```plaintext
> \frac{(x^\rho - 1)}{\rho}

\frac{x^\rho - 1}{\rho}

\ln(x)

\theta \frac{(x^\rho - 1)}{\rho} + \frac{y^\rho - 1}{\rho}

\theta := \text{unapply}(\%, x, y)

u := (x, y) \rightarrow \frac{\theta (x^\rho - 1)}{\rho} + \frac{y^\rho - 1}{\rho}

xmpl := \{\theta = 1\}

xmpl := \{\theta = 1\}

xmpl := \{\theta = 3\}

xmpl := \{\theta = 3\}

\text{plots[implicitplot]} \left[ \text{subs} \left( xmpl, \rho = \frac{1}{2}, u(x, y) = -1 \right), \text{subs} \left( xmpl, \rho = \frac{\frac{15}{10}}{2}, u(x, y) = -1 \right), \text{subs} \left( xmpl, \rho = -2, u(x, y) = -1 \right), \text{subs} \left( xmpl, \rho = -2, u(x, y) = -\frac{\frac{15}{10}}{2} \right) \right], x = 0..2,
y = 0..2, \text{numpoints} = 20000, \text{color} = [\text{red}, \text{red}, \text{blue}, \text{blue}], \text{title} = \text{"Indifference Curves of Two Different CES Utility Functions"} \right]
```
Indifference Curves of Two Different CES Utility Functions
Indifference Curves of Two Different CES Utility Functions

> plots[implicitplot]\left(\begin{array}{l}
\text{subs}\left(xmpl, \rho = -2, u(x, y) = -1\right), \text{subs}\left(xmpl, \rho = \frac{1}{2}, u(x, y) = -1\right), \\
\text{subs}\left(xmpl, \rho = \frac{9}{10}, u(x, y) = -1\right), x = 0..2, y = 0..2, \text{numpoints} = 20000, \text{color} = \text{[red, blue, magenta]}, \text{title} = \text{"Indifference Curves of Three Different CES Utility Functions"}\end{array}\right)
> xmp := xmp2

> plots[implicitplot](
> subs(xmp, rho=-2, u(x, y) = -1),
> subs(xmp, rho = \frac{1}{2}, u(x, y) = -1),
> subs(xmp, rho = \frac{9}{10}, u(x, y) = -1),
> x = 0 .. 2, y = -0 .. 2, numpoints = 20000, color = [red, blue, magenta], title = "Indifference Curves of Three Different CES Utility Functions")
Indifference Curves of Three Different CES Utility Functions

\[ test := \text{subs}(xmpl, \rho = \frac{9}{10}, u(x, y)) \]
\[ test := \frac{10}{9} x^{9/10} - \frac{20}{9} + \frac{10}{9} y^{9/10} \quad (8) \]

\[ \text{plot3d}(test, x = 0..3, y = 0..3, orientation = [ -145, 45, -30 ], shading = zhue, style = patchcontour, title = "Utility Surface, with contours (indifference curves)") \]
Utility Surface, with contours (indifference curves)

> `test := subs(xmpl, rho=-1, u(x, y))`

\[
\text{test} := -\frac{1}{x} + 2 - \frac{1}{y}
\]  

(9)

> `plot3d(test, x=0..1, y=0..1, orientation = [-145, 45, -30], shading = zhue, style = patchcontour, title = "Another CES utility surface, with indifference curves ")`
Another CES utility surface, with indifference curves

Special Case of CES (Cobb — Douglas)

\[ test := \text{subs}(xmp1, \text{limit}(u(x, y), \rho = 0)) \]
\[ test := \ln(x) + \ln(y) \]  \hspace{1cm} (10)

\[ \text{plot3d}(test, x = 0 .. 1, y = 0 .. 1, \text{orientation} = [-90, 0, 0], \text{style} = \text{contour}, \text{shading} = \text{zhue}, \text{title} = \text{"Cobb-Douglas Indifference Curves"}) \]
Cobb-Douglas Indifference Curves

```r
plot3d(test, x = 0..1, y = 0..1, orientation = [ -120, 45, 0 ], style = patchcontour, shading = zhue)
```