Can Achievement Peer Effect Estimates Inform Policy? A View from Inside the Black Box*

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Abstract

Empirical studies of peer effects rely on the assumption that peer spillovers can be measured through observables. However, in the education context, many theories of peer spillovers center around unobservables, such as ability, effort or motivation. I show that when peer effects arise from unobservables, the typical empirical specifications will not measure peer effects accurately, which may help explain differences in the magnitude and even sign of peer effect estimates across studies. I further show that under reasonable assumptions these estimates cannot be applied to determine the effects of regrouping students, a central motivation of the literature.

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1 Introduction

Understanding peer effects is important for a large number of educational policies that either directly or indirectly alter the grouping of students such as bussing for racial integration, ability grouping, the introduction of charter schools and private school vouchers. While empirical studies of peer effects in achievement production abound, they provide mixed evidence regarding the magnitude and even sign of these effects, severely limiting the potential to inform policy.[1] The empirical literature is generally based on the assumption that peer effects derive through behaviors that are observed to the researcher.[2] However, unlike much of the broader social interactions literature, in the education context the underlying theories of peer effects center around behaviors that are unobserved to the researcher. In this paper, I consider how this unobservability changes the interpretation of the peer spillovers typically estimated in empirical specifications (e.g., Hanushek et al., 2003). This provides new insight into why estimated spillovers from peer characteristics may differ in sign across studies, while also shedding light on the potential for these models to inform policies related to regrouping students.

The typical empirical specification, which I term the *statistical model*, assumes achievement is potentially a function of both peer characteristics (such as race, sex, socioeconomic status) and peer achievement. Based on the pioneering work of Manski (1993), much of the literature has focused on the challenges associated with identifying the parameters of this model. The central contribution of this paper is to show that even if the statistical model is identified, the estimated peer effects may be difficult to interpret and apply to education policy. The argument centers around the observation that it is not peer achievement per sé that affects a student's own achievement. Rather, the fundamental intuition for includ-

[1]For instance, see Rivkin and Welch (2006) and Schofield (1995) for a review of the mixed evidence regarding the effect of desegregation.

[2]See Brock and Durlauf (2001b) for an overview.

ing peer achievement in the peer effects regression is that something unobservable about the peers, such as their ability, motivation or behavior, matters, and peer achievement can proxy for these unobservable traits.

Recognizing peer achievement as a proxy for unobservables, I show that after conditioning on peer achievement (contemporaneous or lagged), the estimated effects of peer characteristics may be biased toward 0 or even counterintuitive in sign. For example, suppose that having better educated parents is positively correlated with achievement. Then, conditional on a given level of peer achievement, the higher the parental education of peers the lower the peer ability or effort. It is the fact that peer parental education partially proxies for the unobservable input of peers that drives the estimated effect of peer parental education downward and even negative, depending on the relative magnitude of the unobservable peer effect.

Not only does this help explain some of the mixed evidence in the literature, it also suggests that considerable care needs to be taken in determining how to apply empirical estimates to policy. One might conclude that given these observations, a natural way forward for policy may be to focus on reduced form estimates, an approach often pursued in the literature. For instance, for desegregation policy it may be more relevant to know the total effect of racial composition on achievement (not conditioning on peer achievement), which includes any disparities in unobservables across races.[3]

I show that the potential to effectively apply the reduced form empirical models to the question of regrouping students can be quite different depending on the source of unobservable peer spillovers, i.e., whether the spillover derives through predetermined characteristics of the students, such as ability, or through endogenous behaviors, such as effort. Any regrouping of students implies a reassignment to teachers. While this is well-recognized in the literature, the new insight I contribute is that the effect of the teachers may depend directly

[3]For instance, see Ammermueller and Pischke (2009) for a helpful discussion.

on the composition of the peer group. Intuitively, teachers and peers may play an important role in determining student effort. Being assigned a better teacher may improve student achievement both through a direct effect on the student's behavior and an indirect effect deriving through the improvement in her peers' behaviors. The teacher effect then multiplies as the student's behavior affects her peers' behavior and vice versa (a *social multiplier effect*). I show that in common contexts estimates of the social multiplier effect (or the spillovers from peer achievement) are generally necessary (under reasonable assumptions) to determine the effects of regrouping. This is a surprising result, as it is often assumed that estimates of the spillovers from peer achievement are not important when grouping is based on observable peer characteristics. This limitation does not apply to the case where the spillover derives only through predetermined characteristics. However, I argue that existing research and theory provide strong support for a behavioral effect of peers.

While throughout most of the paper I assume the conditions for identification of the statistical model, I also discuss how the potential to identify the spillovers from peer achievement differ across types of unobservables. Thus, I postpone discussion of the identification of the spillovers from peer achievement until I can bring insight from the model of unobservables to bear on the question. The key distinction is that effort spillovers imply a simultaneity problem (i.e., Manski (1993)'s *reflection problem*) whereas ability spillovers do not.

My paper is not the first to observe that peer achievement really proxies for unobservable peer inputs. Hanushek et al. (2003) and others acknowledge unobservable ability as the underlying rationale for controlling for peer achievement in the peer effects regression. To the best of my knowledge, Arcidiacono et al. (2009) is the only paper to make this explicit in their approach to estimating peer effects, and Burke and Sass (2006) use their approach to estimate peer effects in Florida schools. What I contribute to the existing literature is to develop the implications of the peer effects deriving from unobservables for the interpretation of spillovers from observable peer characteristics. I also draw important distinctions between different types of unobservables and discuss the limitations in applying the typical empirical specification to policy.[4]

I begin in Section 2 by providing some background on the statistical model that is typically estimated in the literature and then write down a theoretical model in Section 3 that makes explicit the argument that peer achievement spillovers derive through unobservables. I begin with the simplest case where the unobservable is exogenous (or predetermined) and build to the endogenous unobservable setting in Section 3.1. I consider the capacity of statistical models to inform regrouping policies in Section 4, contrasting grouping policies based on observable and unobservable peer characteristics. While throughout most of the paper I assume a simple setting where peer effects are homogeneous across student types, I show how critical this assumption may be for the capacity of statistical models to inform grouping policies. In Section 5, I discuss identification of endogenous peer effects and how approaches may vary based on the underlying type of unobservable peer effect. I further briefly consider the implications of dynamics in the model in Section 6. In Section 7, I discuss supporting evidence from empirical applications. Section 8 concludes.

2 Background

Suppose a peer group consists of two students, ($i = \{1, 2\}$). The outcome of interest is achievement on a standardized exam, Y_i. Let X_i denote a vector of observable characteristics of the student. The shared group input μ and the individual residual ξ_i are unobservable to the researcher. The basic linear model of achievement with peer effects is then

$$Y_i = X_i\gamma_x + X_j\tilde{\gamma}_x + Y_j\tilde{\gamma}_y + \mu\gamma_\mu + \xi_i,\ i \in \{1, 2\},\ j \neq i, \tag{2.1}$$

[4] Graham et al. (2009) also provide important complementary insight about the potential limitations of existing models of peer spillovers to inform policy.

where the notation $\tilde{\gamma}$ distinguishes peer effect parameters. Subscripts denoting different peer groups are suppressed to simplify notation. Throughout the paper, I refer to this as the *statistical model*. I make the following assumptions on the unobservables:

$$E(\xi_i|X_i, X_j) = 0, \text{ for } i \in \{1, 2\}, \text{ and} \tag{2.2}$$

$$E(\mu|X_i, X_j) = 0. \tag{2.3}$$

The achievement literature generally takes some from of equation (2.1) as the starting point for estimating peer effects. In this model, peer spillovers derive both through peer characteristics X_j (exogenous or contextual effects) and peer achievement Y_j (the "endogenous" effect). Often peer groups are measured as grades or classrooms, in which case the peer measures are generally included as the mean of the peers exclusive of i, i.e., the *linear-in-means* model. I assume two students per group for simplicity, though the arguments below easily extend to the settings with more students. I also ignore dependence on shared observable classroom inputs, such as teacher characteristics, because this does not affect the arguments below.

I maintain assumption (2.2) for simplicity. It would be violated if there is some unobservable, like ability, that is correlated with observable characteristics of the students. Studies often include lagged achievement in the regression to help control for ability and prior inputs. In Section 6, I return to develop briefly the implications of dynamics and repeated observations on student achievement over time.

Assumption (2.3) is a central assumption in the identification of peer effects. In observational data there is often matching between students and unobserved teacher or school quality, which makes this assumption unlikely to hold. One way the literature addresses this problem is through finding contexts where students are randomly assigned to peer groups (e.g. Boozer and Cacciola, 2001; Graham, 2008; Sacerdote, 2001). When longitudinal data are available, studies also exploit plausibly random cross-cohort variation in student characteristics over time, by controlling for various fixed effects related to the school, teacher, etc (e.g. Hanushek et al., 2003; Hoxby, 2000; Lavy et al., 2008; Lavy and Schlosser, 2007). For most of the paper, I maintain (2.3) to illustrate the challenges associated with unobservable peer effects in the simplest context. However, I relax the assumption in Section 4 to illustrate the potential importance of matching for understanding regrouping.

Plugging in for peer j's achievement, we can solve for the reduced form as

$$Y_i = X_i\pi_x + X_j\tilde{\pi}_x + \mu\pi_\mu + \zeta_i, \tag{2.4}$$

where $\pi_x \equiv \frac{\gamma_x+\tilde{\gamma}_y\tilde{\gamma}_x}{1-\tilde{\gamma}_y^2}$, $\tilde{\pi}_x \equiv \frac{\tilde{\gamma}_x+\tilde{\gamma}_y\gamma_x}{1-\tilde{\gamma}_y^2}$, $\pi_\mu = \gamma_\mu\frac{1+\tilde{\gamma}_y}{1-\tilde{\gamma}_y^2}$, $\zeta_{ik} \equiv \frac{\xi_{ik}+\tilde{\gamma}_y\xi_{jk}}{1-\tilde{\gamma}_y^2}$. The convention in the literature is to describe $\tilde{\pi}_x$ as the *social effect* of peers (the combination of endogenous and contextual effects), following Manski (1993).

Assumptions (2.2) and (2.3) are sufficient to identify the social effect, $\tilde{\pi}_x$. However, it is not possible to separately identify endogenous effects because i and j's achievement are simultaneously determined. Formally, suppose $dim(X_i) = dim(X_j) = K$. The above reduced form equation yields $2K$ parameter estimates, but there are $2K + 1$ structural parameters in equation (2.1). One approach to identifying $\tilde{\gamma}_y$ is to find an exclusion restriction that shifts i's achievement independently of peer j's achievement.[5] A common alternative approach to identification of $\tilde{\gamma}_y$ is to replace peer achievement with its lagged value, under the assumption that the lagged value of peer achievement is conditionally mean independent of μ and ξ_i.[6]

However, unlike other branches of the social interactions literature where the type of specification in (2.1) is widely-used, it is not immediately evident why peer achievement

[5]See Blume et al. (2010) for a more complete discussion of alternative identification strategies. For instance, they show that the endogenous effect can be identified in more general settings when there is variation in the size of peer groups.

[6]See Hanushek et al. (2003), Hoxby (2000) among many others.

belongs in the equation. Annual standardized exams are often the outcome of interest, and, in the absence of cheating, are not a group effort. Thus, peer achievement per sé may not affect a student's achievement. In contrast, the decision of a teenager to smoke or drink alcohol might be readily affected by having peers that engage in these behaviors. Below, I formalize the intuition in the literature that in reality, peer achievement signals something about peers that affects achievement production, e.g., students may benefit from being exposed to more able or more engaged peers.

The method for identifying $\tilde{\gamma}_y$ depends the underlying source of the spillover. In Section 3, I assume that the parameters of the statistical model are identified. This is to provide the simplest exposition of the interpretation of the parameters in equation (2.1) in a setting with unobservable peer spillovers. I then revisit the question of identification of $\tilde{\gamma}_y$ in Sections 5 and 6, at which point I can bring important insight from the model of unobservables into the discussion.

3 Sources of Peer Spillovers and Contextual Effects

Let u_i denote an unobservable characteristic of the student is exogenous or predetermined, like ability. Taking a step back from the statistical model in (2.1), suppose the structural achievement production function is

$$Y_i = X_i\alpha_x + X_j\tilde{\alpha}_x + u_i + u_j\tilde{\alpha}_u + \mu + \epsilon_i, \tag{3.1}$$

for each $i \in \{1, 2\}$, $i \neq j$. In this production function, peer j affects i's achievement through observable and unobservable characteristics, (X_j, u_j). In addition to (2.3) above, assume that

$$E(\epsilon_i|X_i, X_j) = 0, \text{ for } i \in \{1,2\}, \text{ and} \tag{3.2}$$

$$E(u_i|X_i, X_j) = 0, \text{ for } i \in \{1,2\}. \tag{3.3}$$

Assumptions (3.2) and (3.3) operate similarly to (2.2) above. Assumptions (2.3), (3.2) and (3.3)are sufficient to identify α_x and $\tilde{\alpha}_x$, thus the contextual effects parameter in this model.

However, suppose we also want to estimate the effect of peer unobservable ability, $\tilde{\alpha}_u$. This specification imposes that achievement is increasing in the unobservable characteristic of the student. Thus, peer achievement can serve as a proxy for the peer unobservable.[7] Solving for u_j as a function of peer achievement and other inputs and substituting for u_j in equation (3.1) yields

$$\begin{aligned} Y_i =& X_i\left(\alpha_x - \tilde{\alpha}_x\tilde{\alpha}_u\right) + X_j\left(\tilde{\alpha}_x - \alpha_x\tilde{\alpha}_u\right) + Y_j\tilde{\alpha}_u \\ &+ u_i(1-\tilde{\alpha}_u^2) + \mu(1-\tilde{\alpha}_u) + \epsilon_i - \epsilon_j\tilde{\alpha}_u \\ \equiv& X_i\gamma_x + X_j\tilde{\gamma}_x + Y_j\tilde{\gamma}_y + u_i\gamma_u + \mu\gamma_\mu + \epsilon_i - \epsilon_j\tilde{\gamma}_y. \end{aligned} \tag{3.4}$$

This formalizes the justification often used in the literature for the inclusion of peer achievement in the statistical model (2.1).

Assume the parameters of the statistical model are identified, using one of the methods discussed in Section 2.[8] The key question is then how do we correctly interpret the contextual

[7]Throughout I make the assumption that the econometrician chooses to use achievement to proxy for the unobservable. This contrasts with the question of whether peer achievement satisfies the conditions needed to make it a *good* proxy. The fact that it is not a good proxy in fact underlies some of the identification problems and the counterintuitive interpretation of contextual effects described below. I return to this in Section 5.1.

[8]Note that in addition to assumptions (2.3), (3.2), and (3.3), we need one of the methods of identifying $\tilde{\gamma}_y$ as described in Section 2 and further detailed in Section 5.

effects parameter, $\tilde{\gamma}_x$, given the underlying model of peer spillovers in (3.1).[9]

Without loss of generality, assume that the covariates are constructed such that $\alpha_x \geq 0$. Assume further that peer ability spillovers exist (otherwise the exercise is uninteresting), and they are positive, i.e., $\tilde{\alpha}_u > 0$. Given also the intuitive assumption that the marginal effect of peer j's ability does not exceed the marginal effect of i's own ability ($1 \geq \tilde{\alpha}_u$), the endogenous effect parameter $\tilde{\gamma}_y \in (0, 1]$.

If the direct effect of i's observable characteristics is at least as large as the effect of peer j's characteristics, i.e., $\alpha_x \geq \tilde{\alpha}_x$, it follows that $\gamma_x \geq 0$. In other words, the estimated effect of i's characteristics in the empirical specification will have the right sign. It is however biased toward 0 if $\tilde{\alpha}_x > 0$. A similar intuition holds for estimates of the effect of shared classroom inputs, μ.

However, under similarly intuitive assumptions, the sign of the effect of peer characteristics is ambiguous. To illustrate, begin by assuming that there is no direct effect of peer characteristics on achievement, i.e., $\tilde{\alpha}_x = 0$. Then, $\tilde{\gamma}_x = -\alpha_x \tilde{\gamma}_y$. Assuming that $\alpha_x > 0$, so that the individual characteristic matters for achievement, the peer characteristic still enters the statistical model as a proxy for peer ability but negatively ($\tilde{\gamma}_x < 0$), taking the opposite sign of the individual effect α_x. For example, if peer j's parental education has no direct effect $\tilde{\alpha}_x = 0$ on i's achievement, but a positive effect on j's own achievement ($\alpha_x > 0$), then the estimated contextual peer effect in the statistical model after conditioning on peer achievement is negative ($\tilde{\gamma}_x < 0$). This runs counter to the usual intuition that the sign of the contextual effects matches the sign of the individual effects. Intuitively, conditional on a given level of peer achievement, a higher level of peer characteristics actually predicts a lower level of unobserved peer ability.

[9] Note that I treat own and peer unobservables somewhat asymmetrically here. This is for two reason. First, I maintain the assumption that peer ability spillovers are an object of interest, so that the econometrician is interested in recovering $\tilde{\gamma}_y$. Second, this produces an empirical model that is most similar to models estimated in the literature.

If there are direct spillovers from peer characteristics in achievement production the sign of the contextual effect in the statistical model is ambiguous because of the countervailing influences of the indirect effect of peer characteristics as proxying for unobserved peer ability and the direct effect of peer characteristics in achievement production. The stronger the spillovers from peer ability ($\tilde{\alpha}_u$), the stronger the direct effect of the individual characteristic (α_x), and the weaker the direct effect of peer characteristics ($\tilde{\alpha}_x$), the more likely is the contextual effect in the statistical model ($\tilde{\gamma}_x$) to take a "counterintuitive" sign.

A necessary condition for the statistical parameter ($\tilde{\gamma}_x$) to take the same sign as the direct effect of peer characteristics ($\tilde{\alpha}_x$) is that $\tilde{\alpha}_x > \tilde{\gamma}_y \alpha_x$ or $\frac{\tilde{\alpha}_x}{\alpha_x} > \tilde{\alpha}_u$. In words, it must be that the relative effect of the observable characteristic of peer j to the direct effect of the observable characteristics for i must exceed the spillovers from the peer unobservable. Table 1 summarizes the discussion above.

Table 1: Parameter Assumptions with Exogenous Unobservable

	Structural Parameters	Statistical Model Parameters
(1)	$\alpha_x \geq 0$	without loss of generality
(2)	$1 \geq \tilde{\alpha}_u > 0$	(2) $\Rightarrow \tilde{\gamma}_y \in (0, 1]$
		(2)$\Rightarrow \tilde{\gamma}_\mu \in [0, 1)$
(3)	$\alpha_x \geq \tilde{\alpha}_x$	(1)-(3) $\Rightarrow \gamma_x \geq 0$
		(1)-(3) $\Rightarrow \tilde{\gamma}_x \lesseqgtr 0$
(4)	$\frac{\tilde{\alpha}_x}{\alpha_x} > \tilde{\alpha}_u$	(1), (4) $\Rightarrow \tilde{\gamma}_x$ same sign as $\tilde{\alpha}_x$

3.1 Endogenous Unobservable

The basic intuition developed above extends to the more general setting where the unobservable is permitted to be an endogenous choice of the student. While the empirical literature does not distinguish between spillovers deriving through unobservable exogenous characteristics of the student and behavioral choices, both have a place in theories of peer spillovers.

First, in terms of direct spillovers to achievement production, as in equation (3.1), the tracking literature generally finds that being grouped with higher ability peers benefits students (e.g. Figlio and Page, 2002, among others). This could occur through many channels. For instance, higher ability students may ask better questions, from which their classmates benefit, or they may help teach their classmates.[10]

Previous studies also support an effect of peer behavior or effort on achievement production. For instance, Lazear (2001)'s model of peer influence predicts that the disruptive behavior of a student imposes negative externalities on other students in the classroom. Figlio (2007), Lavy and Schlosser (2007) and Kinsler (2006) present empirical evidence that disruptive peers may negatively affect achievement. Equally plausible is the potential positive externality of being grouped with more engaged students.

Beyond direct spillovers to achievement production, peers may also play an important role in shaping student incentives to achieve.[11] While student incentives have received attention elsewhere in the achievement literature (e.g. Bishop and Woessmann, 2004; Costrell, 1994; Fryer, 2010, , among others), for the most part students are treated as passive inputs in achievement production functions. Treating students as decision makers introduces a natural role for peers in setting norms of conduct and providing social pressures against or in favor of achievement.

To solidify intuition for how peers may shape student incentives, it is useful to write down the student's utility in a form similar to other social interaction models estimated in the literature (e.g., Brock and Durlauf, 2001b). Suppose the unobservable u_i is an endogenous choice, such as effort. Suppose the student derives utility from achievement, which is increasing in effort. She also faces a cost to exerting effort that depends on the effort of her

[10]In fact, it is likely that the two effects interact in the sense that high ability students are unlikely to provide positive spillovers if they do not exert effort. The simple, additively separable setting is maintained here to generate the linear-in-means statistical model.

[11]See evidence in Fryer and Torelli (2010) and Bishop (2006).

peers, i.e.,

$$V_i = v_i(Y_i) - c_i(u_i, u_j, \mu),$$

where $\partial v_i(\cdot)/\partial Y_i > 0$, $\partial c_i(\cdot)/\partial u_i > 0$ and $\partial^2 c_i(\cdot)/\partial u_i \partial u_j \neq 0$. If $\partial^2 c_i(\cdot)/\partial u_i \partial u_j \geq 0$, this is a conformity type effect (as discussed by Brock and Durlauf (2001a) and others in the broader social interactions context), where a student seeks to conform to the effort of her peers.[12]

The i subscripts permit utility-maximizing behavior to vary by individual and peer characteristics. I include unobservable teacher quality μ as a potential input into the utility function to capture the idea that teachers can serve an important role in motivating students and encouraging or discouraging behaviors that are conducive to achievement. Thus, teachers can affect behaviors directly and potentially help determine whether a group of students reaches a high or low-achieving equilibrium set of behaviors.

This behavioral model introduces several important new channels for peer effects in the achievement context, beyond the direct externalities in achievement production described above. First, there is the potential for the unobservable peer behavior to affect a student's own behavior. For example, the cost of working hard in a class of non-hard-working peers is likely to be much larger because the student risks standing out as a "nerd" or "teacher's pet."[13] Second, there may be an additional role for peer characteristics, for instance, if having peers with better-educated parents leads students to value achievement more.

Students simultaneously choose their utility-maximizing effort as a best response to peer effort, $u_i^{BR}(u_j)$. Plugging the effort best response into the achievement production function,

[12] In principle, the effect could go in the opposite direction, if having better peers leads to discouragement, as I discuss in my example in Appendix A

[13] For instance, see Bishop et al. (2003). In a competitive environment, students may value achievement mostly as it relates to their peers' achievement, producing a similar style peer effect in determining optimal behavior.

results in an achievement best response:

$$Y_i^{BR} = X_i\alpha_x + X_j\tilde{\alpha}_x + u_i^{BR}(u_j) + u_j\tilde{\alpha}_u + \mu + \epsilon_i.$$

Maintaining the assumption that achievement is monotonically increasing in effort, we can proceed similarly to above and use peer achievement to proxy for peer effort, u_j. Given that the effort best response is increasing in peer effort $\partial u_i^{BR}(u_j)/\partial u_j \geq 0$, which results in this type of setting with conformity effects, this is sufficient to guarantee that the achievement best response is also increasing in peer effort. I finally assume that the achievement we observe in the data results from students' utility-maximizing effort, i.e., $Y_i^*(u_i^*, u_j^*)$, where the $*$ denotes utility-maximizing behavior.

Under these assumptions, the intuition for the interpretation of the contextual effects is very similar to the above setting. In Appendix A, I provide a particular functional form of utility that results in linear-in-parameters statistical model described in equation (2.1). I then go through a similar exercise as in the exogenous unobservable setting to describe the ambiguity of the contextual effect parameter under different sets of assumptions.

However, the key point is that whether the peer effect derives through unobservable exogenous characteristics or endogenous behaviors of the student, the implications for the interpretation of contextual effects in the statistical model are similar. Furthermore, the insight extends to more general forms of the achievement production function which maintain the properties that achievement is monotonically increasing in the unobservable and complementarity between own and peer unobservables. The important contrast is that in the ability case, the unobservables are exogenously determined, whereas in the effort case the unobservables are simultaneously determined and more similar in spirit to Manski (1993)'s endogenous effect. The above model is also useful for illustrating the potentially important role that peer behaviors could have in determining a student's achievement, as this has important implications for whether the statistical model can be applied to understand the effects of regrouping students on achievement. I discuss this further below.

The above discussion is based on the assumption that peer achievement itself does not matter for production. In a context where we consider only direct externalities of peers to achievement production this seems justified, i.e., the externalities derive through behaviors rather than achievement. In contrast, when students are treated as optimizing agents and choose behaviors based on peers, as suggested in the tracking and acting white literature, there may be a direct role for prior peer achievement in determining effort. For instance, if students are placed with peers who are higher performing, as they observe through knowledge of prior achievement, they could choose to work harder to maintain a certain status in the class. However, as long as this is accompanied by direct externalities from peer effort or ability on production or responses to peer effort, as supported by findings in Lavy and Schlosser (2007), Bishop (2006), Figlio (2007), and Kinsler (2006), among others, similar insights into the interpretation of contextual effects will hold.

4 Implications for Regrouping Policies

The previous section illustrates how conditioning on peer achievement produces estimates of contextual effects that are difficult to interpret. This has most directly implications for policies related to determining the effects of regrouping students based on variables that are unobservable to the researcher.

It may often be the case that policy makers (such as the teachers or principal) have more information about the student, such as GPA, classroom performance or IQ, than is available to the researcher. The real challenge arises if the policy maker is balancing two objectives of mixing by SES and "ability" that is unobserved to the researcher. Suppose the researcher uses peer achievement to proxy for this ability. Conditional on peer achievement, the coefficient on peers receiving free/reduced price lunch ($\tilde{\gamma}_x$) includes both a direct effect ($\tilde{\alpha}_x$) and the indirect effect deriving from both variables proxying for peer ability ($\alpha_x \tilde{\alpha}_u$). This contrasts with a model that holds unobserved peer quality constant, where the marginal effect of changing free/reduced price lunch status of peers is $\tilde{\alpha}_x$. Thus, the most direct implication for policy is that it would be incorrect to extending insights from contextual effects parameters in statistical models that control for observable achievement (such as in equation (2.1)) to regrouping policies based on other "ability" measures that are not observed to the researcher (such as in equation (3.1)).

That said, often policy questions of interest center around characteristics of students that are observable to the researcher. For instance, if increased school choice leads to exit of the children with better-educated or higher-income parents, does this hurt the students left behind? Does racial integration improve the performance of black students? In these cases, the reason why the the peer characteristic matters, i.e., whether it is race per sé or the unobservable characteristics correlated with race may be of secondary importance. This observation is often used to motivate focusing on estimating the reduced form effect of peers (as in equation (2.4)). In fact, the analysis above could lend further support that conditioning on peer achievement in the specification is not useful.

With these observations in mind, I consider in Sections 4.1 and 4.2 the potential for reduced form estimates of peer effects to inform grouping policies. Research increasingly recognizes the limitations of the linear-in-means framework and explores how peer effects vary based on student characteristics.[14] Section 4.2 expands the model to allow for heterogeneous peer effects.

To fix ideas, I center the discussion around determining the effects of racial integration. The literature on racial composition effects is extensive and of continued concern to policy

[14]See, for instance, Hoxby and Weingarth (2005), Hoxby (2006), Hanushek et al. (2009), Cooley (2009), Lavy et al. (2008), among others.

makers. However, the arguments below can certainly be extended to other contextual effects of interest.

4.1 Grouping on Observable Contextual Effects

Consider a simple setting with two classrooms, $g \in \{c, d\}$, with two students each. Classrooms c and d are distinguished by their teacher qualities μ_c and μ_d. Initially the allocation g_0 is such that students $\{1, 2\}$ are in c and $\{3, 4\}$ in d. The characteristics X_i are understood to include a dummy variable for whether the student is white or nonwhite. As in Section 2, the reduced form equations are

$$Y_{1c} = X_1\pi_x + X_2\tilde{\pi}_x + \mu_c\pi_\mu + \zeta_{1c},$$

$$Y_{2c} = X_2\pi_x + X_1\tilde{\pi}_x + \mu_c\pi_\mu + \zeta_{2c},$$

$$Y_{3d} = X_3\pi_x + X_4\tilde{\pi}_x + \mu_d\pi_\mu + \zeta_{3d},$$

$$Y_{4d} = X_4\pi_x + X_3\tilde{\pi}_x + \mu_d\pi_\mu + \zeta_{4d}.$$

I maintain the assumption that $E(\zeta_{ig}|X_i, X_j) = 0$ and that consistent estimates of the reduced form parameters $\pi_x, \tilde{\pi}_x$, are available. I consider the case both where $E(\mu_g|X_i, X_j) = 0$ as maintained above and where there is matching so that quasiexperimental methods are used to identify the parameters.

In the current setting, average achievement does not change regardless of the grouping because any gains to one student are perfectly offset by losses to others. Therefore, I focus on equity implications, taking the average achievement of nonwhite students as the outcome of interest. Suppose students $\{1, 2\}$ are nonwhite. I consider how nonwhite achievement changes when moving from the observed segregated grouping g_0 to an integrated grouping g_1.

Suppose g_1 is such that we group students $\{1, 3\}$ in c and $\{2, 4\}$ in d. The average change in achievement for nonwhite students is then

$$E(Y_1 + Y_2|g_1, \vec{X}) - E(Y_1 + Y_2|g_0, \vec{X}) = [(X_3 + X_4) - (X_1 + X_2)]\tilde{\pi}_x + E(\pi_\mu(\mu_d - \mu_c)|\vec{X}),$$

where $\vec{X} = (X_1, X_2, X_3, X_4)$.

Under the assumption of random assignment of students to teacher quality (and hence $E(\mu_c|\vec{X}) = E(\mu_d|\vec{X}) = 0$), the expected change in achievement depends only on the contextual effects parameter. Thus, reduced form estimates are sufficient to predict the effect of moving from the extreme of a perfectly segregated setting to an integrated setting.

When the initial condition is one of matching rather than random assignment, which is most often the case in observational data,[15] the fact that regrouping students implicitly also involves reallocating teacher quality takes on important meaning. First, it is important to draw out the contrast between the case where the unobservable peer effect derives through ability (an exogenous effect) versus effort (an endogenous effect). As emphasized by Manski (1993) and others, endogenous and contextual peer effects potentially have quite different implications for policy. For instance, suppose we redistribute resources, among students. In the context where there are endogenous peer effects, this creates *social multiplier* effect, whereby the improvement, or loss, to one student's achievement spills over to other students in the classroom, multiplying the effect of the resource shift.

Social multiplier effects may occur as a result of the regrouping if students respond to teachers.[16] In the ability setting the classrooms that receive the higher teacher quality experience higher achievement only through the direct effect of the improvement in teacher quality. In the effort setting, there is also an indirect effect deriving through the effect of

[15]For instance, Clotfelter et al. (2006) find evidence that more highly qualified teachers tend to be matched with more affluent schools or schools with fewer minority students.

[16]Roderick and Engel (2001), for instance, show that teachers play a significant role in determining students' effort responses to high stakes testing.

increased teacher quality on student effort and the social multipliers created.

Restating this in terms of the parameters of the model, in the reduced form above the social multiplier is captured by π_μ. Suppose $\mu_d > \mu_c$, so that the white students initially have higher teacher quality than the nonwhites. When student 2 is assigned to classroom d his achievement improves both because of higher μ and because of his new peer. If the unobservable peer quality is exogenous, the social multiplier does not exist. This can be seen by plugging in for $\gamma_\mu = (1-\tilde{\gamma}_y)$, so that $\pi_\mu \equiv \gamma_\mu \frac{1+\tilde{\gamma}_y}{1-\tilde{\gamma}_y^2} = \frac{(1-\tilde{\gamma}_y)(1+\tilde{\gamma}_y)}{1-\tilde{\gamma}_y^2} = 1$. However, when effort varies with teacher inputs as in the set up described in Section 3.1 and the simple example in the Appendix in equation (A.2), the social multiplier exists because teacher quality affects effort and peer effort affects the student's own effort.[17]

Return now to the case of observational data where teachers are matched to students and $\mu_d > \mu_c$. Suppose that we still have consistent estimates of the reduced form contextual effects, perhaps through some of the quasi-experimental methods described above. For instance, if teacher quality is fixed we can approximate $\mu_c^* \equiv \pi_\mu E(\mu_c|X_1, X_2)$ and $\mu_d^* \equiv \pi_\mu E(\mu_d|X_3, X_4)$, as the fixed effect from the above regressions when panel data on different peer groups with the same teacher are available. This is sufficient for separating out the change in average achievement of nonwhite students deriving from the change in teacher quality under the regrouping, i.e., $\pi_\mu E((\mu_d - \mu_c)|\vec{X}) = \mu_d^* - \mu_c^*$.

Thus, reduced form estimates are sufficient to determine the effect of regrouping even in the presence of social multipliers. However, this relies heavily on the assumption that, if the social multiplier exists, it is constant across student types, which I relax below.

[17] In the example in Appendix A $\pi_\mu = \frac{1-\tilde{\gamma}_y^2+\delta\beta_\mu(1-\tilde{\gamma}_y)}{1-\tilde{\gamma}_y^2}$, where β_μ captures how teacher quality affects a student's value of achievement.

4.2 Heterogeneous Peer Effects

Evidence suggests that nonwhites and whites may respond differently to peers, and these disparities may have important implications for the effect of desegregation on the racial achievement gap.[18] Returning to the previous example but introducing heterogeneity, let the subscript w denote white and n nonwhite. The reduced form equations are then

$$
\begin{aligned}
Y_{1c} &= X_1 \pi_{xnn} + X_2 \tilde{\pi}_{xnn} + \mu_c \pi_{\mu nn} + \zeta_{1c}, \\
Y_{2c} &= X_2 \pi_{xnn} + X_1 \tilde{\pi}_{xnn} + \mu_c \pi_{\mu nn} + \zeta_{2c}, \\
Y_{3d} &= X_3 \pi_{xww} + X_4 \tilde{\pi}_{xww} + \mu_d \pi_{\mu ww} + \zeta_{3d}, \\
Y_{4d} &= X_4 \pi_{xww} + X_3 \tilde{\pi}_{xww} + \mu_d \pi_{\mu ww} + \zeta_{4d},
\end{aligned}
$$

where $\pi_{xrr'} \equiv \frac{\gamma_{xr}+\tilde{\gamma}_{yr}\tilde{\gamma}_{xr'}}{1-\tilde{\gamma}_{yr}\tilde{\gamma}_{yr'}}$, $\tilde{\pi}_{xrr'} \equiv \frac{\tilde{\gamma}_{xr}+\tilde{\gamma}_{yr}\gamma'_{xr}}{1-\tilde{\gamma}_{yr}\tilde{\gamma}_{yr'}}$, $\pi_{\mu rr'} = \frac{\gamma_{\mu r}+\tilde{\gamma}_{yr}\gamma_{\mu r'}}{1-\tilde{\gamma}_{yr}\tilde{\gamma}_{yr'}}$, and $r, r' \in \{n, w\}$.

If students are reassigned to create mixed-race classes as before, the contextual effects are different from in the segregated setting. The reduced form parameters for nonwhites are now $\pi_{xnw} \equiv \frac{\gamma_{xn}+\tilde{\gamma}_{yn}\tilde{\gamma}_{xw}}{1-\tilde{\gamma}_{yn}\tilde{\gamma}_{yw}}$, $\tilde{\pi}_{xnw} \equiv \frac{\tilde{\gamma}_{xn}+\tilde{\gamma}_{yn}\gamma_{xw}}{1-\tilde{\gamma}_{yn}\tilde{\gamma}_{yw}}$. Importantly, the social multiplier also varies by the composition of the classroom, i.e., $\pi_{\mu nw} \equiv \frac{\gamma_{\mu n}+\tilde{\gamma}_{yn}\gamma_{\mu w}}{1-\tilde{\gamma}_{yn}\tilde{\gamma}_{yw}}$ for nonwhites. Thus, the change in the expected achievement for nonwhites in moving from the segregated to the mixed-race

[18] For instance, see Fordham and Ogbu (1986), Cooley (2009), Hanushek et al. (2009), Hoxby and Weingarth (2005), Fryer and Torelli (2010), among others. Similarly, evidence suggests the the effect of peer achievement may vary by the "ability" of the students. See, for instance, Cooley (2009), Ding and Lehrer (2007), Hanushek et al. (2003), Hoxby and Weingarth (2005) for evidence of nonlinearities. Ammermueller and Pischke (2009) is an exception. Given that nonwhite students are more highly concentrated in the lower tails of the achievement distribution, heterogeneous peer effects by "ability" could have important implications for desegregation as well.

setting with heterogeneous peer effects is

$$E(Y_1+Y_2|g_1,\vec{X})-E(Y_1+Y_2|g_0,\vec{X})=(X_1+X_2)\pi_{xnw}+(X_3+X_4)\tilde{\pi}_{xnw}$$
$$-(X_1+X_2)(\pi_{xnn}+\tilde{\pi}_{xnn})+E((\mu_d+\mu_c)\pi_{\mu nw}-2\mu_c\pi_{\mu nn}|\vec{X}).$$

This suggests at least two challenges associated with using the reduced form to estimate effects of regrouping. The first is a support assumption. Given heterogeneous peer effects, it may not be possible to infer the contextual peer effects associated with racially mixed classrooms $(\tilde{\pi}_{xwn},\tilde{\pi}_{xnw})$ if we only observe homogenous classrooms, from which we can recover $(\tilde{\pi}_{xww},\tilde{\pi}_{xnn})$. Given a sufficiently rich support, however, and a sufficiently flexible estimator, this problem may be mitigated.[19] Suppose that we observe both racially mixed and segregated classrooms and that we have consistent estimates of the reduced form parameters for both settings.

Again, begin by assuming that students are randomly assigned to teacher quality so that $E(\mu_g|\vec{X})=0$. As in the example above, the problem is then trivial, as the change in expected achievement on nonwhite students depends only on the change in peer composition.

However, a second challenge emerges when $E(\mu_g|\vec{X})\neq 0$ and teacher quality creates social multipliers. As above, to recover the expected change in average nonwhite achievement, we need to account for the fact that nonwhite students would also be assigned to higher quality teachers in the mixed-race setting. Thus, it is also necessary to approximate $E((\mu_d+\mu_c)\pi_{\mu nw}-2\mu_c\pi_{\mu n}|\vec{X})$ using the residuals from the reduced form equations. As above, we can estimate of $\mu_c^*\equiv E(\mu_c\pi_{\mu n}|\vec{X})$ and $\mu_d^*\equiv E(\mu_d\pi_{\mu w}|\vec{X})$ in the segregated setting, perhaps as a teacher fixed effect when panel data are available.

However, we also need to obtain estimates of the effect of teacher qualities μ_c and μ_d

[19] Note that this gets even more complicated when there are more than 2 students per classroom, as the reduced form parameter depends on the number of students of each race in the classroom.

on nonwhite students in racially mixed settings in order to approximate $E(\mu_d \pi_{\mu nw})$ and $E(\mu_c \pi_{\mu nw})$. While in the homogenous peer effects setting this is not a concern as the effect of teacher quality is the same for white and nonwhite students, in the heterogeneous peer effects setting, the effect of teacher quality is a function of the racial composition of the classroom. Thus, it is not possible to infer the effect of regrouping *unless* we observe the same teacher quality in expectation in the mixed-race classrooms as we do in racially segregated classrooms. This follows simply because the residual from the reduced form confounds the teacher quality effect and the social multiplier, which is a function of the racial composition of the classroom.

Intuitively, in the perfectly integrated system, nonwhite students would receive higher teacher quality on average than the initial observed racially-segregated assignment. If this reallocation to teachers creates social multiplier effects, it is not possible to separate an effect of racial integration from a teacher effect without estimates of the social multiplier (or endogenous) effect.

In summary, the support assumptions become particularly stringent if we seek to apply reduced form estimates to determine the effect of regrouping when peer spillovers vary by race. We need to observe not only segregated and racially-mixed classrooms, but also the same teacher quality in expectation across the two sets of classrooms. The latter challenge only arises in the context where the reallocation of teachers creates social multipliers that are a function of the composition of the peer group. If the unobservable peer effect simply arises through ability, the social multiplier does not exist, i.e., $\tilde{\pi}_{\mu nw} = \tilde{\pi}_{\mu nn} = 1$.

In this context, even though grouping is based on observables, the example shows that it may be important to estimate the deeper structural parameters (the endogenous or social multiplier effect) that are not recovered by reduced forms. Thus, it provides another justification why ignoring the endogenous unobservables and focusing on the reduced form specification may not be an appropriate way forward. It illustrates that identifying the endogenous effect may in many cases (that fit most observational settings) may be critical to determining the effects of regrouping.

5 Identification in the Context of the Model

As suggested in the previous section, identification of the endogenous effect may be important for policy. And, similar determining the effects of regrouping, whether the unobservable peer effect is predetermined or endogenous is also critical for identification of the endogenous peer effect. I first consider how the model informs identification of the endogenous peer effect in the statistical model. Because of the problems illustrated above with interpretation of the contextual effect parameters, I then consider in Section 5.1 some useful ways forward that move away from the statistical model and focus on the underlying structural achievement production function or effort best response function.

Recall from Section 2 that the challenge for identifying the endogenous effect, $\tilde{\gamma}_y$, is that Y_i and Y_j are simultaneously determined. One way to address the simultaneity problem is to find an exclusion restriction, a variable that shifts peer achievement (or effort) independently of i's achievement, so that equation 2.1 takes the form

$$Y_i = X_i\gamma_x + X_j\tilde{\gamma}_x + Y_j\tilde{\gamma}_y + Z_i\gamma_Z + \mu\gamma_\mu + \xi_i.$$

The model provides important insight into potential sources of exclusion restrictions. The use of peer achievement to proxy for the unobservable eliminates any potential exclusion restrictions deriving through direct inputs to the achievement production function. This follows because even if the structural production function (3.1) takes the form where, for instance, a student is affected by his own parent's education but not the parental education of his peers (i.e., $\tilde{\alpha}_x = 0$ in equation (3.1) above), parental education is not a valid exclusion restriction after conditioning on peer achievement. As discussed previously, peer parental education still "affects" achievement in the statistical model as a proxy for the peer unobservable, even in the absence of a direct contextual effect in the structural production function.

However, a potential exclusion restriction could derive from a characteristic or policy that affects a student's utility-maximizing effort, but does not affect peer effort or achievement production directly. In this case, a peer j has a Z that acts as a utility shifter and provides a potential exclusions, in the sense that it only affect i's achievement indirectly through peer j's utility maximizing effort.[20] Given $dim(Z_i) \geq 1$ and Z_i satisfies the typical independence assumptions with both the unobserved group effect and the individual level residual (e.g., $E(\mu|Z_i) = 0$ and $E(\xi_i|Z_i) = 0$), $\gamma_{\tilde{y}}$ is identified.

Note that this argument cannot be applied to the case where peer achievement spillovers derive only through an unobserved predetermined or exogenous peer attribute such as peer ability simply because the students are not maximizing anything. In fact, the introduction of peer achievement to proxy for "ability" creates a simultaneity problem that is not present in the model structural model, i.e., ability is not simultaneously determined.

To consider some examples of potential exclusion restrictions, a policy or program that affects the incentives of some students in the peer group but not others may be useful. Cooley (2009) offers one example–the introduction of student accountability standards, which threaten students with retention if they do not perform above a certain level. Relying on the idea that only "low-achievers" suffer the threat of this policy, the instrument is then the percentage of peers held accountable. Another potential exclusion restriction is a family-level characteristic that affects choice of effort. One example that might not affect achievement

[20]For clarity, Appendix A shows an example of where these exclusions arise in the behavioral model under a particular functional form assumption on the utility that leads to the statistical model in equation (5). Cooley (2009) describes assumptions in a more general setting.

production directly could be the presence of a high-achieving sibling. Papers using social networks, peers of peers, can be extended to provide exclusion restrictions that are effectively of this type, i.e., student A affects B and B affects C, but A only affects C indirectly through his effect on B, i.e., A does not enter C's production function directly.[21]

Throughout I have maintained assumptions that generalize to a linear-in-means model of achievement production with peer spillovers which provides a sort of worst-case scenario for identification, as emphasized by Brock and Durlauf (2001b). Cooley (2009) shows how the identifying assumptions can be extended to a more general (semi-parametric) framework using a more general form of the behavioral model described in Section 3.1.

5.1 Other Approaches

The above discussion on the statistical model that is commonly used in the literature and describes how the model of peer effects inform interpretation of parameters of the statistical model and identification. If we take as given that peer achievement only enters the statistical model as a proxy for unobservables, a more natural way forward for estimating peer effects might be to estimate the underlying production function in equation (3.1) or the effort best response described in Section 3.1.

This framework is a natural setting to apply the analytic tools developed in measurement error models (See Cunha et al., 2010, for an example in the education context). Most administrative data sets in the United States have several test scores, generally reading and math performance, and several states have well-known data sets that also follow students over time providing repeated measures of student performance. Under certain assumptions, it would be possible to use the multiple test measures to identify underlying student unobservables. This may help provide a way forward for identifying the effect of peer unobservables as well,

[21]See Bramoulle et al. (2009) and Laschever (2008) for the broader social interactions context and Giorgi et al. (2007) for the education context.

without condition on observed peer achievement.

Furthermore, alternative measures of peer effort may be available in these data, such as disciplinary infringements, days absent or tardy, hours of television watched or time spent reading for fun, as in the case of North Carolina administrative data. Also, recent work in an experimental setting that seeks to understand student incentives, such as Fryer (2010), Kremer et al. (2009) may provide useful contexts for examining peer effects, given different measures of effort available and policies aimed at shifting effort by incentivizing students.

While switching to some form of these measurement error models is promising in terms of eliminating the bias in contextual effects parameters introduced by conditioning on peer achievement directly, the identification problems of the Manski (1993) model still apply. Effort is simultaneously determined, so some type of exclusion would be helpful for identifying the effect of peer effort on a student's own effort and achievement. However, unlike the statistical model in equation (5), exclusions in the production function, such as parental characteristics that might affect my own achievement but not my peers directly, could be valid instruments in this context because achievement net of these characteristics is not being used to proxy in the same way. In Cooley and Navarro (2010), we explore the potential for prior inputs to achievement to serve as exclusions.

In observational contexts, non-random assignment to peer groups would still need to be addressed, perhaps convincingly, through variations of the panel data methods described above. The work of Cunha et al. (2010) and others in developing ways of dealing with endogeneity of inputs in education technology when inputs are unobserved would extend naturally to the context of peer effects, with non-trivial modifications. We explore the potential to exploit multiple measurements of ability and effort to identify peer effects in achievement production in greater detail in Cooley and Navarro (2010).

6 Dynamics

As discussed Section 5, the fact that achievement and peer achievement are simultaneously determined in equilibrium and share common unobserved inputs (such as teacher quality) leads to an identification problem. While this paper focuses on a static setting for illustrative purposes, much of the literature relies on panel data techniques to identify peer effects. The literature often addresses this problem by using a lagged measure of peer achievement to proxy for the unobservable.[22] Lagged achievement most often corresponds to achievement at the end of the prior academic year, which is the assumption for this discussion.

The use of lagged peer achievement is best justified in a model where students are passive inputs to the production process, i.e., as described in Section 3 above. Using once-lagged peer achievement to capture ability relies on the assumption that only peer ability accumulated in the prior year $t-1$ matters for achievement at time t. In the more common specification, twice-lagged peer achievement serves as the proxy for the unobservable,[23] so that the relevant peer ability needs to be from two years prior. Particularly if prior teacher and/or peer quality affect the level of ability acquired and the quality of teachers or peers varies across grades, this may be a less realistic assumption. Of course, a special case where this holds is when ability is a fixed innate characteristic. This may be a less desirable assumption in that it effectively negates the role of schools, i.e., there is no human capital accumulation.

When the unobservable is endogenous as in Section 3.1, it is more difficult to support the use of lagged peer achievement as a proxy. This is restrictive particularly if effort is perfectly determined by ability and current peers and teachers affect effort. Stinebrickner and Stinebrickner (2008) provide support that ability is not generally a good predictor of effort, but rather that effort varies considerably over time and has a large effect on achievement.

[22]See Hanushek et al. (2003) for discussion.

[23]Two lags are chosen because of controlling for a child's once-lagged achievement in value-added specifications.

Furthermore, the intuition that teachers in particular may affect student effort is supported by studies such as Roderick and Engel (2001) who show that teachers play an important role in determining how students respond to high stakes testing. Furthermore, unless a student's behavior is constant over time, this rules out the spillovers from peer disruptive behavior, as discussed above.[24]

Given the restrictiveness of these assumptions and evidence in the literature to the contrary, the more likely interpretation of a model that replaces contemporaneous with lagged peer achievement seems to be that of a reduced form specification (as in equation (2.4) above), where the coefficient on prior peer achievement captures both a direct effect of unobserved peer characteristics and the endogenous effect deriving through current peer behavior, or a *social effect*.

6.1 Interpreting Contextual Effects

It is worth emphasizing that the inclusion of lagged peer achievement in the statistical model still leads to similar problems in the interpretation of $\tilde{\gamma}_x$ as discussed in Section 3. In fact, if the X_{it} are time-invariant, as is the case with most characteristics available in administrative data, the interpretation of the contextual effect parameter is the same whether conditioning on lagged or contemporaneous peer achievement. However, if characteristics change over time, the indirect effect of conditioning on lagged achievement is weaker to the extent that average peer characteristics then vary over time.

Consider a simple example to illustrate. I extend the model in equation (3.1) to include

[24] Figlio (2007), for instance, finds that the propensity to be disruptive varies with age.

time subscript t and assume that students 1 and 2 are in classroom c at time t, i.e.,

$$Y_{1ct} = X_{1t}\alpha_x + X_{2t}\tilde{\alpha}_x + u_1 + u_2\tilde{\alpha}_u + \mu_{ct} + \epsilon_{1ct},$$
$$Y_{2ct} = X_{2t}\alpha_x + X_{1t}\tilde{\alpha}_x + u_2 + u_1\tilde{\alpha}_u + \mu_{ct} + \epsilon_{2ct},$$

where I assume that the unobservable is fixed over time for simplicity.

Suppose student 2 was in classroom d in the previous period with a different peer, student 3. Student 2's achievement for the previous period is

$$Y_{2dt-1} = X_{2t-1}\alpha_x + X_{3t-1}\tilde{\alpha}_x + u_2\alpha_u + u_3\tilde{\alpha}_u + \mu_{dt-1} + \epsilon_{2dt-1}$$

Solving for student 2's ability in terms of prior achievement and plugging in for 2's ability in student 1's achievement at time t yields

$$\begin{aligned} Y_{1ct} =& X_{1t}\alpha_x + X_{2t}\tilde{\alpha}_x - X_{2t-1}\alpha_x\tilde{\alpha}_u + Y_{2dt-1}\tilde{\alpha}_u - X_{3t-1}\tilde{\alpha}_x\tilde{\alpha}_u \\ &+ u_1\alpha_u - u_3\tilde{\alpha}_u^2 + \mu_{ct} - \mu_{dt-1}\tilde{\alpha}_u + \epsilon_{1ct} - \epsilon_{2dt-1}\tilde{\alpha}_u. \end{aligned}$$

If the students' characteristics are time-invariant, the contextual effect estimated in the statistical model controlling for lagged peer achievement take the same form as in equation (3.4), i.e., $\tilde{\gamma}_x = \tilde{\alpha}_x - \alpha_x\tilde{\alpha}_u$. Even if characteristics are time-varying, to the extent that they are highly correlated over time, a similar result will hold.

Finally, note that the fact that peer groups change over time does not mitigate the problem with the interpretation of $\tilde{\gamma}_x$ in this example.

6.2 Regrouping with Lagged Achievement

Even if controlling for lagged peer achievement does not solve the identification problem or fix the interpretation of the contextual effect in the statistical model, it may be useful for informing policy. In some cases lagged achievement may be used explicitly in assigning students to classrooms. For instance, public schools in Wake County, North Carolina, attempted to maintain racial balance by integrating schools based on free/reduced price lunch status and ensuring that low-achievers, as determined by prior year test scores, were distributed somewhat evenly across schools.

This provides an interesting contrast to the grouping on unobservables motivation for controlling for peer achievement as discussed in Section 4. In this case, because I know the policy that the district is pursuing, the coefficient on peer free/reduced price lunch status conditional on prior peer achievement is precisely the object of interest to policy makers. In other words, it does not matter that the empirical model does not recover the contextual effect in the structural production function. Of course, if peer effects do in fact differ across student types, the criticism of Section 4.2 still applies.

7 Evidence from Empirical Applications

Interpreting Contextual Effects Table 2 provides some evidence that achievement proxy argument may help explain mixed findings regarding the sign and magnitude of contextual effects in the literature. It is not intended to be a representative survey of the literature, but rather includes a sample of studies that report both the estimated effect of average peer achievement and peer characteristics.

Generally, the contextual effect parameter appears more likely to take on the "counterintuitive" sign or be smaller in magnitude when the spillover from peer achievement is larger in magnitude, as the model suggests. For instance, Kang (2007) finds that the effect of average peer achievement is 0.304 and the effect of average father's education is negative. Gibbons and Telhaj (2006) find that the effect of average peer achievement is 0.218 and the effect of the percentage of classroom peers who are white is insignificant. Vigdor and Nechyba (2007) find a relatively small effect of peer achievement, 0.072, and the effect of percentage black is negative, -0.137.

Recall from the discussion of the interpretation of contextual effects in the statistical model, that the bias toward 0 is larger the larger is $\tilde{\gamma}_y$. Studies which use lagged measures of peer achievement generally find smaller spillovers from peer achievement. One way to reconcile this discrepancy is that lagged measures do not capture some of the more salient peer effects which derive through contemporaneous behavioral choices as discussed above. These studies are often less likely to find estimates of contextual effects that are counterintuitive in sign.

To provide additional evidence for how contextual effects vary across settings, I use administrative data from North Carolina public schools grades 3 to 8, years 1998 to 2006 to estimate a linear-in-means version of the statistical model described above,

$$Y_{igst} = \gamma_0 + X_{igst}\gamma_x + \bar{P}_{-igst}\tilde{\gamma}_x + \bar{Y}_{-igst-k}\tilde{\gamma}_y + \delta_{st} + \delta_g + u_{igst}, \tag{7.1}$$

where g indexes grade and s school. Achievement is measured by reading scores on standardized end of grade exams and are normalized to have mean 0 and standard deviation 1. X_{igst} includes dummies for whether the student is black, Hispanic or receives free-reduced price lunch (a proxy for low income), as well as i's lagged achievement to control for prior inputs into the achievement process. $\bar{P}_{-igst}$ includes percentage black, Hispanic or free/reduced price peers in i's grade (excluding i). $\bar{Y}_{-igst-k}$ denotes average contemporaneous achievement of peers or twice-lagged achievement (i.e., $k = 0, 2$).[25] I include school-by year fixed effects

[25]Studies that estimate value-added specifications use twice-lagged peer achievement because once-lagged peer achievement is correlated with the lag of i's own achievement.

to control for potential selection into schools that is correlated with peer group composition. Peer groups are measured at the grade level, so that identification relies on plausibly exogenous variation in cohort composition across grades within a school.[26]

Table 3 illustrates how estimates of the contextual peer effects change across specifications with different measures of average peer achievement controlling for the peer unobservable. Column 1 conditions on contemporaneous peer achievement. As discussed above, because of simultaneity in achievement, this model should overstate the magnitude of the spillovers from peer achievement. The estimated endogenous peer effect ($\tilde{\gamma}_y$) is 0.41, which means that a 1 standard deviation increase in average peer achievement (0.29) raises a student's achievement by about 0.12 of a standard deviation. In this example, the estimated contextual peer effects take counterintuitive signs: the effect of percentage black is 0.22, percentage Hispanic is 0.11 and percentage free/reduced price lunch is 0.09.

In Column 2, I avoid the simultaneity problem by using twice-lagged peer achievement to proxy the unobservable peer effect. Just as estimates in Column 1 are likely to overstate the effect of the peer unobservable because of simultaneity, these estimates may considerably understate the effect if behavioral spillovers are salient. Not surprisingly, the estimated effect of peer achievement is much smaller in magnitude, 0.05, meaning that increasing the lagged average peer achievement raises a students achievement by only 0.01 of a standard deviation. In this case, $\tilde{\gamma}_y$ is small, and while the estimated contextual effect may be biased toward 0, they do not take counterintuitive signs: the effect of percentage black is not statistically significantly different from 0, percentage Hispanic is -0.06 and percentage free/reduced price lunch is -0.02.

Finally, in Column 3, I consider the reduced form version of (7.1) that does not condition on peer achievement, i.e., the linear-in-means version of equation (2.4). Recall that this case

[26]Similar strategies are used by Hanushek et al. (2009),Hoxby (2000), Lavy and Schlosser (2007), Lavy et al. (2008), among many others.

estimates a social effect, in that the effect of peer characteristics includes any endogenous peer effects and they are not biased toward 0 from conditioning on peer achievement. Consistent with intuition, these estimates are slightly larger in magnitude than in column 2, though in most cases not statistically significantly different. The estimated effect of percentage black is now -0.02, though still not statistically significantly different from 0. The estimated effect of percentage Hispanic is -0.07 and percentage free/reduced price lunch doubles in size to -0.04. The small disparity is not surprising given the relatively small magnitude of $\tilde{\gamma}_y$ in Column 2.

Regrouping Policies In Section 4.2, I illustrate the potential importance of estimating social multiplier effects for regouping policy in observational setting when there is heterogeneity in spillovers. In complementary work, Cooley (2009), I estimate the effect of contemporaneous peer achievement for North Carolina elementary school students, using an exclusion restriction. I estimate a more general version of (7.1) that permits heterogeneity in responses across the percentiles of the conditional achievement distribution and by race, while also permitting students to respond differently to classroom peers of their own race. I find evidence of large behavioral spillovers from peers of the same race, but not from peers of other races. I further find that the magnitude of the spillovers diminishes across the percentiles of the achievement distribution. Both types of heterogeneity suggest that it would be inappropriate to use the reduced form model to predict the effect of regrouping, particularly given evidence of considerable evidence that black students face lower teacher quality on average than white students.[27] Taken together with the large magnitude of endogenous peer effects, also supported in Graham (2008), suggests that accounting from the social multipliers introduced by reassigning teachers and peers could be quantitatively important.

This is supported by the regrouping simulation I perform in Cooley (2009). I estimate

[27]For instance, see Clotfelter et al. (2005).

the predicted achievement benefits from creating racially diverse classrooms by merging a higher, predominately white and lower-achieving racially mixed district. I compare estimates that capture the social multiplier effects deriving through peer achievement to a reduced form model, which also permits heterogeneity by race and conditional percentiles of the achievement distribution but does not attempt to separate out the effect of contemporaneous peer achievement. I find that failure to account for the social multiplier effect of reallocating resources across different types of groups can severely misstate the effect of desegregation. In this example, it leads to an overstatement of the effect of resource redistribution on the narrowing of the achievement gap. Intuitively, this is because the reallocation of resources is multiplied by the peers, which is not separately identified in the reduced form. I further find that the reduced form predicts very different achievement benefits from the merger. Generally, the reduced form overstates the increases in the achievement gap for the students in the higher-achieving district and overstates the narrowing of the gap for students in the lower-achieving district.

Overall, the basic evidence from this simulation suggests that contemporaneous spillovers may play an important role in understanding regrouping policies in an observational setting, a fact that has been overlooked in the literature to date.

8 Conclusion

In this paper, I clarify the rationale for endogenous peer effects, i.e., the inclusion of peer achievement in the achievement production function. I take as an underlying premise that peer achievement per sé does not matter in achievement production, but rather serves as a proxy for an unobserved quality of the peer group, as generally argued in the literature. I contrast two types of peer spillovers that peer achievement could capture–unobserved effort and ability. The important distinction is that only the former is truly endogenous, the latter being predetermined.

I highlight three reasons that make standard empirical estimates of peer effects in educational achievement difficult to apply to answer central policy questions regarding the effects of regrouping students. The first lies in the interpretation of the statistical model, i.e., estimates of the spillovers from peer characteristics conditional on peer achievement. Using peer achievement to proxy for unobserved peer "quality" suggests that peer characteristics may appear to be correlated with achievement even if they do not directly affect achievement, but only indirectly as a proxy for peer quality. Furthermore the indirect proxy channel works in opposition to the direct externality that is commonly assumed, suggesting that the intuition that a student should be positively affected by peers with characteristics conducive to achievement may not always bear out in estimates. This finding may help explain mixed evidence in the literature regarding the sign and magnitude of contextual effects.

Second, I show that under reasonable assumptions that are supported elsewhere in the literature, the reduced form estimates of the social effect of peers are not sufficient to determine the effects of regrouping students even when grouping is based on observable characteristics. If unobservable peer effort spillovers exist and effort responds to teacher inputs, reassigning students to teachers creates social multiplier effects. Estimating these social multipliers (or the endogenous effect) is then central to developing viable policy implications of large scale reallocations of students. Empirical evidence suggests that this is likely to be quantitatively as well as qualitatively important.

Third, if effort spillovers exist, the tendency to ignore the reflection problem, minimizing the importance of simultaneity concerns for identification of peer spillovers, may be misguided. Lagged measures of peer achievement are generally preferred to contemporaneous peer achievement because they are less likely to be correlated with unobserved group effects, such as teacher quality. The model highlights how lagged peer achievement may miss important aspects of the unobservable peer effect. Thus, these estimates cannot be interpreted as causal, limiting their applicability to policy. Though the simultaneity concerns associated with including contemporaneous peer achievement are often thought to be insoluble, the theoretical model further suggests natural exclusion restrictions that permit the identification of endogenous peer effects that are not available in the ability-based framework that is commonly assumed. Errors-in-variables type models may be a natural way forward for solving some of these problems.

While I focus on the achievement context, the insight for peer behaviors deriving from unobservables may be important for other areas of the social interactions literature, such as understanding the effect of peers on obesity.

References

Andreas Ammermueller and Jŏrn-Steffen Pischke. Peer effects in European primary schools: Evidence from PIRLS. *Journal of Labor Economics*, 27:315–348, July 2009.

Peter Arcidiacono, Gigi Foster, Natalie Goodpaster, and Josh Kinsler. Estimating spillovers in the classroom with panel data, with an application to the classroom. Working Paper, August 2009.

John Bishop. *Drinking from the Fountain of Knowledge: Student Incentive to Study and Learn - Externalities, Information Problems and Peer Pressure*, volume 2 of *Handbook of the Economics of Education*, chapter 15, pages 909–944. Elsevier, 2006.

John H. Bishop and Ludger Woessmann. Institutional effects in a simple model of educational production. *Education Economics*, 12(1):17–38, 2004.

John H. Bishop, Matthew Bishop, Lara Gelbwasser, Shanna Green, and Andrew Zuckerman. Nerds and freaks: A theory of student culture and norms. Brookings Papers on Education Policy, 2003.

Lawrence E. Blume, William A. Brock, Steven N. Durlauf, and Yannis M. Ioannides. *Identification of Social Interactions*, volume 1A of *Handbook of Social Economics*. North-Holland, November 2010.

Michael A. Boozer and Stephen E. Cacciola. Inside the black box of Project Star: Estimation of peer effects using experimental data. Yale Economic Growth Center Discussion Paper No. 832, June 2001.

Yann Bramoulle, Habiba Djebbari, and Bernard Fortin. Identification of peer effects through social networks. *Journal of Econometrics*, 150:41–55, 2009.

William A. Brock and Steven N. Durlauf. Discrete choice with social interactions. *The Review of Economic Studies*, 68(2):235–260, April 2001a.

William A. Brock and Steven N. Durlauf. Interactions-based models. In James Heckman and Edward Leamer, editors, *Handbook of Econometrics*, volume 5, pages 3297–3380. Elsevier, Amsterdam, 2001b.

Mary A. Burke and Tim R. Sass. Classroom peer effects and student achievement. Working papers, Department of Economics, Florida State University, February 2006.

Clotfelter, T. Charles, Ladd, F. Helen, Vigdor, and L. Jacob. Teacher-student matching and the assessment of teacher effectiveness. *Journal of Human Resources*, 41(4):778–820, 2006.

Charles T. Clotfelter, Helen F. Ladd, and Jacob Vigdor. Who teaches whom? race and the distribution of novice teachers. *Economics of Education Review*, 24(4):377–392, August 2005.

Jane Cooley and Salvador Navarro. A hidden markov model of human capital accumulation. Working Paper, September 2010.

Jane C. Cooley. Desegregation and the achievement gap: Do diverse peers help? Working Paper, May 2009.

Robert M Costrell. A simple model of educational standards. *American Economic Review*, 84(4):956–71, September 1994.

Flavio Cunha, James J. Heckman, and Susanne M. Schennach. Estimating the technology of cognitive and noncognitive skill formation. *Econometrica*, 78(3):883–931, 05 2010.

Weili Ding and Steven F Lehrer. Do peers affect student achievement in china's secondary schools? *The Review of Economics and Statistics*, 89(2):300–312, 2 2007.

David N. Figlio. Boys named sue: Disruptive children and their peers. *Education Finance and Policy*, 2(4):376–394, 2007.

David N. Figlio and Marianne E. Page. School choice and the distributional effects of ability tracking: Does separation increase equality? *Journal of Urban Economics*, 51:497–514, 2002.

Signithia Fordham and John Ogbu. Black students' school success: Coping with the "burden of acting white". *The Urban Review*, 18:176–206, 1986.

Roland G. Fryer, Jr. Financial incentives and student achievement: Evidence from randomized trials. NBER Working Papers 15898, National Bureau of Economic Research, Inc, 2010.

Roland G. Fryer, Jr. and Paul Torelli. An empirical analysis of [']acting white'. *Journal of Public Economics*, 94(5-6):380 – 396, 2010.

Stephen Gibbons and Shqiponje Telhaj. Peer effects and pupil attainment: Evidence from secondary school transition. LSE, May 2006.

Giacomo De Giorgi, Michele Pellizzari, and Silvia Redaelli. Be as careful of the books you read as of the company you keep: Evidence on peer effects in educational choices. Iza discussion papers, Institute for the Study of Labor (IZA), 2007.

Bryan S. Graham. Identifying social interactions through conditional variance restrictions. *Econometrica*, 76(3):643–660, 2008.

Bryan S. Graham, Guido Imbens, and Geert Ridder. Measuring the average outcome and inequality effects of segregation in the presence of social spillovers. Working Paper, November 2009.

Eric A. Hanushek, John F. Kain, Jacob M. Markman, and Steven G. Rivkin. Does peer ability affect student achievement? *Journal of Applied Econometrics*, 18(5):527–544, 2003.

EricA. Hanushek, JohnF. Kain, and StevenG. Rivkin. New evidence about brown v. board of education: The complex effects of school racial composition on achievement. *Journal of Labor Economics*, 27(3):349–383, 2009.

Gary T. Henry and Dana K. Rickman. Do peers influence childrens skill development in preschool? *Economics of Education Review*, 26:100–112, 2007.

Caroline Hoxby. Peer effects in the classroom: Learning from gender and race variation. Working Paper 7867, National Bureau of Economic Research, August 2000.

Caroline M. Hoxby. Economics of education. *NBER Reporter*, Fall 2006.

Caroline M. Hoxby and Gretchen Weingarth. Taking race out of the equation: School reassignment and the structure of peer effects. Working Paper, 2005.

Changhui Kang. Classroom peer effects and academic achievement: Quasi-randomization evidence from south korea. *Journal of Urban Economics*, 61(3):458 – 495, 2007.

Josh Kinsler. Suspending the right to an education or preserving it? a dynamic equilibrium model of student behavior, achievement, and suspension. Working Paper, November 2006.

Michael Kremer, Edward Miguel, and Rebecca Thornton. Incentives to learn. *The Review of Economics and Statistics*, 91(3):437–456, 02 2009.

Ron Laschever. The doughboys network: Social interactions and the employment of World War I veterans. Working Paper, April 2008.

Victor Lavy and Anala Schlosser. Mechanisms and impacts of gender peer effects at school. NBER Working Papers 13292, National Bureau of Economic Research, Inc, August 2007.

Victor Lavy, M. Daniele Paserman, and Analia Schlosser. Inside the black box of ability peer effects: Evidence from variation in low achievers in the classroom. NBER Working Papers 14415, National Bureau of Economic Research, Inc, October 2008.

Edward P. Lazear. Educational production. *Quarterly Journal of Economics*, 116(3):777–803, 2001.

Charles Manski. Identification of endogenous social effects: The reflection problem. *The Review of Economic Studies*, 60(3):531–542, 1993.

Steven Rivkin and Finis Welch. volume 2 of *Handbook of the Economics of Education*, chapter 27, pages 1019–1049. Elsevier, 2006.

Melissa Roderick and Mimi Engel. The grasshopper and the ant: Motivational responses of low-achievemetn students to high-stakes testing. *Educational Evaluation and Policy Analysis*, 23(3):197–227, 2001.

Bruce I. Sacerdote. Peer effects with random assignment: Results for Dartmouth roommates. *Quarterly journal of economics*, 116:681–704, 2001.

Janet Ward Schofield. Review of research on school desegregation's impact on elementary and secondary school students. In James A. Banks, editor, *Handbook of Research on Multicultural Education*, pages 597–616. Simon and Schuster Macmillan, New York, NY, 1995.

Todd R. Stinebrickner and Ralph Stinebrickner. The causal effect of studying on academic performance. *Frontiers in Economic Analysis and Policy, Berkeley Electronic Press*, 9(1): 1868–1868, 2008.

Jacob Vigdor and Thomas Nechyba. Peer effects in North Carolina Public Schools. In P.E. Peterson and L. Woessmann, editors, *Schools and the Equal Opportunity Problem*. MIT Press, 2007.

Table 2: Recent Studies on Peer Achievement and Composition.

Studies	Context	Source/ Specification	Peer Achievement	Est. Effect	Contextual Effect	Est. Effect
Cooley (2009)	Public Schools, N. Carolina	Table 3, Col. 1; White Students; IV School-Year FEs	Avg. Reading (T)	0.499	% Nonwhite in Classroom	-.046
Gibbons and Telhaj (2006)	Public Schools, England	Table 5, Col. 5	Avg. Key Stage (T-3)	0.218	% White in Grade	-0.004 (insig.)
Hanushek et al. (2003)	Public Schools Texas	Table 2, Col. 3; Student FEs	Avg. Math (T-2)	0.15	% Reduced Price Lunch	0.1
Hanushek et al. (2004)	Public Schools Texas	Table 1, Col. 7; Student FEs	Avg. Math (T-2)	0.01	% Black in Grade	-0.25
Henry and Rickman (2007)	Preschool Georgia	Table 4, Col. 1	Avg. Cognitive (WJ-AP)	0.39	% Black in Classroom	0.12
Kang (2007)	TIMSS countries	Table 5, Col. 7; IV School FEs	Avg. Math (T)	0.304	Avg. Father Education	-.042
Vigdor and Nechyba (2007)	Public Schools N. Carolina	Table 2, Col. 3; School FEs	Avg. Math (T-2)	0.072	% Black in Classroom	-0.137

Table 3: Peer Effects in North Carolina
(N=2,292,532)

	Contemporaneous	Twice-Lagged	Reduced Form
Avg. peer reading score	0.4054***	0.0469***	
	[0.0043]	[0.0034]	
Black	-0.0981***	-0.0990***	-0.0991***
	[0.0009]	[0.0009]	[0.0009]
Hispanic	-0.0259***	-0.0267***	-0.0267***
	[0.0018]	[0.0018]	[0.0018]
Free/reduced price lunch	-0.0920***	-0.0922***	-0.0922***
	[0.0008]	[0.0008]	[0.0008]
% Black	0.2229***	0.0087	-0.021
	[0.0138]	[0.0137]	[0.0135]
% Hispanic	0.1110***	-0.0626**	-0.0686**
	[0.0268]	[0.0267]	[0.0267]
% FRP lunch	0.0929***	-0.0189**	-0.0351***
	[0.0094]	[0.0094]	[0.0093]
R^2	0.6354	0.634	0.634

Standard errors in brackets *significant at 10%; ** significant at 5%; *** significant at 1%. The date are from North Carolina public schools end of grade exams in reading for grades 3 to 8 and academic years 1997/98-2005/06. Constant, grade and school by year fixed effects and lagged achievement also included in the regression.

A Deriving Achievement Best Response

In Section 3.1, I discuss how moving to a model with an endogenous unobservable leads to a similar interpretation of the contextual peer effects as above. Here, I provide the details of the argument using a particular functional form of utility produces the statistical model of equation (2.1) that is linear-in-parameters in achievement and peer characteristics as an equilibrium outcome of students' utility maximizing efforts.

Suppose that utility takes the form

$$V_i = \beta_{yi} Y_i - \frac{\beta_u}{2} u_i^2 + \tilde{\beta}_u u_i u_j,$$

where $\beta_{yi} \geq 0$, $\beta_u \geq 0$. The cost of effort in this model takes the form that Brock and Durlauf (2001a) term the *proportional spillovers* case, permitting complementarity in effort if $\tilde{\beta}_u \geq 0$ and discouragement type effects otherwise. Allowing the marginal utility of achievement to vary across individuals permits variation in utility-maximizing effort.[28] Note that this type of heterogeneity is new to the literature on social interactions, where heterogeneity is generally driven by terms in the residual. As I demonstrate below, it also proves to be an important generalization for identification.

Given that students simultaneously choose effort to maximize expected utility, a student i's best response to any given level of peer effort is

$$u_i^{BR} = \frac{\beta_{yi}}{\beta_u} + \frac{\tilde{\beta}_u}{\beta_u} u_j.$$

Utility-maximizing effort is a function of the marginal utility of effort relative to the cost and is increasing in the average effort of peers as a result of the conformity effect.[29] In previous work, Cooley (2009), I show the informational assumptions and other conditions needed for a Nash equilibrium to exist in a more general setting. In the present context, the equilibrium described below is consistent with various types of informational assumptions

[28] Note that as it is the marginal rate of substitution that matters, restricting β_u to be homogeneous is without loss of generality. However, in a model of endogenous reference group formation it seems likely that $\tilde{\beta}_u$ might also be individual-specific, i.e., where individuals place more weight on the actions of peers more "like" themselves. While this has interesting implications, it is beyond the scope of the present paper.

[29] Note that allowing for effort and peer effort complementarities in the achievement production function would suggest that the best response is increasing in average peer effort even in the absence of the conformity effect.

given the additive separability in the residual and other classroom inputs.

Recall that achievement is monotonically increasing in effort by assumption. Thus, the effort best response maps into an achievement best response, which is observable to the econometrician. Solving for the unobservable as a function of achievement using the production function in (3.1) yields

$$u_i = \frac{1}{1-\tilde{\alpha}_u^2}(Y_i - Y_j\tilde{\alpha}_u - X_i(\alpha_x - \tilde{\alpha}_x\tilde{\alpha}_u) - X_j(\tilde{\alpha}_x - \alpha_x\tilde{\alpha}_u) \\ - \mu(1-\tilde{\alpha}_u) - \epsilon_i + \epsilon_j\tilde{\alpha}_u).$$

Plugging i's effort best response into the achievement function and proxying for peer effort through peer achievement and other variables as above results in the achievement best response, i.e.,

$$\begin{aligned} Y_i^{BR} =& \frac{\beta_{yi}(1-\tilde{\alpha}_u^2)}{\beta_u + \tilde{\beta}_u\tilde{\alpha}_u} + X_i\left(\alpha_x - \frac{\tilde{\beta}_u + \beta_u\tilde{\alpha}_u}{\beta_u + \tilde{\beta}_u\tilde{\alpha}_u}\tilde{\alpha}_x\right) + X_j\left(\tilde{\alpha}_x - \frac{\tilde{\beta}_u + \beta_u\tilde{\alpha}_u}{\beta_u + \tilde{\beta}_u\tilde{\alpha}_u}\alpha_x\right) \\ &+ Y_j\left(\frac{\tilde{\beta}_u + \beta_u\tilde{\alpha}_u}{\beta_u + \tilde{\beta}_u\tilde{\alpha}_u}\right) + \mu\left(1 - \frac{\tilde{\beta}_u + \beta_u\tilde{\alpha}_u}{\beta_u + \tilde{\beta}_u\tilde{\alpha}_u}\right) + \epsilon_i - \epsilon_j\left(\frac{\tilde{\beta}_u + \beta_u\tilde{\alpha}_u}{\beta_u + \tilde{\beta}_u\tilde{\alpha}_u}\right), \\ \equiv& \frac{\beta_{yi}(1-\tilde{\alpha}_u^2)}{\beta_u + \tilde{\beta}_u\tilde{\alpha}_u} + X_i(\alpha_x - \tilde{\gamma}_y\tilde{\alpha}_x) + X_j(\tilde{\alpha}_x - \tilde{\gamma}_y\alpha_x) + Y_j\tilde{\gamma}_y \\ &+ \mu(1-\tilde{\gamma}_y) + \epsilon_i - \epsilon_j\tilde{\gamma}_y, \\ \equiv& \gamma_{0i} + X_i\gamma_x + X_j\tilde{\gamma}_x + Y_j\tilde{\gamma}_y + \mu\gamma_\mu + \epsilon_i - \epsilon_j\tilde{\gamma}_y. \end{aligned}$$

Given that the achievement best response is linear-in-parameters, it can be shown that a unique Nash equilibrium achievement exists, (Y_1^*, Y_2^*).[30] The observed equilibrium achieve-

[30]While I assume that students are able to perfectly predict their peers achievement, i.e., they observe $(X_i, X_j, \mu, \epsilon_i, \epsilon_j)$, this may not be the case in reality. If, for instance, students observed only their own ϵ_i prior to choosing effort and some mean 0 prior on the effort of peers, ϵ_j would not enter the best response. While these different types of informational assumptions have different implications for identification, the contrast is beyond the scope

ment as a function of individual and peer characteristics is then

$$Y_i^* = \gamma_{0i} + X_i\gamma_x + X_j\tilde{\gamma}_x + Y_j^*\tilde{\gamma}_y + \mu\gamma_\mu + \epsilon_i - \epsilon_j\tilde{\gamma}_y. \tag{A.1}$$

The achievement best response takes the same form as the statistical model in equation (2.1), a result that motivates the functional form choices above. Furthermore, the second equality shows that the structure of the marginal effects of the covariates is the same to the ability case in Section 3. The only differences are in the constant and the endogenous effect parameter $\tilde{\gamma}_y$, which derive through utility-maximizing effort and the best response.

Table 4: Effort Model Parameter Assumptions

Structural Parameters			Statistical Model Parameters
(1)	$\alpha_x \geq 0$		without loss of generality
(2)	$\beta_u \geq 0$		
(3)	$\tilde{\alpha}_u \geq 0$		
(4i)	$\tilde{\beta}_u \geq 0$		(2)-(3), (4i) $\Rightarrow \tilde{\gamma}_y \geq 0$
(4ii)	$\tilde{\beta}_u < 0$	(a) $\beta_e > \tilde{\beta}_u\tilde{\alpha}_u$	
		(b) $\beta_u\tilde{\alpha}_u \geq -\tilde{\beta}_u$	(2)-(3), (4ii,a&b) $\Rightarrow \tilde{\gamma}_y \geq 0$
		(c) $\beta_u\tilde{\alpha}_u < -\tilde{\beta}_u$	(2)-(3), (4ii,c) $\Rightarrow \tilde{\gamma}_y < 0$
(5)	$\beta_u \geq \tilde{\beta}_u$	$1 \geq \tilde{\alpha}_u$	(5) $\Rightarrow \tilde{\gamma}_y \leq 1$
(6)	$\alpha_x \geq \tilde{\alpha}_x$		(1), (6), $\tilde{\gamma}_y \leq 1 \Rightarrow \gamma_x \geq 0$
			(1), $\tilde{\gamma}_y \in [0,1]$, (5) $\Rightarrow \tilde{\gamma}_x \lesseqqgtr 0$
			(1), $\tilde{\gamma}_y < 0 \Rightarrow \tilde{\gamma}_x$ same sign as $\tilde{\alpha}_x$
(7)	$\frac{\tilde{\alpha}_x}{\alpha_x} \geq \tilde{\gamma}_y$		(1), $\tilde{\gamma}_y \in [0,1]$, (7) $\Rightarrow \tilde{\gamma}_x$ same sign as $\tilde{\alpha}_x$

Similarly to above, Table 4 describes assumptions on the parameters of the structural model and implications for the interpretation of the statistical parameters. I consider first the properties of $\tilde{\gamma}_y$. Assume that $\tilde{\alpha}_u \geq 0$ so that effort and peer effort are weakly complementary inputs to achievement production. This is consistent with the theory that harder working peers create a better learning environment. If $\tilde{\beta}_u \geq 0$, so that i's effort is increasing in

of the present paper.

peer j's effort, it follows that $\tilde{\gamma}_y \geq 0$. Suppose instead i's effort is decreasing in peer j's effort ($\tilde{\beta}_u < 0$). Then if the intuitively appealing constraint that the effects of i's own effort on utility and achievement exceeds the effect of his peers $\beta_u > \tilde{\beta}_u \tilde{\alpha}_u$, the denominator is positive. If in combination with this assumption $\beta_u \tilde{\alpha}_u \geq -\tilde{\beta}_u$, then $\tilde{\gamma}_y \geq 0$. Otherwise, $\tilde{\gamma}_y < 0$.

It is straightforward to show that $\tilde{\gamma}_y \leq 1$ if $\beta_u \geq \tilde{\beta}_u$, i.e., that the disutility from i's own effort exceeds the (dis)utility derived from conforming to j's effort, and $1 \geq \tilde{\alpha}_u$, i.e., the marginal effect of i's own effort exceeds that of her peer. Similarly to the ability setting in Section **??**, this is enough to ensure that shared inputs μ enter the statistical model with the same sign as their marginal product. Furthermore, assuming that $\alpha_x \geq \tilde{\alpha}_x$, then $\gamma_x \geq 0$, taking the same sign as α_x.

The interpretation of $\tilde{\gamma}_x$ also follows similarly to the ability case. A necessary condition for $\tilde{\gamma}_x$ to take the same sign as $\tilde{\alpha}_x$ is that $\frac{\tilde{\alpha}_x}{\alpha_x} \geq \tilde{\gamma}_y$.

If $\tilde{\beta}_u \geq 0$, the larger the individual effect and peer effort spillovers (deriving through the indirect effect of peer effort on i's utility-maximizing effort or the direct effect of peer effort on achievement production), and the smaller the contextual peer effect, the more likely that $\tilde{\gamma}_x$ takes the opposite sign of $\tilde{\alpha}_x$.

If $\tilde{\beta}_u < 0$ and $\beta_u \tilde{\alpha}_u < -\tilde{\beta}_u$, conditions are such that $\tilde{\gamma}_y < 0$. Then, the estimated contextual effect $\tilde{\gamma}_x$ will take the same sign as the structural parameter $\tilde{\alpha}_x$ but will actually overstate the contextual effect, assuming that α_x and $\tilde{\alpha}_x$ have the same sign. Note that this instance of overstating the contextual effect is difficult to justify in the setting where only peer ability matters, unless average peer ability negatively affects achievement production.

As it is written, the current specification is limited because equilibrium effort does not vary explicitly by a student's observable characteristics or classroom inputs. A simple way to incorporate variation in effort across individual and classroom types is to allow the marginal utility of achievement to depend on observable characteristics of the classroom, the individual student, her peers that affect production directly and also potentially other observables Z_i, i.e., $\beta_{yi} = \beta_0 + X_i\beta_x + X_j\tilde{\beta}_x + Z_i\beta_z + \mu\beta_\mu$.

In this case equation (A.1) becomes

$$Y_i^* = \gamma_0 + X_i\gamma_x + X_j\tilde{\gamma}_x + Y_j^*\tilde{\gamma}_y + Z_i\gamma_z + \mu\gamma_\mu + \epsilon_i - \epsilon_j\tilde{\gamma}_y, \tag{A.2}$$

where

$$\begin{aligned}
\gamma_0 &\equiv \beta_0\delta, \\
\delta &\equiv \frac{1(1-\tilde{\alpha}_u^2)}{\beta_u + \tilde{\beta}_u\tilde{\alpha}_u}, \\
\gamma_x &\equiv \alpha_x - \tilde{\gamma}_y\tilde{\alpha}_x + \delta\beta_x, \\
\tilde{\gamma}_x &\equiv \tilde{\alpha}_x - \tilde{\gamma}_y\alpha_x + \delta\tilde{\beta}_x, \\
\gamma_z &\equiv \delta\beta_z \\
\gamma_\mu &\equiv (1 - \tilde{\gamma}_y + \delta\beta_\mu).
\end{aligned}$$

Maintaining the assumptions that $\beta_u > -\tilde{\beta}_u\tilde{\alpha}_u$ and that the marginal effect of i's own effort is greater than the effect of her peer's effort $(1 \geq \tilde{\alpha}_u)$, then $\delta \geq 0$. It follows that the addition of peer characteristics in the utility function makes the spillovers from peer characteristics more likely to take the expected sign. Furthermore, allowing for preferences over achievement and effort to vary by observable characteristics highlights an alternative channel through which individual characteristics, such as parental education, may affect achievement production, i.e., through student motivation rather than as a direct input to production.

One particularly useful feature of this extension is that it provides a more explicit role for parents, other home inputs and classroom inputs. However, complementarities between effort and characteristics are likely to enter the best response in other ways. For instance, complementarities between a student's effort and his own or his peers' characteristics in achievement production would produce similar results to the above case. Utility-maximizing effort would then be increasing in own or peer characteristics because the marginal product of effort is increasing in these inputs. Furthermore, an argument could be made that marginal utility is either increasing or decreasing in achievement. If marginal utility is increasing in achievement, then students with “better” X_i would want to exert relatively more effort, which would produce similar results to the above framework. Alternatively, if the marginal utility of achievement is diminishing, this would lead to the opposite effect.