

# Immigration Restrictions and Labor Market Skills

## *Preliminary and Incomplete*

John Kennan\*

University of Wisconsin-Madison and NBER

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### Abstract

Differences in income levels across countries are generally attributed to differences in productivity. In studies of internal migration (e.g. within the U.S.), it is commonly observed that skilled workers move much more than unskilled workers. The paper analyzes the implications of such differential migration, using a model in which efficiency differences are labor-augmenting, and free trade in product markets leads to factor price equalization, so that wages are equal across countries when measured in efficiency units. Relaxation of immigration restrictions increases the effective supply of labor on the world market, and differential migration implies that there may be an increase in the ratio of skilled to unskilled labor; but skill levels are relatively low in less productive countries, and the data indicate that the net effect is that free migration in fact decreases the ratio of skilled to unskilled labor on the world market. The effects on wages depend on elasticities of substitution, but these effects are in any case surprisingly small, while the income gains for migrants (net of migration costs) are very large.

## 1 Introduction

Immigrants account for a substantial fraction of the population in many developed countries (for example, about 13% in the U.S., about 20% in Canada). The number of immigrants would undoubtedly be much larger were it not for pervasive legal restrictions. There is a large literature on the economics of immigration, mainly dealing with questions about labor productivity and wages.<sup>1</sup> But the most interesting question has not received as much attention as it deserves: what would happen if we just let people choose where they want to live? Clearly, the immigrants who would not otherwise have moved would be better off. By how much? Who would lose, and how much?<sup>2</sup>

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\*Department of Economics, University of Wisconsin, 1180 Observatory Drive, Madison, WI 53706; jkennan@ssc.wisc.edu. I thank Ted Bergstrom and many seminar participants for helpful comments.

<sup>1</sup>See, for example, [Borjas \(1999\)](#) and [Card \(2009\)](#).

<sup>2</sup>See [Hamilton and Whalley \(1984\)](#), [Moses and Letnes \(2004\)](#), [Walmsley and Winters \(2005\)](#), [Iregui \(2005\)](#), [Freeman \(2006\)](#), [Klein and Ventura \(2007\)](#), [Klein and Ventura \(2009\)](#), [Clemens \(2011\)](#) and [Kennan \(2013\)](#).

In order to analyze the effects of restrictions on migration, it is necessary to have some understanding of why so many people want to move from one country to another; that is, it is necessary to have a theory of cross-country wage differentials. It is not enough to attribute these differentials to restrictions on labor mobility, since standard international trade models imply that differences in wages (and other factor prices) are implicitly arbitrated through free trade in product markets.

To some extent, differences in income levels across countries can be accounted for by differences in physical and human capital, with the remainder being attributed to differences in productivity. Such accounting exercises generally pay no attention to any theory of factor prices.<sup>3</sup> There are big wage differences<sup>4</sup>, but the return on capital is roughly equal across countries, as [Caselli and Feyrer \(2007\)](#) point out. This is consistent with factor price equalization, given labor-augmenting technology differences across countries.<sup>5</sup>

The main question considered in this paper is what happens to the wages of workers at different skill levels when immigration increases. One key element in the analysis is that there are many goods, so it is possible that not much happens, because of the Rybczynski theorem: changes in the endowments of different factors can be absorbed by changes in the mix of products produced, given different factor intensities for different products.<sup>6</sup>

But the Rybczynski theorem applies to a small open economy. What happens in a big open economy?

## 2 Immigration Restrictions, Wages and Welfare

[Hamilton and Whalley \(1984\)](#) initiated the quantitative analysis of the costs of immigration restrictions. Their analysis (and subsequent work by [Moses and Letnes \(2004\)](#) and [Iregui \(2005\)](#), for example) was based on a multi-country version of the model illustrated in [Figure 1](#).<sup>7</sup> This is a standard textbook model of a segmented labor market (used for example to show the relative wage effects associated with unions). If two labor markets (say Mexico and the U.S.) have different downward-sloping curves showing the marginal product of labor as a function of the number of workers employed (here the number of workers in Mexico is measured from left to right, and the number in the U.S. is measured from right to left), then migration from Mexico to the U.S. reduces the wage in the U.S. and increases the wage in Mexico (the initial position being indicated by

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<sup>3</sup>See [Hall and Jones \(1999\)](#), [Hendricks \(2002\)](#), [Caselli \(2005\)](#), [Hsieh and Klenow \(2010\)](#) and [Schoellman \(2012\)](#), for example. [Lagakos, Moll, Porzio, and Qian \(2013\)](#) argue that previous work has failed to account for cross-country differences in the returns to experience. When age-earnings profiles are estimated for less-developed countries, the returns to experience are found to be much lower than the returns estimated using data from developed countries. But even when differences in measured human capital stocks are adjusted for these differences in the returns to experience, more than one third of the differences in income levels are attributed to differences in productivity.

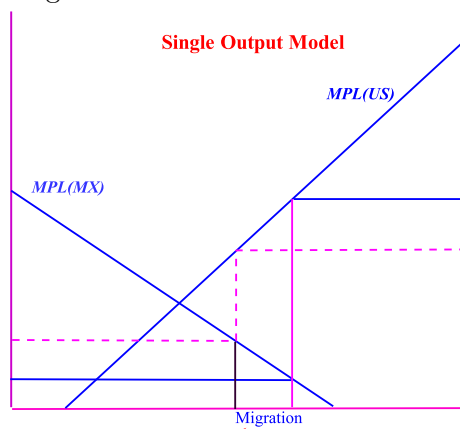
<sup>4</sup>See [Clemens, Montenegro, and Pritchett \(2008\)](#).

<sup>5</sup>See [Trefler \(1993\)](#) and [Kennan \(2013\)](#).

<sup>6</sup>[Lewis \(2013\)](#) reviews the evidence on immigrant-native worker substitution, in the context of a model with many goods; the log-linear relationship between relative prices and relative quantities is undone by Rybczynski effects; see also [Ciccone and Peri \(2011\)](#).

<sup>7</sup>See [Bhagwati \(1984\)](#)

Figure 1: Labor Allocation Effects



the solid lines, and the final position by the dashed lines). Since total output in each country is the area under the marginal product curve, migration raises total output. In the [Hamilton and Whalley \(1984\)](#) analysis, the result when immigration restrictions are removed is at the point where marginal products are equal, and total output is maximal.<sup>8</sup>

[Hamilton and Whalley \(1984\)](#) divide the world economy into 7 regions (EEC, U.S., Japan, other developed countries, OPEC, LDCs and Newly Industrialized Countries). Each region has its own CES technology, with different TFP levels, and different elasticities of substitution between capital and labor. The elasticities are set at “extraneous” values (meaning that calculations are made for a range of possible values), and the TFP levels are inferred from the data (World Bank data on GNP and population, and factor shares data from U.N sources). The results indicate that there are massive costs due to labor misallocation: total world output could be roughly doubled if all immigration restrictions were removed.

There are two major limitations of this segmented labor market model, in the present context. One is that the people in the model are not attached to any particular place, whereas there is a great deal of evidence indicating that, other things equal, most people would rather live their lives in the place where they grew up. Thus in the absence of immigration restrictions, one would not expect that wages would be equalized across countries, but rather that in the new equilibrium, every potential migrant would view the wage gain from moving to a high-wage country as insufficient to offset the cost of living away from home. The implied increase in output would then be less than the maximal amount (i.e. the amount calculated by [Hamilton and Whalley \(1984\)](#)), and even this increase overstates the gains from relaxing immigration restrictions, because it neglects the migration cost.<sup>9</sup>

The second limitation is that there is no consideration of the extent to which wage differences

<sup>8</sup> In exactly the same way, union relative wage effects are understood as the result of limiting the number of workers in one sector, which drives up the marginal product, while pushing more workers into the nonunion sector, which drives down the marginal product there.

<sup>9</sup>For example, the marginal migrant is more productive after migrating, but has a net gain of zero.

are arbitrated through product markets. [Hamilton and Whalley \(1984\)](#) acknowledged this, with the comment that “most trade economists appear to regard factor price equalization as a theoretical possibility rather than an empirical proposition”, and proceeded to use a model in which only a single composite consumption good is produced, so that the factor price equalization theorem is moot. But of course the same comment applies to the idea that there is only one consumption good. The factor price equalization theorem does not apply directly in models which assume that every firm produces a distinct good, as in [Dixit and Stiglitz \(1977\)](#) and [Krugman \(1980\)](#), but product market arbitrage also implies strong restrictions on cross-country wage differences in such models.<sup>10</sup>

The [Klein and Ventura \(2009\)](#) analysis differs from [Hamilton and Whalley \(1984\)](#) in that the gains from migration are attributed to differences in TFP (as opposed to differences in the numbers of workers allocated to each country). The model used in [Kennan \(2013\)](#) attributes wage differences to differences in labor productivity; in this model the factor price equalization theorem holds when labor is measured in efficiency units, as in [Trefler \(1993\)](#). In contrast to the Klein-Ventura model, the gains have nothing to do with reallocating capital across countries, because it is assumed, in line with the evidence presented by [Caselli and Feyrer \(2007\)](#), that there are no differences in the productivity of capital, and factor price equalization implies that the return to capital is the same in all countries.<sup>11</sup>

### 3 Model

The model has many produced goods, and it is assumed that the production technologies are similar, but not identical. Factor prices are equalized, but elasticities of substitution are not constant at the market level, even though the technology for each product is a (nested) CES.

#### 3.1 Factor Price Equalization with Labor-Augmenting Technology Differences

Suppose there are  $J$  countries, with common technologies, but different productivity levels. If the productivity differences are labor-augmenting (i.e. Harrod-neutral), then the technology for

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<sup>10</sup>For example, in the basic model in [Krugman \(1980\)](#), and also in the [Melitz \(2003\)](#) model, wages are equal across countries, because everything is symmetric. When transport costs are introduced, wages are higher in larger countries. Each good is consumed in all countries, and if production is moved to a larger country, the cost of delivering the good to consumers is reduced (because the consumers are closer). This pushes down the wage in smaller countries, to the point where the lower wage just offsets the higher transport cost. But transport costs are hardly a plausible explanation for the huge cross-country wage differences seen in the data.

<sup>11</sup>Even this modified version of the factor price equalization theorem is viewed skeptically as an empirical proposition by trade economists – see [Davis and Weinstein \(2004\)](#), for example. One reason for this skepticism, as explained by [Goldberg and Pavcnik \(2007\)](#), is that there is no evidence that the increased exposure of developing countries to international trade seen in recent years has led to a reduction in the skill premium. But of course no amount of evidence can repeal a theorem, and there seems to be no fully satisfactory model that accounts for large cross-country differences in wages paid to workers whose output is sold in competitive international markets. At this point the efficiency units version of factor price equalization seems to be the best model available.

product  $s$  in country  $j$  can be specified as

$$Q_s^j = F_s(K, a_j S, b_j U)$$

where  $(a_j, b_j)$  represent efficiency units of skilled and unskilled labor per worker in country  $j$ .

Let  $c_s^0$  be the unit cost function for product  $s$  when the labor inputs are measured in efficiency units, so that the production function is  $Q_s = F_s(K, S, U)$ . Then the cost function for product  $s$  in country  $j$  is

$$c_s^j(v, w) = c_s^0\left(v, \frac{w^S}{a_j}, \frac{w^U}{b_j}\right)$$

where  $w$  is the wage per efficiency unit of (skilled and unskilled) labor, and  $v$  is the price of capital.

If there is free trade in the product markets, with no transportation costs, then the zero-profit condition implies

$$p_s = c_s^0\left(v, \frac{w^S}{a_j}, \frac{w^U}{b_j}\right)$$

If three products  $\{s_1, s_2, s_3\}$  are produced in country  $j$ , then

$$c_s^0\left(v, \frac{w^S}{a_j}, \frac{w^U}{b_j}\right) = p_s, \quad s \in \{s_1, s_2, s_3\}$$

These equations determine the factor prices in country  $j$ . If the marginal rates of technical substitution satisfy a single-crossing condition, the factor prices are uniquely determined. Then if country  $\ell$  also produces these same products, the same equations determine factor prices in country  $\ell$ , with  $a_\ell$  in place of  $a_j$ . This implies  $v_j = v_\ell$ , and

$$\frac{w_j}{a_j} = \frac{w_\ell}{a_\ell}$$

Thus

$$\begin{aligned} w_j^S &= a_j w_0^S \\ w_j^U &= b_j w_0^U \end{aligned}$$

where  $w_0^S$  and  $w_0^U$  are reference wage levels that can be normalized to 1.

### 3.2 General Equilibrium

Given the factor prices, the prices of consumer goods are determined by the cost functions. Then the quantities are determined by these goods prices, and by preferences and total income (where income depends on factor prices). Given the quantities to be produced, and the factor prices, producers determine the profit-maximizing factor quantities. This gives demand curves for the factors, and factor prices are determined so as to clear the factor markets.

For simplicity, it is assumed that: (1) preferences are identical in all countries, and are described by a loglinear utility function with expenditure share parameters  $\theta_s$ ; and (2) the production function for each good is a nested CES, and is the same in all countries, with substitution elasticities that are the same for all goods (and with the understanding that the labor input is measured in efficiency units). The consumption goods are produced using capital and labor, where labor is a composite of skilled and unskilled labor, and where the skill mix is allowed to differ across products.

It is also assumed that each worker supplies one time-unit of labor (inelastically). This time-unit implies different amounts of effective labor in different countries, for two reasons: human capital endowments  $h_j$  may differ across countries, and each unit of human capital in country  $j$  means  $a_j^S$  efficiency units of skilled labor, or  $a_j^U$  efficiency units of unskilled labor.

The nested CES treats labor as a composite of two components, skilled and unskilled. The composite is a power-linear function of skilled and unskilled labor<sup>12</sup>:

$$L^\kappa = \gamma S^\kappa + (1 - \gamma) U^\kappa$$

with  $\kappa < 1$ , where  $\gamma \in [0, 1]$  is a parameter reflecting the relative importance of skilled and unskilled labor in the composite. The elasticity of substitution between skilled and unskilled labor is  $\zeta = \frac{1}{1-\kappa}$ . It is assumed that this elasticity of substitution is the same for all products, but  $\gamma$  may differ across products, with some products being more skill-intensive than others.

Output is a power-linear function of capital and (composite) labor. Write this as

$$Y^\rho = \alpha K^\rho + (1 - \alpha) L^\rho$$

with  $\rho < 1$ , where  $\alpha \in [0, 1]$  is a parameter reflecting the relative importance of capital and labor. The elasticity of substitution between skilled and unskilled labor is  $\sigma = \frac{1}{1-\rho}$ . It is assumed that this elasticity of substitution is the same for all products, but  $\alpha$  may differ across products, with some products being more capital-intensive than others. Note that the production function for each good has constant returns.

The marginal product of labor satisfies

$$Y^{\rho-1} MPL = (1 - \alpha) L^{\rho-1}$$

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<sup>12</sup>The function is linear if  $\kappa = 1$ , and it is log-linear if  $\kappa = 0$ .

so

$$\left(\frac{MPL}{1-\alpha}\right)^\sigma = APL$$

Similarly

$$\begin{aligned}\left(\frac{MPL_u}{1-\gamma}\right)^\zeta &= \frac{L}{U} \\ \left(\frac{MPL_s}{\gamma}\right)^\zeta &= \frac{L}{S}\end{aligned}$$

where  $MPL_u$  and  $MPL_s$  are the marginal products of skilled and unskilled labor in the labor composite.

The price of good  $r$  is given by

$$p_r^{1-\sigma} = \alpha_r \left(\frac{v}{\alpha_r}\right)^{1-\sigma} + (1-\alpha_r) \left(\frac{W_r}{1-\alpha_r}\right)^{1-\sigma}$$

where  $W_r$  is the price of the labor composite in efficiency units, which is determined by the CES cost function for labor:

$$W_r^{1-\zeta} = \gamma_r \left(\frac{w^S}{\gamma_r}\right)^{1-\zeta} + (1-\gamma_r) \left(\frac{w^U}{1-\gamma_r}\right)^{1-\zeta}$$

The conditional factor demand functions are the derivatives of the cost functions, by Shephard's lemma. The derivatives are determined by

$$\begin{aligned}c_r^{-\sigma} \frac{\partial c_r}{\partial v} &= \left(\frac{v}{\alpha_r}\right)^{-\sigma} \\ c_r^{-\sigma} \frac{\partial c_r}{\partial w} &= \left(\frac{w}{1-\alpha_r}\right)^{-\sigma}\end{aligned}$$

The demands for unskilled and skilled labor are obtained from the labor cost function . Thus

$$\begin{aligned}W_r^{-\zeta} \frac{\partial w_r}{\partial w^U} &= \left(\frac{w^U}{\gamma_r}\right)^{-\zeta} \\ W_r^{-\zeta} \frac{\partial w_r}{\partial w^S} &= \left(\frac{w^S}{1-\gamma_r}\right)^{-\zeta}\end{aligned}$$

where  $w_r$  is the unit price of labor relative to the production technology for good  $r$ . Thus the factor demands are given by

$$\begin{aligned}
K_r &= Q_r c_r^\sigma \left( \frac{v}{\alpha_r} \right)^{-\sigma} \\
U_r &= L_r W_r^\zeta \left( \frac{w^U}{\gamma_r} \right)^{-\zeta} \\
S_r &= L_r W_r^\zeta \left( \frac{w^S}{\gamma_r} \right)^{-\zeta}
\end{aligned}$$

### 3.3 Equilibrium Characterization

The equilibrium conditions can be described as follows. First the labor share in each industry is given by

$$\frac{1 - \lambda_r}{\lambda_r} = \left( \frac{\alpha_r}{1 - \alpha_r} \right)^\sigma \left( \frac{W_r}{v} \right)^{\sigma-1}$$

and the share of skilled labor in total labor income in each industry is given by

$$\frac{1 - \eta_r}{\eta_r} = \left( \frac{1 - \gamma_r}{\gamma_r} \right)^\zeta \left( \frac{w^S}{w^U} \right)^{\zeta-1}$$

Skilled labor earnings in industry  $r$  can be written as

$$w^S S_r = \eta_r W_r L_r$$

Similarly, labor cost in industry  $r$  can be written as

$$W_r L_r = \lambda_r p_r Q_r$$

The quantities to be produced are determined by the expenditure shares  $\theta_r$  applied to total income

$$p_r Q_r = \theta_r \left( w^S \bar{S} + w^U \bar{U} + v \bar{K} \right)$$

where  $\bar{S}$  and  $\bar{U}$  are the aggregate amounts of skilled and unskilled labor in the world (in efficiency units), and  $\bar{K}$  is the aggregate amount of capital.

Putting these pieces together gives skilled labor earnings in industry  $r$  as

$$w^S S_r = \eta_r \lambda_r \theta_r \left( v \bar{K} + w^S \bar{S} + w^U \bar{U} \right)$$

and adding over industries gives the market-clearing condition for skilled labor

$$w^S \bar{S} = \sum_r \eta_r \lambda_r \theta_r \left( v \bar{K} + w^S \bar{S} + w^U \bar{U} \right)$$



with a similar equation for unskilled labor. Thus the market-clearing equations can be written as

$$\begin{aligned} A_S(x)(1+x_1+x_2) &= x_1 \\ A_U(x)(1+x_1+x_2) &= x_2 \end{aligned}$$

where

$$x = (x_1, x_2) = \left( \frac{w^S \bar{S}}{v \bar{K}}, \frac{w^U \bar{U}}{v \bar{K}} \right) \quad (1)$$

and

$$\begin{aligned} A_S(x) &= \sum_r \theta_r \lambda_r(x) \eta_r(x) \\ A_U(x) &= \sum_r \theta_r \lambda_r(x) (1 - \eta_r(x)) \end{aligned}$$

To complete this characterization of the equilibrium conditions, it must be shown that the shares can be expressed in terms of  $x$ . The skilled labor share can immediately be written as

$$\eta_r(x) = \frac{1}{1 + \left( \frac{1-\gamma_r}{\gamma_r} \right)^\zeta \left( \frac{x_1 \bar{U}}{x_2 \bar{S}} \right)^{\zeta-1}} \quad (2)$$

To write the share of the labor composite as a function of  $x$  first note that the factor price ratio in industry  $r$  satisfies

$$\left( \frac{W_r}{v} \right)^{1-\zeta} = \gamma_r^\zeta \left( x_1 \frac{\bar{K}}{\bar{S}} \right)^{1-\zeta} + (1-\gamma_r)^\zeta \left( x_2 \frac{\bar{K}}{\bar{U}} \right)^{1-\zeta}$$

Thus

$$\lambda_r(x) = \frac{1}{1 + \left( \frac{\alpha_r}{1-\alpha_r} \right)^\sigma \left( \gamma_r^\zeta \left( x_1 \frac{\bar{K}}{\bar{S}} \right)^{1-\zeta} + (1-\gamma_r)^\zeta \left( x_2 \frac{\bar{K}}{\bar{U}} \right)^{1-\zeta} \right)^{\frac{\sigma-1}{1-\zeta}}}$$

Adding the two market-clearing equations gives

$$(1+x_1+x_2) \sum_r \theta_r \lambda_r(x) = x_1+x_2$$

so

$$1 - \sum_r \theta_r \lambda_r(x) = 1 - \frac{x_1+x_2}{1+x_1+x_2}$$

and

$$\sum_r \theta_r (1 - \lambda_r(x)) = \frac{1}{1 + x_1 + x_2} \quad (3)$$

Substituting this in the market-clearing equations gives

$$\begin{aligned} \sum_r \theta_r \lambda_r(x) \eta_r(x) &= x_1 \sum_r \theta_r (1 - \lambda_r(x)) \\ \sum_r \theta_r \lambda_r(x) (1 - \eta_r(x)) &= x_2 \sum_r \theta_r (1 - \lambda_r(x)) \end{aligned}$$

and these equations can be rearranged as

$$\begin{aligned} \sum_r \theta_r \lambda_r(x) (\eta_r(x) + x_1) - x_1 &= 0 \\ \sum_r \theta_r \lambda_r(x) (1 - \eta_r(x) + x_2) - x_2 &= 0 \end{aligned} \quad (4)$$

Thus an equilibrium is a point  $x = (x_1, x_2)$  that solves these two (nonlinear) equations. It is straightforward to show that a solution exists. But showing uniqueness by direct analysis of these equations is surprisingly difficult. And showing uniqueness indirectly using the economic structure that generates the equations is surprisingly easy.

### 3.4 Uniqueness

The proof of uniqueness involves the following steps, each of which is quite straightforward.

1. Every solution of the market-clearing equations is associated with a competitive equilibrium.
2. Every competitive equilibrium is Pareto optimal.
3. Every Pareto optimal allocation maximizes the utility of a consumer who owns everything.
4. If two allocations are Pareto optimal, they must have the same total output vector, because preferences are strictly convex.
5. The production function for each good is strictly quasiconcave.
6. If two Pareto optimal production plans yield the same output, they must use the same input vectors.

These steps are discussed in turn.

1. Suppose that  $x$  solves the market-clearing equations 4. Normalize the price of capital so that  $v = 1$ , and set wages as

$$(w^S, w^U) = \bar{K} \left( \frac{x_1}{\bar{S}}, \frac{x_2}{\bar{U}} \right)$$

and set product prices equal to the production costs, meaning that

$$p_r^{1-\sigma} = \alpha_r^\sigma + (1 - \alpha_r)^\sigma W_r^{1-\sigma}$$

with

$$W_r^{1-\zeta} = \gamma_r^\zeta (w^S)^{1-\zeta} + (1 - \gamma_r)^\zeta (w^U)^{1-\zeta}$$

Using these factor and product prices, set the quantities of the consumption goods so that

$$p_r Q_r = \theta_r (w^S \bar{S} + w^U \bar{U} + \bar{K})$$

and set the input quantities so that

$$\begin{aligned} \left( \frac{Q_r}{K_r} \right)^\rho &= \alpha_r + (1 - \alpha_r) \left( \frac{W_r}{v} \frac{\alpha_r}{1 - \alpha_r} \right)^{1-\sigma} \\ \left( \frac{Q_r}{L_r} \right)^\rho &= \alpha_r \left( \frac{v}{W_r} \frac{1 - \alpha_r}{\alpha_r} \right)^{1-\sigma} + (1 - \alpha_r) \end{aligned}$$

This fully specifies the equilibrium quantities and (relative) prices. It is easy to check that the consumption and production choices are optimal at these prices, and the factor markets clear because  $x$  solves the market-clearing equations.

2. This step is just the first welfare theorem.<sup>13</sup>
3. Given that preferences are homothetic, and identical for all agents, the competitive equilibrium must maximize the utility of a consumer who has these preferences and owns everything. In any competitive equilibrium, by homotheticity, there is a number  $\beta^t$  for each consumer  $i$  such that the individual consumption plans are given by

$$q^i = \beta^i Q$$

where  $Q$  is the equilibrium aggregate output vector. The utility function representing a

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<sup>13</sup>The proof can be recited so as to emphasize that this step is elementary. If there is an alternative allocation that is a Pareto improvement, the value of aggregate consumption at the equilibrium prices is strictly larger in this alternative allocation (someone is doing strictly better, so the value of this person's consumption bundle might be strictly greater, or it would have been chosen before; and no one is doing worse, and if they could have achieved this by spending less money, then local nonsatiation implies that they could have done better). The value of consumption is the value of net production plus the value of the endowment. But the value of net production can't be higher in the alternative plan, because if it were, some producer was not maximizing profit. And since the value of the endowment is unchanged, this gives a contradiction.

homothetic preference ordering can be chosen so that it is linear homogeneous, implying that

$$u(q^i) = \beta^i u(Q)$$

Then if  $u(\hat{Q}) > u(Q)$  for some feasible aggregate output vector  $\hat{Q}$ , and if this output is divided over consumers so that  $\hat{q}^i = \beta^i \hat{Q}$ , then

$$\begin{aligned} u(\hat{q}^i) &= \beta^i u(\hat{Q}) \\ &> u(q^i) \end{aligned}$$

which gives a contradiction. Therefore if  $Q$  is the aggregate output vector in a competitive equilibrium, then  $u(Q)$  is maximal over the set of feasible output vectors.

4. From step 3, any Pareto optimal allocation maximizes the utility function over the set of feasible aggregate consumption plans, and since the feasible set is convex and the utility function is strictly quasiconcave, there is a unique utility-maximizing consumption vector.
5. Any constant-returns CES production function with a nonnegative and finite elasticity of substitution is concave and strictly quasiconcave. Suppose  $f : \mathbb{R}^n \rightarrow \mathbb{R}^k$  and  $g : \mathbb{R}^m \rightarrow \mathbb{R}^\ell$  are strictly quasiconcave functions, and suppose  $h : \mathbb{R}^{k+\ell} \rightarrow \mathbb{R}$  is a strictly increasing and strictly quasiconcave function, and let  $H(x, y) = h(f(x), g(y))$  for  $x \in \mathbb{R}^n$  and  $y \in \mathbb{R}^m$ . The question is whether  $H$  is strictly quasiconcave. Take two distinct points  $(x^a, y^a)$  and  $(x^b, y^b)$  in  $\mathbb{R}^n \times \mathbb{R}^m$ , with  $X^a = f(x^a)$  and  $X^b = f(x^b)$ , and  $Y^a = g(y^a)$  and  $Y^b = g(y^b)$ . Let  $\bar{x} = \delta x^a + (1 - \delta)x^b$  and  $\bar{y} = \delta y^a + (1 - \delta)y^b$  and  $\bar{X} = \delta X^a + (1 - \delta)X^b$  and  $\bar{Y} = \delta Y^a + (1 - \delta)Y^b$ , with  $\delta \in (0, 1)$ . First suppose  $(X^a, Y^a) \neq (X^b, Y^b)$ . Then since  $h$  is strictly quasiconcave, we have

$$h(\bar{X}, \bar{Y}) > \min(h(X^a, Y^a), h(X^b, Y^b))$$

By concavity of the functions  $f$  and  $g$  we have

$$\begin{aligned} f(\bar{x}) &\geq \bar{X} \\ g(\bar{y}) &\geq \bar{Y} \end{aligned}$$

Since  $h$  is increasing, this implies

$$h(f(\bar{x}), g(\bar{y})) \geq h(\bar{X}, \bar{Y})$$

Thus

$$\begin{aligned}
H(\bar{x}, \bar{y}) &= h(f(\bar{x}), g(\bar{y})) \\
&\geq h(\bar{X}, \bar{Y}) \\
&> \min\left(h(X^a, Y^a), h(X^b, Y^b)\right) \\
&= \min\left(H(x^a, y^a), H(x^b, y^b)\right)
\end{aligned}$$

Now suppose  $(X^a, Y^a) = (X^b, Y^b)$ . Either  $x^a \neq x^b$ , which implies  $f(\bar{x}) > \min(f(x^a), f(x^b)) = \bar{X}$ , or  $y^a \neq y^b$  which implies  $g(\bar{y}) > \bar{Y}$ , and in either case  $H(\bar{x}, \bar{y}) > h(\bar{X}, \bar{Y})$  since  $h$  is strictly increasing, and  $h(\bar{X}, \bar{Y}) = H(x^a, y^a) = H(x^b, y^b)$ . Thus in any case

$$H(\bar{x}, \bar{y}) > \min\left(H(x^a, y^a), H(x^b, y^b)\right)$$

which proves that  $H$  is strictly quasiconcave.

6. If two Pareto optimal production plans yield the same output vector, then any convex combination of these plans is also feasible, and yields at least as much output of each good; and if the two plans involve distinct input vectors for some product, the convex combination yields a strictly greater quantity of this product, since the production function for each good is strictly quasiconcave (by step 4).

### 3.4.1 Cobb-Douglas Final Goods

The equilibrium analysis is simplified considerably when the production technology at the top level is Cobb-Douglas. In this case  $\lambda_r = 1 - \alpha_r$ , and the market-clearing equation for capital is

$$v\bar{K} = \sum_r \theta_r \alpha_r (v\bar{K} + w^S \bar{S} + w^U \bar{U})$$

Rearrange this equation as

$$\psi = \frac{w^S \bar{S}}{v\bar{K}} + \frac{w^U \bar{U}}{v\bar{K}}$$

where

$$\psi = \frac{1}{\sum_r \theta_r \alpha_r} - 1$$

This gives the following relationship between relative factor quantities and prices

$$r_S x_S + r_U x_U = 1$$

where

$$(r_S, r_U) = \frac{1}{\psi \bar{K}} (\bar{S}, \bar{U})$$

Define  $z_1 = r_S x_S$ , so that  $1 - z_1 = r_U x_U$ . Then the skill premium is

$$\xi = \frac{x_S}{x_U} = \frac{z_1}{\varrho(1 - z_1)}$$

where  $\varrho = \frac{R_S}{R_U} = \frac{\bar{S}}{\bar{U}}$ . Using this notation, the market-clearing equation for skilled labor can be written as

$$z_1 = \sum_r \theta_r (1 - \alpha_r) \eta_r \left( \frac{1}{\psi} + 1 \right)$$

The skilled labor share (of total labor income) in product  $r$  is

$$\eta_r(x) = \frac{1}{1 + G_r \xi^{\zeta-1}} \quad (5)$$

where

$$G_r = \left( \frac{1 - \gamma_r}{\gamma_r} \right)^\zeta$$

The market-clearing equation for skilled labor then reduces to

$$z_1 = \sum_r \frac{\Theta_r}{1 + G_r \xi^{\zeta-1}} \quad (6)$$

where

$$\Theta_r = \frac{\theta_r (1 - \alpha_r)}{\sum_t \theta_t (1 - \alpha_t)}$$

Here  $G$  and  $\Theta$  depend only on (preference and technology) parameters, while  $\varrho$  depends on endowments.

When  $\zeta > 1$ , the function on the right side of equation 6 decreases monotonically from 1 to 0 as  $z$  increases from 0 to 1 (since  $\xi$  increases from 0 to infinity); also, this function is increasing in  $\varrho$  (because  $\varrho^{1-\zeta}$  is decreasing in  $\varrho$ ). Thus there is a unique solution for  $z_1$ , which is increasing in  $\varrho$ . An increase in the endowment of unskilled labor has no direct effect on  $z_1$ ; meanwhile  $\varrho$  decreases, and this means that  $z_1$  decreases, so  $x_S$  decreases, meaning that the wage of skilled workers falls relative to the price of capital. If the skill premium falls when  $\varrho$  falls, then the right side of equation 6 increases; but this is a contradiction, since the left side decreases. Thus an increase in the endowment of unskilled labor increases the skill premium. And since the skill premium rises

while the relative wage of skilled workers falls, the relative wage of unskilled workers must also fall when the endowment of unskilled labor rises.

These effects are of course symmetric, so when  $\zeta > 1$ , an increase in the endowment of skilled labor implies a fall in the skill premium and in the relative wages of both types of labor.<sup>14</sup>

### Inelastic Substitution Between Skilled and Unskilled Labor

When  $\zeta < 1$ , the equilibrium analysis is more complicated. The equilibrium equations are

$$\begin{aligned} z_1 + z_2 &= 1 \\ z_1 &= \sum_r \frac{\Theta_r}{1 + H_r \frac{z_1}{z_2} \zeta^{-1}} \end{aligned}$$

where  $H_r = \varrho^{1-\zeta} G_r$ .

Substituting from the first equation in the second and rearranging gives

$$\sum_r \frac{\Theta_r}{1 + H_r \frac{z_1}{z_2} \zeta^{-1}} = z_1$$

Since  $\sum_r \Theta_r = 1$ , this equation can be written as

$$\sum_r \Theta_r \left( \frac{1}{1 + H_r \frac{z_1}{z_2} \zeta^{-1}} - z_1 \right) = 0$$

or

$$\sum_r \Theta_r \frac{1 - z_1 - H_r z_1^\zeta z_2^{1-\zeta}}{z_1 + H_r z_1^\zeta z_2^{1-\zeta}} = 0$$

Also,  $z_2 = 1 - z_1 \neq 0$ , so the equation reduces to

$$\sum_r \Theta_r \frac{1 - H_r z_1^\zeta z_2^{-\zeta}}{z_1 + H_r z_1^\zeta z_2^{1-\zeta}} = 0$$

or

---

<sup>14</sup>An increase in the endowment of skilled labor has no direct effect on  $1 - z = r_U x_U$ ; meanwhile  $\varrho$  increases, and this means that  $z$  increases and  $1 - z$  decreases, so  $x_U$  decreases, meaning that the wage of unskilled workers falls relative to the price of capital. Since  $\varrho$  increases, the skill premium falls. And since the skill premium falls while the relative wage of unskilled workers falls, the relative wage of skilled workers must also fall when the endowment of skilled labor rises.

$$\sum_r \Theta_r \frac{1 - H_r y^\zeta}{1 + H_r y^{\zeta-1}} = 0$$

where  $y = \frac{z_1}{1-z_1}$ .

It is easy to see that this equation has a solution if  $\zeta > 0$ , since each term in the sum starts at zero, with a positive slope, and is negative for large  $y$ . An increase in  $\bar{U}$  implies a decrease in  $H_r$ , shifting the function up, so if the function is downward-sloping at the equilibrium point (as will be shown below), then the equilibrium value of  $y$  increases. This means that an increase in the endowment of unskilled labor increases the skilled wage relative to the price of capital. Since the skill premium is  $\xi = \frac{\bar{U}}{\bar{S}}y$ , an increase in  $\bar{U}$  increases the skill premium.

The equilibrium value of  $1 - z = \frac{\bar{U}}{\psi K} \frac{w^U}{v}$  falls when  $\bar{U}$  increases, so the unskilled wage must fall relative to the price of capital.

Given that  $z_1 = 0$  is not an admissible solution, the equilibrium equation can be written as

$$v_0(t) = \sum_r \Theta_r v_r(t) = 0$$

where

$$v_r(t) = \frac{1 - H_r t^{\beta-1}}{H_r + t}$$

with  $t = \left(\frac{z_1}{1-z_1}\right)^{1-\zeta}$  and  $\beta - 1 = \frac{\zeta}{1-\zeta} > 0$ .

The derivative of  $v_r$  is

$$\begin{aligned} v_r'(t) &= \frac{-(\beta - 1) H_r t^{\beta-2}}{H_r + t} - \frac{1}{H_r + t} v_r(t) \\ &\leq -\frac{1}{H_r + t} v_r(t) \end{aligned}$$

If  $v_r(t) > 0$  then  $H_t < t^{1-\beta}$  and  $\frac{1}{H_r+t} v_r(t) > \frac{1}{t^{1-\beta}+t} v_r(t)$  and  $-\frac{1}{H_r+t} v_r(t) < -\frac{1}{t^{1-\beta}+t} v_r(t)$ .

If  $v_r(t) < 0$  then  $H_t > t^{1-\beta}$  and  $\frac{1}{H_r+t} < \frac{1}{t^{1-\beta}+t}$  and  $\frac{1}{H_r+t} (-v_r(t)) < \frac{1}{t^{1-\beta}+t} (-v_r(t))$ .

Thus (in either case)

$$v_r'(t) < -\frac{1}{t^{1-\beta} + t} v_r(t)$$

Then



$$\begin{aligned}
v_0'(t) &= \sum_r \Theta_r v_r'(t) \\
&< - \sum_r \Theta_r \frac{1}{t^{1-\beta} + t} v_r(t) \\
&= - \frac{1}{t^{1-\beta} + t} v_0(t)
\end{aligned}$$

Thus the slope of the function  $v_0$  is negative at every root of the function, so there is a unique root.<sup>15</sup>

The effect of a change in factor endowments can be analyzed as follows. Suppose the total supply of unskilled labor  $\bar{U}$  increases, with the supplies of skilled labor and capital held fixed. Since  $H_r = \sqrt{\frac{\bar{S}}{\bar{U}}} G_r$ , this implies a decrease in  $H_r$ . Each function  $v_r$  is decreasing in  $H_r$ , since

$$v_r(t) = \frac{1 + t^\beta}{H_r + t} - t^{\beta-1}$$

Suppose  $t^*$  is the solution for the original value of  $H_r$ , with  $v_0(t^*; H_r) = 0$ , and  $t^{**}$  is the solution for the new value of  $H_r$ , with  $v_0(t^{**}; H_r - \Delta H_r) = 0$ . Then  $v_0(t^*; H_r - \Delta H_r) > 0$ , since  $v_0$  is decreasing in  $H_r$ , and this implies  $t^{**} > t^*$ ,<sup>16</sup> meaning that an increase in the total supply of unskilled labor increases the equilibrium value of  $t$ , and therefore increases the equilibrium value of  $z_S = \frac{1}{\psi} \frac{\bar{S}}{K} \frac{w^S}{v}$ . This means that the skilled wage rises relative to the price of capital. Also, since  $z_1 + z_2 = 1$ , and  $z_1$  increases,  $z_2 = \frac{1}{\psi} \frac{\bar{U}}{K} \frac{w^U}{v}$  must fall, which implies that the relative wage of unskilled labor falls (as would be expected). And this of course implies that the skill premium rises.

In summary, in the case where the elasticity of substitution between capital and labor at the top level is unity, it has been shown that the skill premium rises when the effective supply of unskilled labor rises relative to the supply of skilled labor. And it has been shown that the effect on relative wages depends on the elasticity of substitution between skilled and unskilled labor. In the elastic case ( $\zeta > 1$ ) more unskilled labor reduces both wages, relative to the price of capital, while in the inelastic case the skilled wage rises while the unskilled wage falls.

<sup>15</sup>The function is initially positive, and eventually negative, because  $v_0(0) = \sum_r \frac{\Theta_r}{H_r} > 0$  and  $v_r(t) < 0$  for  $t > \bar{t}_r$ , so  $v_0(t) < 0$  for  $t > \max_r \bar{t}_r$ . If there are two distinct roots, there is another root between these two, since the function has a negative slope at both roots, and therefore it must turn positive somewhere in between, and since the function is continuous it must take the value zero at some point. So either there is just one root, or there are infinitely many. If there is an infinite sequence of roots they must all lie in the interval  $(0, \max_r \bar{t}_r)$ , and there must be a cluster point of this sequence in this interval. Since this cluster point can be approached along an infinite sequence of root points, the cluster point must itself be a root, and the slope of the function at the cluster point is the limit of the change in the function from each point in the sequence to the cluster point, but this change is zero, whereas the slope is negative, which is a contradiction. Therefore there is a unique root.

<sup>16</sup>If  $t^{**} < t^*$  then  $v_0(t; H_r - \Delta H_r)$  is negative for  $t$  just above  $t^{**}$ , but positive for  $t$  just below  $t^*$ , implying that the function has another root above  $t^{**}$ , a contradiction, since it has been shown that there is only one root.

### 3.5 Goods Prices

The price of good  $r$  is given by

$$p_r^{1-\sigma} = \alpha_r \left( \frac{v}{\alpha_r} \right)^{1-\sigma} + (1 - \alpha_r) \left( \frac{W_r}{1 - \alpha_r} \right)^{1-\sigma}$$

where  $W_r$  is the price of the labor composite in efficiency units, which is determined by the CES cost function for labor:

$$W_r^{1-\zeta} = \gamma_r \left( \frac{w^S}{\gamma_r} \right)^{1-\zeta} + (1 - \gamma_r) \left( \frac{w^U}{1 - \gamma_r} \right)^{1-\zeta}$$

The price ratio between any two consumer goods is given by

$$\frac{p_r^{1-\sigma}}{p_t^{1-\sigma}} = \frac{\alpha_r \left( \frac{v}{\alpha_r} \right)^{1-\sigma} + (1 - \alpha_r) \left( \frac{W_r}{1 - \alpha_r} \right)^{1-\sigma}}{\alpha_t \left( \frac{v}{\alpha_t} \right)^{1-\sigma} + (1 - \alpha_t) \left( \frac{W_t}{1 - \alpha_t} \right)^{1-\sigma}}$$

or

$$\frac{p_r^{1-\sigma}}{p_t^{1-\sigma}} = \left( \frac{W_r}{W_t} \right)^{1-\sigma} \frac{\alpha_r^\sigma \left( \frac{v}{W_r} \right)^{1-\sigma} + (1 - \alpha_r)^\sigma}{\alpha_t^\sigma \left( \frac{v}{W_t} \right)^{1-\sigma} + (1 - \alpha_t)^\sigma}$$

Thus an increase in the price of capital relative to labor implies an increase in the relative price of capital-intensive goods.

### 3.6 Immigration and Wages

The effective total supply of labor aggregated over countries is

$$\bar{L} = \sum_j a_j h_j N_j$$

where  $N_j$  is the labor force in country  $j$ . When workers move to countries where labor is more efficient, the effective supply of labor increases, and if the world capital stock is taken as fixed, this reduces the capital-labor ratio. Thus if  $M_{jk}$  workers migrate from  $j$  to  $k$ , the change in the effective labor supply is

$$\Delta \bar{L} = \sum_j \sum_k (a_k h_k - a_j h_j) M_{jk}$$

The amount of effective labor time needed to earn enough to buy one unit of good  $r$  is  $\frac{p_r}{W_r}$ .

This is determined by

$$\left(\frac{p_r}{W_r}\right)^{1-\sigma} = \alpha_r^\sigma \left(\frac{v}{W_r}\right)^{1-\sigma} + (1-\alpha_r)^\sigma$$

and in the Cobb-Douglas case ( $\sigma = 1$ ) this reduces to

$$\log\left(\frac{p_r}{W_r}\right) = \alpha_r \log\left(\frac{v}{W_r}\right) - \alpha_r \log(\alpha_r) - (1-\alpha_r) \log(1-\alpha_r)$$

The composite wage  $W_r$  is given by

$$W_r^{1-\zeta} = \gamma_r \left(\frac{w^S}{\gamma_r}\right)^{1-\zeta} + (1-\gamma_r) \left(\frac{w^U}{1-\gamma_r}\right)^{1-\zeta}$$

and in the Cobb-Douglas case ( $\zeta = 1$ ) this reduces to

$$\log(W_r) = \gamma_r \log\left(\frac{w^S}{\gamma_r}\right) + (1-\gamma_r) \log\left(\frac{w^U}{1-\gamma_r}\right)$$

Then in the case where the technology is Cobb-Douglas at both levels

$$\log(p_r) = \alpha_r \log\left(\frac{v}{\alpha_r}\right) + \bar{\alpha}_r \gamma_r \log\left(\frac{w^S}{\gamma_r}\right) + \bar{\alpha}_r \bar{\gamma}_r \log\left(\frac{w^U}{\bar{\gamma}_r}\right) - \bar{\alpha}_r \log \bar{\alpha}_r$$

where  $\bar{\alpha}_r = 1 - \alpha_r$  and  $\bar{\gamma}_r = 1 - \gamma_r$ . This involves constant terms that can be ignored (since they don't change when the endowments change). Thus

$$\log(p_r) = \alpha_r \log v + \bar{\alpha}_r \gamma_r \log w^S + \bar{\alpha}_r \bar{\gamma}_r \log w^U - \log(p_r^0)$$

where

$$\log(p_r^0) = \alpha_r \log \alpha_r + \bar{\alpha}_r \gamma_r \log \gamma_r + \bar{\alpha}_r \bar{\gamma}_r \log \bar{\gamma}_r + \bar{\alpha}_r \log \bar{\alpha}_r$$

When immigration restrictions are relaxed, the capital-labor ratios and factor-price ratios change, and this leads to changes in real wages. These changes affect all countries in exactly the same way (regardless of whether they are sending or receiving countries). Factor price equalization holds both before and after the migration of labor, but migration reduces the wage per efficiency unit of labor (and therefore also reduces the wages of all workers who do not migrate).

The indirect utility function is the log of the real wage

$$u^* = \sum_r \theta_r \log\left(\frac{w}{p_r}\right)$$

Then (ignoring the constant terms) the indirect utilities for unskilled and skilled workers are

$$\begin{aligned} u_U^* &= \sum_r \theta_r \alpha_r \log \left( \frac{w^U}{v} \right) + \sum_r \theta_r \bar{\alpha}_r \gamma_r \log \left( \frac{w^U}{w^S} \right) \\ u_S^* &= \sum_r \theta_r \alpha_r \log \left( \frac{w^S}{v} \right) + \sum_r \theta_r \bar{\alpha}_r \bar{\gamma}_r \log \left( \frac{w^S}{w^U} \right) \end{aligned}$$

The income ratios in the Cobb-Douglas case are fixed by the technology:

$$\begin{aligned} \sum_r \theta_r \bar{\alpha}_r \gamma_r &= \alpha_0 \frac{w^S S_0}{v K_0} \\ \sum_r \theta_r \alpha_r \bar{\gamma}_r &= \alpha_0 \frac{w^U U_0}{v K_0} \end{aligned}$$

where  $\alpha_0 = \sum_r \theta_r \alpha_r$ . So (again ignoring constant terms), the indirect utilities are

$$\begin{aligned} u_U^* &= \alpha_0 \log \left( \frac{K_0}{U_0} \right) + \sum_r \theta_r \bar{\alpha}_r \gamma_r \log \left( \frac{S_0}{U_0} \right) \\ u_S^* &= \alpha_0 \log \left( \frac{K_0}{S_0} \right) + \sum_r \theta_r \bar{\alpha}_r \bar{\gamma}_r \log \left( \frac{U_0}{S_0} \right) \end{aligned}$$

Now consider a change in the endowment of unskilled labor. The utility changes can be stated in terms of equivalent changes in real wages. Thus

$$\begin{aligned} \Delta \log \tilde{w}^U &= - \left( \alpha_0 + \sum_r \theta_r \bar{\alpha}_r \gamma_r \right) \Delta \log U_0 \\ \Delta \log \tilde{w}^S &= \left( \sum_r \theta_r \bar{\alpha}_r \bar{\gamma}_r \right) \Delta \log U_0 \end{aligned}$$

where  $\tilde{w}^S, \tilde{w}^U$  are the wages that would yield utilities  $u_S^*, u_U^*$  if the prices had remained at the original levels.

For example, if the unskilled labor endowment doubles, and if the labor share is  $\bar{\alpha}_0 = \frac{2}{3}$ , and if the share of skilled labor in the labor composite in each industry is  $\gamma_r = \frac{1}{2}$  the real wage of skilled workers rises by about 25%, while the real wage of unskilled workers falls by about 37%.<sup>17</sup>

<sup>17</sup>In logs, the skilled wage increases by  $\frac{1}{3} \log 2$ , so the wage level increases by the factor  $2^{\frac{1}{3}} \approx 1.26$ , and the unskilled wage falls by  $\frac{2}{3} \log 2$ , so the wage level falls by the factor  $2^{-\frac{2}{3}} \approx 0.63$ .

### 3.7 Wages in the Long Run

Migration increases the return on capital, since the effective capital-labor ratio decreases. In steady state equilibrium with a constant returns technology

$$f'(k^*) = \rho + \delta$$

where  $f'$  is the marginal product of capital,  $\rho$  is the rate of time preference,  $\delta$  is the depreciation rate and  $k^*$  is the effective capital-labor ratio. In the short run, migration increases the effective labor supply, so the capital-labor ratio falls below  $k^*$ , and the marginal product of capital rises above  $\rho + \delta$ . The investment rate therefore increases, and this continues until the effective capital-labor ratio returns to  $k^*$ , and the real wage returns to its original level. Thus migration does not reduce wages in the long run. And if immigration restrictions are removed gradually, in such a way that the effective labor supply grows at the same rate as the capital stock, then wages do not fall even in the short run.

### 3.8 Migration Decisions

One might initially expect that in a world with open borders, everyone would move to the most productive location. But this ignores the strong attachment to home locations that is evident in the data.<sup>18</sup>

For a worker at skill level  $s$ , let  $a_{js} = \frac{y_{js}}{y_{0s}} \leq 1$  be the level of income in the home location ( $y_{js}$ ), relative to the highest income available elsewhere ( $y_{0s}$ ), and assume that migration involves a utility cost  $\delta$ , which is drawn from a distribution  $F_s$ . Since the utility function is loglinear, the indirect utility function can be expressed as  $\log(y)$ . Then it is optimal to stay in the home location if

$$\log(y_{0s}) - \delta \leq \log(y_{js})$$

If the distribution of  $\delta$  is exponential,  $F_s(t) = 1 - e^{-\zeta_s t}$ , then the probability of staying is

$$\text{Prob}\left(\delta \geq \log\left(\frac{y_0}{y_j}\right)\right) = e^{-\zeta_s \log\left(\frac{y_0}{y_j}\right)} = (a_j)^{\zeta_s}$$

So if the proportion who stay is  $\mathcal{S}_{js}$  then

$$\log(\mathcal{S}_{js}) = \zeta_s \log(a_{js})$$

This model can be “calibrated” using data on the proportion of Puerto Ricans who choose to stay in Puerto Rico, even though the U.S. border is open (to Puerto Ricans), and wages in

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<sup>18</sup>For example, [Kennan and Walker \(2011\)](#) show that attachment to home is an important determinant of internal migration decisions in the U.S.

the U.S. are considerably higher. The results using data from the 2000 Census are as follows.

Schooling	<9	9-11	12	13-15	16	17	9-17	9-12	13-17
Wage Ratio	0.46	0.49	0.53	0.60	0.67	0.72	0.57	0.52	0.64
Stay	0.68	0.53	0.62	0.69	0.73	0.64	0.64	0.60	0.70
$\zeta$	0.49	0.88	0.75	0.72	0.78	1.34	0.80	0.79	0.82
N	218,715	203,138	515,421	254,483	134,023	56,929	1,070,231	718,559	445,435

The average moving cost is the reciprocal of  $\zeta$ . For people with very little education, the moving cost seems to be high. The estimates below assume that  $\zeta_s = .5$  for people with less than 9 years of schooling, and  $\zeta_s = .8$  at higher schooling levels.

## 4 Labor Supply and Wages with Open Borders: Magnitudes

### 4.1 The Effective Supply of Labor

Given that the proportion of stayers is  $a^{\zeta}$ , the average supply of effective labor after migration (to the most productive location, where the efficiency level is normalized to 1) is  $a_j^{\zeta} \times a_j y_{0s} + (1 - a_j^{\zeta}) \times y_{0s}$ . Thus the increase in effective labor per person is  $(1 - a_j^{\zeta}) (1 - a_j) \frac{y_{js}}{a_j}$ , and the aggregate increase in effective labor due to migration is

$$\Delta \bar{L} = \sum_{j=1}^J (1 - a_j^{\zeta}) (1 - a_j) \frac{y_{js}}{a_j} N_{js}$$

where  $N_{js}$  is the supply of labor at skill level  $s$  in country  $j$ .

### 4.2 Effective World Labor Supply Estimates

Data on the supply of labor at different schooling levels are available from [Barro and Lee \(2010\)](#), and relative wages can be estimated using U.S. Census data in combination with the relative wage data from [Clemens, Montenegro, and Pritchett \(2008\)](#).

Using the above approximations, the effective world labor supply with open borders increases by the amounts shown below.

Schooling Years	0-8	9-12	13-16	
Percentage Increase	149%	101%	42%	

There are two points here. One is that there is a big increase in labor supply. The other is that there is a big decrease in the ratio of skilled to unskilled workers.

One conclusion from the analysis in [Kennan \(2013\)](#) is that the effect of open borders on real wage rates is small (even in the short run, with capital held fixed). There is no reason to expect that this

result would change much when workers at different skill levels are not perfect substitutes, simply because the result is basically driven by the assumption of constant returns, with a realistic value for the capital share. In a dynamic model, the effect on real wages would be very much attenuated, even in the short run. For example, in the [Kennan and Walker \(2011\)](#) model of internal migration within the U.S., it takes about 10 years before the response to a simulated (permanent) increase in the real wage in one location is more or less complete.

## 5 Net Gains from Migration

When a worker of type  $s$  moves from  $j$  to a frontier country (where  $a = 1$ ), the (gross) income gain is

$$\begin{aligned}\Delta y &= (1 - a_{js}) y_{0s} \\ &= \frac{1 - a_{js}}{a_{js}} y_{js}\end{aligned}$$

For the average migrant, the net gain is roughly the average of this and zero, if the lowest migration cost is zero. The proportion of people who do not migrate is  $a_{js}^{\zeta_s}$  (according to the simple model of migration decisions described in Section 3.8), so the income gain for the average person (including nonmigrants) is

$$\bar{g}_{js} = \frac{1}{2} \frac{(1 - a_{js}^{\zeta_s})(1 - a_{js})}{a_{js}} y_{js}$$

The average net gain over a group of countries is given by

$$\bar{g} = \frac{\sum_j N_j \bar{g}_j}{\sum_j N_j}$$

For the countries in the [Clemens, Montenegro, and Pritchett \(2008\)](#) this gives the following (rough) estimates (in PPP dollars, at 2013 prices).

Schooling Years	0-8	9-12	13-16	
Net Gain	4396	4340	4349	
Average Income	3420	3867	4628	
Percentage Gain	129%	112%	94%	

These are very large gains, and they do not differ much across skill levels.

## 6 Conclusion

The conclusion in Kennan (2013) was that if labor is homogeneous, open borders could yield huge welfare gains despite large costs associated with uprooting people from their home countries. In practice of course there are large differences in workers' skill levels, and it is commonly argued that workers of different types are not close to being perfect substitutes. Moreover, it is well known that migration rates are much higher for workers at higher skill levels, indicating that migration costs may be much lower for skilled workers. This paper considers the effects of differential migration by workers at (two) different skill levels in the context of a world economy in which factor price equalization is achieved through arbitrage in competitive product markets.

The general equilibrium effects of migration are quite complicated when labor is not homogeneous, and when many final products are produced with variable factor intensities. Given a simple specification of preferences, with nested CES production functions for the consumption goods, it has been shown that there is a unique equilibrium. The effects of differential migration rates depend on the elasticities of substitution, between capital and the labor composites used for the various goods, and between skilled and unskilled labor. In the case where the elasticity of substitution between capital and the labor composites is unity, so that the overall labor share of income is fixed for each product (though not fixed in the aggregate), it was shown that when the world supply of unskilled labor increases (due to migration) the wages of skilled workers rise if the elasticity of substitution between skilled and unskilled labor is less than unity (and the skilled wage falls in the opposite case).

Given that skilled workers are more mobile, a relaxation of immigration restrictions leads to increased migration of skilled relative to unskilled workers. Nevertheless, the effect on world labor supplies is dominated by the flows of unskilled workers, simply because the proportion of unskilled workers is very high in low-productivity countries. Rough estimates suggest that the effective supply of unskilled would increase by considerably more than 100% in a world with open borders. If changes in the skill premium associated with these increased migration flows are neglected, the welfare gains are very large. In particular, the gain for unskilled workers is comparable to an income increase of about 130% in poorer countries, while even for skilled workers the gain is comparable to a 90% increase in income.



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