mu)}{\delta} \cdot \frac{f(\mu)}{\bar{F}(\mu)}. \]

(39)

These differential equations together with the boundary condition in \( G(\mu_m) = 0 \) fully characterize the distribution of workers.
Given the distribution $G$, the firm’s initial belief that a match of quality $\mu \geq \mu_m$ is accepted can be expressed as

$$A(\mu) = \frac{u + G(\min(\mu, \mu^*))}{u + G(\mu_a)}. \quad (40)$$

Using Bayes’ rule, the probability density function corresponding to the distribution $H_0$ can be written as

$$h_0(\mu) = \frac{A(\mu)f(\mu)}{P_a}, \quad (41)$$

where $P_a$ is the probability that a worker accepts a match, which can be written as

$$P_a = \int_{\bar{\mu}}^{\mu} A(\mu)f(\mu)d\mu. \quad (42)$$
### 3.3 Summary of equilibrium conditions

For the numerical exercise below, it is useful to note that two parameters, \(c_1\) and \(m_0\) can be eliminated from the equilibrium conditions. In other words, two more normalizations are possible. Let then \(\hat{s}(\mu) = s(\mu)c_1^{\frac{1}{1+\rho}}\), \(\hat{\lambda} = \lambda c_1^{-\frac{1}{1+\rho}}\), and \(\hat{c}_f = c_f \left(\frac{c_1^{\frac{1}{1+\rho}}}{m_0}\right)^{\frac{1}{1+\alpha}}\). Then the complete set of equilibrium conditions can be rewritten as

\[
0 = (1 + \rho) \left[c_0 \frac{1}{\rho}\right]^{\frac{1+\rho}{1}} - \frac{\hat{\lambda}}{r + \delta} \int_{\mu_s}^{\hat{\mu}} \hat{F}(\mu')d\mu',
\]

\[
\hat{s}(\mu_s) = \left[c_0 \frac{1}{\rho}\right]^{\frac{1+\rho}{\rho}}
\]

\[
\hat{s}'(\mu) = -\frac{\hat{\lambda}\hat{F}(\mu)\hat{s}(\mu)^{1-\rho}}{(1 + \rho)\rho \left[r + \delta + \hat{\lambda}\hat{s}(\mu)\hat{F}(\mu)\right]}
\]

\[
\mu_m = b - w.
\]

\[
0 = \hat{c}_f\hat{\lambda}\hat{s}(\mu_s)^{\frac{1}{1+\rho}} - \int_{\mu_m}^{\hat{\mu}} (J(\mu) - K)A(\mu)f(\mu)d\mu
\]

\[
0 = \beta \int_{\mu_m}^{\hat{\mu}} J(\mu)A(\mu)f(\mu)d\mu - (1 - \beta) \int_{\mu_m}^{\hat{\mu}} (W(\mu) - U)A(\mu)f(\mu)d\mu
\]

\[
A(\mu) = \frac{u + G(\mu)}{u + G(\mu_s)} \text{ if } \mu \leq \mu_s.
\]

\[
G'(\mu) = \begin{cases} 
\frac{u\hat{\lambda}\hat{s}(\mu_m)}{\delta + \hat{\lambda}\hat{s}(\mu)\hat{F}(\mu)} - \frac{\delta G(\mu)}{\hat{F}(\mu)} f(\mu) & \text{if } \mu \leq \mu_s \\
\frac{u\hat{\lambda}\hat{s}(\mu_m)}{\delta + \hat{\lambda}\hat{s}(\mu)\hat{F}(\mu)} - \frac{\delta G(\mu)}{\hat{F}(\mu)} f(\mu) & \text{if } \mu > \mu_s
\end{cases}
\]

\[
u = \frac{\delta}{\delta + \hat{\lambda}\hat{s}(\mu_m)\hat{F}(\mu_m)}.
\]

Using the asset equations, the value of employment to a worker and a firm can be written as

\[
J(\mu) = \begin{cases} 
\frac{p - w}{r + \delta + \lambda \hat{s}(\mu)\hat{F}(\mu)} & \text{if } \mu \leq \mu_s \\
\frac{p - w}{r + \delta} & \text{if } \mu > \mu_s
\end{cases}
\]

\[
W(\mu) = \begin{cases} 
U + \int_{\mu_m}^{\mu} \frac{1}{r + \delta + \lambda \hat{s}(\mu)\hat{F}(\mu')}d\mu' & \text{if } \mu \leq \mu_s \\
W(\mu_s) + \frac{\mu - \mu_s}{r + \delta} & \text{if } \mu > \mu_s
\end{cases}
\]
4 Representative simulations

In this section, I simulate the above economy, and look at what happens when I change the aggregate productivity parameter $p$ and the exogenous job destruction rate $\delta$.

For the distribution of match qualities, I use a normal distribution with mean zero and variance 1. For the choice of the other parameters, unless otherwise mentioned, I follow Shimer (2004) as closely as possible, to facilitate direct comparison of the results. The model is set to generate quarterly series, so $r$ is chosen to be 1.2%, giving an annual discount rate of 4.8%. The aggregate productivity is chosen to be $p = 5$, implying that in terms of the total payoff from production, the probability of drawing a match quality that is a half as important as output (i.e. equal to 2.5) is 0.62%. Of course, the endogenous distribution of match qualities first-order stochastically dominates the distribution of the draw of match qualities, which is standard normal, so the question of how important match quality is compared to output is determined endogenously. The value of leisure or of unemployment insurance, $b$, is set to 2.5 at 50% of the match output. This is higher than the number used by Shimer (2004) (he uses 40%), but recall that in this model the flow payoff from unemployment is $b$ less the cost of search, where this cost is strictly positive in this model, while it is 0 in Shimer’s work. In fact, taking into account the search cost, the flow payoff from unemployment is less than 40% of output. The exogenous job destruction rate is set to 6%.

The cost of posting a vacancy is set to 2.5% of output, though recall that this number is affected by the normalization of $c_1$ and $m_0$. The fixed cost of creating a match is set to 10, or 2 periods of output. The fixed cost of search for a worker is set close to 0 at 0.001, though even such a small fixed cost leads to no more search once the 70th percentile in the quality distribution is reached. The parameter $\rho$ is set equal to 0.2.

The matching function is chosen to be Cobb-Douglas

$$m(s, v) = m_0 s^\alpha v^{1-\alpha}, \quad (53)$$
with elasticity with respect to unemployment $\alpha$ of 0.62. The workers’ share of the expected surplus is also set to equal 0.62.

For these parameter values, the endogenous values of interest are reported in Table 1.

<table>
<thead>
<tr>
<th>Baseline case</th>
</tr>
</thead>
<tbody>
<tr>
<td>unemployment rate</td>
</tr>
<tr>
<td>vacancy rate</td>
</tr>
<tr>
<td>fraction of unemployed searchers</td>
</tr>
<tr>
<td>fraction of unemployed new hires</td>
</tr>
<tr>
<td>job-to-job transition rate</td>
</tr>
<tr>
<td>wage rate</td>
</tr>
<tr>
<td>lowest accepted match quality</td>
</tr>
<tr>
<td>search threshold match quality</td>
</tr>
<tr>
<td>average match quality</td>
</tr>
</tbody>
</table>

### 4.1 Comparative statics results — aggregate productivity

Next, I allow aggregate productivity to vary. As stated earlier, I rely on comparisons of stationary equilibria to assess the response of the model to aggregate shocks. In the standard search model such a comparative static exercise invariably gives results that are very close to the dynamic response of the full stochastic model. This is due to the fact that transition dynamics are very swift in the standard model due to the forward-looking and instantaneous adjustment of the vacancy-unemployment ratio and the resulting high job-finding rate. Due to the presence of on-the-job search, the full stochastic version of the model of this paper is much more complex. In particular, the complete distribution of match qualities across employed workers enters the state space, and hence the dynamics become more gradual. This is due to the fact that the vacancy-unemployment ratio does not adjust instantaneously to its new long-run value, and while the job-finding rate of unemployed workers is equally high as in the standard model, that is not true of the job-to-job transition rate of employed workers, and hence the adjustment towards the new steady state could be much more prolonged.
These simulations show that on-the-job search and job-to-job transitions vary positively with aggregate productivity shocks when comparing stationary equilibria for different aggregate productivity levels.

The amplification mechanism embedded in the model shows up clearly when considering the response of the unemployment and the vacancy rate to changes in aggregate productivity. In the standard model, as Shimer (2004) has shown, the elasticity of the vacancy-unemployment ratio with respect to labor productivity (which in the standard model is equal to market tightness) is below 2 for reasonable parameter values. Here, the elasticity of the vacancy-unemployment ratio is 9.07, which is both due to the decline in unemployment (elasticity of –5.44) and to an increase in the vacancy rate (elasticity of 3.63).8

8With regards to the level of the vacancy rate, recall again that these numbers correspond to the normalization where $c_1 = 1$ and $m_0 = 1$. Varying these parameters — which do not have clear equivalents in the data — would result in varying the level of the vacancy rate, but not its elasticity.
### 4.2 Comparative statics results — destruction shocks

<table>
<thead>
<tr>
<th></th>
<th>1% decline in destruction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>stationary equil. value</td>
</tr>
<tr>
<td>unemployment rate</td>
<td>9.35%</td>
</tr>
<tr>
<td>vacancy rate</td>
<td>0.348</td>
</tr>
<tr>
<td>fraction of unemployed searchers</td>
<td>17.42%</td>
</tr>
<tr>
<td>fraction of unemployed new hires</td>
<td>29.89%</td>
</tr>
<tr>
<td>job-to-job transition rate</td>
<td>2.10%</td>
</tr>
<tr>
<td>wage rate</td>
<td>4.19</td>
</tr>
<tr>
<td>lowest accepted match quality</td>
<td>-1.6904 ( \tilde{F} = 95.45% )</td>
</tr>
<tr>
<td>search threshold match quality</td>
<td>0.432 ( \tilde{F} = 33.50% )</td>
</tr>
<tr>
<td>average match quality</td>
<td>.5141</td>
</tr>
</tbody>
</table>

In these simulations the vacancy-unemployment ratio varies negatively with job destruction shocks and with the unemployment rate. Shimer (2004) argues that destruction shocks induce a positive correlation between vacancies and unemployment rate in the standard model, thereby causing the vacancy-unemployment ratio to have a positive elasticity with respect to adverse destruction shocks. That conclusion no longer holds up in this model in the presence of job-to-job movements. This brings back destruction shocks into the picture as a potential source of business cycle variation in the vacancy-unemployment ratio.

### 5 Conclusions

INCOMPLETE.
References


6 Appendix

6.1 Proof of Proposition 1

Define conditional distributions

\begin{align}
H^m(\mu) &= \frac{H_0(\mu)}{H_0(\mu_s)} \quad \mu \leq \mu_s \\
H^n(\mu) &= \frac{H_0(\mu) - H_0(\mu_s)}{1 - H_0(\mu_s)} \quad \mu \geq \mu_s
\end{align}

(54) (55)

Then the belief of firm in match of tenure $T$

\[ H_T(\mu) = \begin{cases} 
H^m(\mu)\omega_T & \text{if } \mu \leq \mu_s \\
H^n(\mu)(1 - \omega_T) & \text{if } \mu > \mu_s
\end{cases} \]

(56)

where $\omega_0 = H_0(\mu_s)$. Applying Bayes’s rule, the differential equation for the evolution of $\omega_T$ is

\[ \dot{\omega}_T = -\eta \omega_T (1 - \omega_T), \]

(57)

where $\eta$ is the rate at which workers below match quality $\mu_s$ find better matches. Solving the differential equation in Equation (57) gives

\[ \omega_T = \frac{\omega_0}{\omega_0 + e^{\eta T}(1 - \omega_0)}. \]

(58)

Clearly, $\omega_T$ is decreasing in $T$, hence $H_T$ first-order stochastically dominates $H_0$. This in turn means that a firm is always willing to continue a relationship at the initial wage.

**Corollary 4.** There is a single wage in equilibrium.