Asymmetric information, information externalities, and efficiency: the case of oil exploration

Kenneth Hendricks*
and
Dan Kovenock**

In this article we examine the effect of private information and information externalities on the ex post efficiency of investment in oil exploration. We show that too much drilling tends to occur if firms believe that the area is likely to contain a sizeable pool of oil, and too little drilling occurs if the opposite is true. Bargaining with well-defined property rights to the information externality can eliminate underinvestment, but overinvestment remains a problem because firms have an incentive not to disclose their private information.

1. Introduction

The leasing practices of the federal government frequently produce situations in which two or more firms lease adjoining tracts in an area that may contain an oil deposit. Each firm has invested in a private, seismic survey which provides a random, but informative signal about the probability of finding a deposit. The existence of the deposit and its size can only be determined by drilling an exploratory well. Each firm prefers to let the other firm incur the expense of this well. But neither firm can delay its drilling decision indefinitely, since its lease expires after a certain length of time (usually five years) and cannot be renewed. Which firm drills first, and when? How efficient is the resulting allocation? What incentives do firms have to trade? How does private information affect the negotiation process?

The purpose of this article is to analyze the above situation and address these questions. A number of previous studies (Gilbert, 1981; Miller, 1972; Peterson, 1975; Stiglitz, 1975; and Leitzinger and Stiglitz, 1984) have discussed the importance of information externalities in oil exploration and their rent-dissipating effects. Their analyses, however, ignore the dynamic aspects of the externality and the role of private information. The latter feature is particularly important since, if firms knew each other’s valuations, there is every reason to suppose that they would bargain to an efficient allocation.

There is strong evidence that the drilling information externality is an important factor

* University of British Columbia.
** Purdue University.

We thank Robert Porter, Charles Wilson, and an associate editor for their comments and suggestions, but take full responsibility for any errors. Hendricks gratefully acknowledges financial support from the National Science Foundation, and Kovenock thanks the Purdue Research Foundation and the Krannert Endowment for their financial support.
in the investment decisions of oil firms. Hendricks, Porter, and Boudreau (1987) report that, during the period of 1954 to 1970, oil firms allowed leases on 308 federal tracts off the coasts of Texas and Louisiana (29% of the total sample) to expire without drilling any wells. The average price paid for these leases was $800,000 (in 1972 dollars). Thus, either firms expected to earn positive returns from drilling these leases at the time of purchase, and subsequent information caused them to revise their beliefs and abandon them, or firms valued the option to drill. Since oil and gas prices were quite stable during this period, a positive option value is an indication that firms anticipated the possibility of obtaining information from subsequent drilling activity which would cause beliefs to be revised upward. Both explanations rely upon the presence of information spillovers.

The information externality problem is not limited to common pools. The discovery of a deposit is usually regarded as a signal that more deposits are likely to be found in the area, and firms behave accordingly. This issue has been the focus of two recent court cases. The state governments of Texas and Louisiana contended that discoveries of oil pools on their offshore leases substantially increased winning bids on nearby federal leases, even when these leases did not cover the pools in question. The states argued that the language of the Outer Continental Shelf Lands Act Amendment of 1978 should be interpreted to apply to these situations and that they were therefore entitled to any increases in federal lease values caused by the information generated on state leases. The judge concurred, and as a result, the federal government had to make payments in excess of a billion dollars.

We study the drilling decisions of oil firms in the context of a two-firm model in which each firm independently chooses when and whether to drill. The essential features of our model, which is presented in Section 2 and is motivated by a discussion of the leasing practices of the federal government, are that each firm has a private, informative signal about the value of the leases and has two periods to drill its lease. Therefore, if a firm chooses not to drill today, it can always drill tomorrow; but if it does not drill then, its lease expires. The two-period structure is sufficient to capture the basic trade-off in delaying the drilling decision: the costs from waiting arise from discounting, as the realization of profits is deferred; the benefits come from the acquisition of information which permits a more informed drilling decision.

In Section 3 we characterize the set of Bayesian Nash equilibria of the drilling game and study their efficiency properties. We prove that the model possesses a symmetric equilibrium in which each firm drills with positive probability in each period. This equilibrium captures the possibility of coordination mistakes, which occur when both firms take the same action (either drill or wait) in the first period instead of drilling sequentially. Asymmetric equilibria may also exist, including, in particular, ones in which firms coordinate their drilling decisions. In this case, one firm makes its drilling decision in period 1 and drills only in that period, while the other firm waits until period 2 to make its drilling decision. While these equilibria are not uninteresting and may in some cases describe drilling behavior, we argue that the data on drilling patterns on offshore leases suggests that firms generally fail to coordinate their drilling decisions.

Our efficiency analysis consists of comparing the allocations associated with the symmetric equilibrium to those which would be implemented by a social planner if he knew the firms’ signals. We establish two main results. First, if each firm believes on the basis of its own signal that the area is likely to contain oil, then overdrilling may occur. This happens either because firms fail to coordinate their decisions and drill simultaneously instead of sequentially, or because firms are not informed of each other’s signal and therefore drill when they should not. Second, if firms believe that their leases are at best marginal investments, then too little drilling may occur. This situation arises because drilling information has positive social value, but zero private value.

An interesting aspect of this outcome is that it can occur even though the private returns from drilling, conditional on the signals of both firms, are positive. The difficulty here is
that each firm convinces the other by its inactivity that it possesses a relatively poor signal. The subsequent revision of beliefs then causes both firms to become overly pessimistic about the returns to drilling and to let their leases expire without drilling any wells.

In Section 4, we examine the incentives for firms to bargain to an efficient allocation. To keep the analysis simple, we assume that the right to drill second is allocated to one of the firms. This means that if no agreement exists, the firm with this right can force the other firm to make its drilling decision first.\(^1\) We also assume that seismic reports are verifiable (in the sense of Milgrom (1981)). We then construct a persuasion game in which we ask, Should the firm with the property right to the information externality disclose any information to the other firm?

The standard situation in oil exploration is that firms can sell their leases. We show that in this case there is no sequential equilibrium in which the firm with the property right to the externality fully discloses its signal. Moreover, there is a sequential equilibrium in which that firm conveys no information and makes a negligible offer to buy the lease of the other firm. The other firm accepts the offer if it has no intention of drilling first, and drills otherwise. Thus, it sometimes drills when it is not efficient to do so. If the leases are non-transferable, then the results of Milgrom and Roberts (1986) apply, and one can show that the firm with the property right fully discloses its information. But, in this case, the information externality cannot be internalized. We conclude from this analysis that, even with well-defined property rights, bargaining does not eliminate all of the inefficiencies resulting from the decentralization of drilling decisions.

We close in Section 5 with some remarks about the implications of our analysis for government policy.

2. A model of decentralized exploration

The federal government transfers production rights to oil and gas deposits on its offshore lands to the private sector by means of a sequence of lease sales. The organization of a lease sale begins when the government makes one or more areas available for exploration and invites nominations from the oil industry as to which tracts should be offered for sale. The size of a tract is typically 5,000 acres, which is substantially less than the acreage required to ensure exclusive ownership of any deposits that might be present. Firms are permitted to gather information about the area using seismic surveys and off-site drilling, but they are not allowed to drill on-site wells. After the nominations are made, the government constructs a final list of tracts, which is then offered to the public through an auction procedure.

The tracts sold in a lease sale usually number well over one hundred, and they are auctioned off simultaneously. A firm submits a separate bid on each tract that it is interested in acquiring. A bid is a dollar figure which the firm promises to pay the government at the time of the sale if it is awarded the tract. The firm submitting the highest bid is awarded the tract. The results of the bidding on all tracts, as well as the identities of the bidders and the values of their bids, are announced at a public meeting.

If it wins a tract, a firm has five years to drill exploratory wells. If no work is done during this period, ownership of the lease reverts to the government, which may subsequently auction off the tract in some future sale. If oil and/or gas is discovered in sufficient quantities to warrant production, the lease is automatically renewed for as long as it takes the firm to extract the hydrocarbons.

From our perspective, there are two points to note about this sale mechanism. First, firms that are awarded tracts have private information about these tracts. The revelation of bids yields some information about the firms' priors, but it is unlikely to remove all of the

---

\(^1\) One circumstance under which this situation can occur is if the lease of one of the firms expires before that of the other firm.
uncertainty which each firm may have about the others’ valuations, particularly if firms engage in tract-specific seismic work after the auction is held. Second, leases in a specific area are generally sold at the same sale and, as a result, expire at the same time. Leases that are not sold, either because no bids were submitted or because the government rejected the high bid, may be offered in a subsequent sale. But this generally does not occur until several years later, and then only if oil and/or gas has been found on some of the leases in the initial offering.

The salient features of the post-auction drilling decisions on offshore leases can be captured in a model with two firms, labeled a and b, and two periods, indexed by $t = 1, 2$. In what follows, $a$ will refer to an arbitrary firm, and $b$ will refer to the other firm (i.e., $a \neq b$). Let $X$ be a random variable denoting the common, but random amount of oil on each of the tracts leased by the firms, where $X \in [0, \infty]$. At the beginning of period 1, each firm independently decides whether to explore its tracts in that period or wait until period 2. If $a$ drills and $b$ waits, then $b$ observes the outcome $x$ and makes a riskless investment decision in period 2. If neither firm explores in period 1, then, at the beginning of period 2, each firm once again independently decides whether or not to explore. If a firm does not explore its tract by the end of period 2, its lease expires.

Each firm $a$ is endowed with private information about $X$. We will represent this information by a signal, $s_a$, that takes a value in the unit interval $[0, 1]$. We assume that, given any realization of $X$, $(s_a, s_b)$ is distributed over the unit square according to a distribution function $F$ with continuous density function $f$. The marginal distribution of $X$ is assumed to be continuously differentiable everywhere except at $x = 0$. That is, there may be a positive probability that the tracts contain no oil.

We will need to impose some structure on $F$. The following restrictions will prove sufficient for our purposes.

**Assumption 1.** (a) $f(s_a, s_b | x) = g(s_a | x)g(s_b | x)$; (b) $g(s_a | x)$ possesses the strict monotone likelihood ratio property (MLRP).

Assumption 1(a) states that, given any $x$, the firms’ signals are independently and identically distributed with density function $g$. Thus, firms have different, but equally informative signals about the amount of oil in the area. Assumption 1(b) imposes several restrictions on the posterior distribution, $H(\cdot | s)$, which describes the beliefs of a firm about $x$ when it is endowed with signal $s$, and on the corresponding density function, $h(\cdot | s)$. These restrictions are identified in the following lemma.

**Lemma 1.** If $g(s | x)$ possesses the MLRP then, (i) for all $x > 0$, $h(x | s)$ possesses the MLRP, and (ii) $H(\cdot | s)$ stochastically dominates in the first-order sense $H(\cdot | s')$ for every $s > s'$.

The MLRP of $h(x | s)$ follows directly from Bayes’ rule and the definition of the MLRP. The proof of the stochastic dominance property is well-known (see Milgrom (1981)) and is not repeated here.

We consider a very simple model of returns. If the area contains oil, then the pools on the tracts leased by the firms are distinct, and their sizes are identical. Consequently, if firm $a$ drills first or simultaneously with the other firm in period 1, and if $X = x$, it will earn

$$L(x) = \Pi(x) - C,$$

where $\Pi(x)$ is the present value of production profits and $C$ is the cost of drilling the well. We shall assume that $\Pi$ is a nondecreasing, continuous function of $x$. If firm $a$ waits in period 1, and firm $b$ drills, then firm $a$ will drill in period 2 if its profits net of drilling costs are revealed to be positive; otherwise, it will let its lease expire. Therefore, it will earn

$$\delta W(x) = \delta \max [0, \Pi(x) - C],$$

where the discount rate $\delta$ satisfies $0 < \delta < 1$. If neither firm drills in period 1, and firm $a$
drills in period 2, it will earn $\delta L(x)$. Finally, if firm $\alpha$ does not drill in either period 1 or period 2, it will earn zero.

To decide whether and when to drill, each firm needs to compare the costs and benefits of waiting. The cost of waiting in our model arises from discounting, as expected profits are deferred. The main benefit from waiting arises from the possibility that the other firm will drill first, thereby revealing that there is not enough oil to make drilling a well a profitable investment. In these instances, the firm that waits will avoid the losses that it would have incurred had it drilled in period 1.

Notice that in our model the amount of oil which each firm can extract from its lease does not depend upon whether and when the other firm drills. This condition is not likely to hold if the leases are on a common pool. For example, suppose the range of $X$ includes an interval where the pool can profitably support at most one well. The firm which drills first in such a pool gets to extract all of the reserves, since the optimal response of the follower is not to drill. But if both firms drill simultaneously, drilling costs are sunk and, as a result, the firms must share the reserves. Thus, expected returns from drilling first are larger than the expected returns from drilling simultaneously with the other firm.

Incorporating the production externality into the model complicates the analysis considerably. The presumption of our model is that the information externality is relatively more important in deciding whether and when to drill an exploratory well. That is, the expected value of any first-mover advantages is small in comparison to the expected gain from following. This is not always true, but it is likely to be valid whenever the probability of finding commercial-size pools is low and the production lag, if such a pool is discovered, is short.

### 3. The drilling game

We shall initially assume that firms do not bargain before making their drilling decisions. The main justification for making this assumption is that the incidence of leases being sold by one firm to another, or of joint ventures being formed solely for exploratory purposes, appears to be relatively low. A possible explanation for this fact is the presence of asymmetric information, which greatly complicates the negotiation process.\(^2\) We shall return to this issue in a later section.

We define a period 1 strategy for firm $\alpha$ in the drilling game as a function $m_{\alpha_i} : [0, 1] \rightarrow \{0, 1\}$. Here $m_{\alpha}(s_{\alpha}) = 1$ means that firm $\alpha$ plans to explore its tracts in period 1 if its signal is $s_{\alpha}$, and $m_{\alpha}(s_{\alpha}) = 0$ means that firm $\alpha$ plans not to explore its tracts in period 1 if its signal is $s_{\alpha}$. Given any strategy $m_{\alpha}$, define $\Omega_{\alpha} = \{s_{\alpha} : m_{\alpha}(s_{\alpha}) = 1\}$ to be the set of signals that will induce firm $\alpha$ to drill in period 1.

The payoff to firm $\alpha$ with signal $s_{\alpha}$ from drilling in period 1 is independent of $m_{\beta}$ and is given by

$$P(1; s_{\alpha}) = \int_0^\infty L(x) dH(x | s_{\alpha}).$$

The payoff to firm $\alpha$ if it does not drill in period 1 depends upon $m_{\beta}$. If $s_{\beta} \in \Omega_{\beta}$, then $x$ will be revealed, and firm $\alpha$ can anticipate a return of $W(x)$ in period 2. Define

$$G(\Omega_{\beta} | x) = \int_{\Omega_{\beta}} g(s_{\beta} | x) ds_{\beta} = P(s_{\beta} \in \Omega_{\beta} | x).$$

The expected return from waiting, conditional on signal $s_{\alpha}$ and on the event that firm $\beta$ drills in period 1, is

$$\int_0^\infty \delta W(x) G(\Omega_{\beta} | x) dH(x | s_{\alpha}) / \int_0^\infty G(\Omega_{\beta} | x) dH(x | s_{\alpha}).$$

---

\(^2\) Wiggins and Libecap (1985) and Libecap and Wiggins (1985) provide evidence that imperfect information explains why oil firms frequently fail to unitize production on common pools. For theoretical discussions of the problems of bargaining under asymmetric information, see Samuelson (1985) and Farrell (1987).
If \( s_\beta \notin \Omega_\beta \), then firm \( \beta \) does not drill in period 1, and firm \( \alpha \) updates its beliefs about \( x \) accordingly. Given its revised beliefs, the optimal strategy for firm \( \alpha \) in period 2 is to drill if the expected returns from drilling are positive and to let the lease expire otherwise. Therefore, the expected return from waiting, conditional on signal \( s_\alpha \) and on the event that firm \( \beta \) does not drill in period 1, is

\[
\delta \max \left[ 0, \int_0^\infty L(x)(1 - G(\Omega_\beta | x))dH(x|s_\alpha) / \int_0^\infty (1 - G(\Omega_\beta | x))dH(x|s_\alpha) \right].
\]

Putting the above two expressions together yields the payoff to firm \( \alpha \) from waiting in period 1 and following its optimal plan in period 2:

\[
P_\alpha(0, m_\beta, s_\alpha) = \delta \int_0^\infty W(x)G(\Omega_\beta | x)dH(x|s_\alpha) + \max \left[ 0, \delta \int_0^\infty L(x)(1 - G(\Omega_\beta | x))dH(x|s_\alpha) \right].
\] (4)

We will impose the following assumptions on the payoff function. They are designed to eliminate the uninteresting cases.

Assumption 2. (a) \( \int_0^\infty L(x)dH(x|s_\alpha = 0) < 0 \); (b) \( \int_0^\infty L(x)dH(x|s_\alpha = 1) > 0 \).

Assumption 2(a) states that if firm \( \alpha \) receives the worst possible signal, its expected return from drilling conditional on this information is negative. Similarly, Assumption 2(b) states that if firm \( \alpha \) receives the best possible signal, its expected return from drilling conditional on this information is positive.

We define a Bayesian Nash equilibrium for the game as a pair of strategies \((m^*_\alpha, m^*_\beta)\) such that for all strategies \( m_\alpha \), \( P_\alpha(m_\alpha(s_\alpha), m^*_\beta, s_\alpha) \geq P_\alpha(m_\alpha(s_\alpha), m^*_\beta, s_\alpha) \) for every \( s_\alpha \in [0, 1] \), \( \alpha = a, b \), \( \alpha \neq \beta \).

4. Characterization of equilibria

In this section we characterize the set of Bayesian Nash equilibria. We show first that, given any strategy by firm \( \beta \), firm \( \alpha \)'s best response is to partition the unit interval into two subintervals. Thus, there exists a reservation signal such that the firm drills in period 1 if its signal is greater than this signal and waits otherwise. This property allows us to analyze equilibrium behavior in terms of reaction functions in which one firm’s reservation signal is a function of that of the other firm. We then prove that a symmetric Bayesian Nash equilibrium always exists.\(^3\)

Proposition 1. If \( m_\alpha \) is a best response to \( m_\beta \), there exists a unique signal, \( \hat{s}_\alpha(m_\beta) \), such that

\[
m_\alpha(s_\alpha; m_\beta) = \begin{cases} 
1 & \text{if } s_\alpha > \hat{s}_\alpha(m_\beta) \\
0 & \text{if } s_\alpha \leq \hat{s}_\alpha(m_\beta).
\end{cases}
\]

Define the difference between the expected payoff to firm \( \alpha \) from drilling in period 1 and its expected payoff from waiting in period 1 and making an optimal drilling decision in period 2 as

\[
\Phi(s_\alpha, m_\beta) = \int_0^\infty L(x)dH(x|s_\alpha) - \delta \int_0^\infty W(x)G(\Omega_\beta | x)dH(x|s_\alpha) - \delta \max \left[ 0, \int_0^\infty L(x)(1 - G(\Omega_\beta | x))dH(x|s_\alpha) \right].
\]

\(^3\) Throughout the article, proofs of the propositions are given in the Appendix.
To establish the proposition, we need to show that the solution to the equation \( \Phi(s, m_\beta) = 0 \), assuming one exists, is unique. (If there is no solution, then \( \hat{s}_\alpha \) is set equal to one, since, given Assumptions 2(a) and 2(b), this implies \( \Phi(s_\alpha, m_\beta) \) is negative for all \( s_\alpha \).) The primary difficulty in doing so is that \( \Phi(s_\alpha, m_\beta) \) is not monotone in \( s_\alpha \). An increase in \( s_\alpha \) has two effects: it increases firm \( \alpha \)'s estimate of the value of its lease, and it increases firm \( \alpha \)'s estimate of the probability that firm \( \beta \) will explore in period 1. The latter effect can dominate, thereby causing \( \Phi(s_\alpha, m_\beta) \) to fall.

The proof relies upon the following result, which is of some independent interest.

**Lemma 2.** Let \( f(x) \) be any real-valued function with the property that there exists a value \( x^* \) such that (i) \( f(x^*) = 0 \), and (ii) \( f(x) < 0 \) for \( x < x^* \) and \( f(x) > 0 \) for \( x > x^* \). If \( f(x|s) \) is a density function which has the MLRP, then

\[
\int_{s'}^{\infty} f(x|s) \mu(x|s') dx > 0 \quad \text{for all} \quad s > s',
\]

where \( s' \) solves \( \int_{s'}^{\infty} f(x|s') \mu(x|s') dx = 0 \).

**Proof.** See the Appendix.

From Lemma 1, \( h(x|s) \) has the monotone likelihood ratio property. Furthermore, the function which gives the difference in the expected returns to leading and following conditional on \( x \) satisfies the single-crossing property of Lemma 2. The critical assumption is that \( L(x) \) is at least as large as \( W(x) \) whenever \( L(x) \) is positive. This is satisfied in our model since \( W(x) = \max \{0, L(x)\} \). Proposition 1 then follows.

Given Proposition 1, we can restrict the strategy space of each player to the set of functions with the reservation property without loss of generality. We shall refer to the function \( R(s) = \hat{s}_\alpha \) as firm \( \alpha \)'s reaction function. Notice that the reaction functions of the two firms are identical, since the return functions and the distributions from which they are derived are the same for both firms. Proposition 2 gives some useful properties of the reaction function.

**Proposition 2.** The reaction function has the following properties: (i) \( 0 < R(1) < 1 \), and (ii) \( R(\cdot) \) is continuous and nonincreasing on \([0, 1]\), and differentiable and decreasing when \( R(\cdot) < 1 \).

The set of Bayesian Nash equilibria can be characterized in terms of intersections of the firms' reaction functions. The following existence result is obtained.

**Proposition 3.** A unique, symmetric Bayesian Nash equilibrium always exists.

The argument is illustrated in Figure 1. If it is certain that firm \( \beta \) will not drill in period 1, then firm \( \alpha \)'s best response is to drill if its signal is greater than \( R(1) \), and to abandon its lease otherwise. Since \( R(1) \) is less than one, and \( R(\hat{s}_\beta) \) is a continuous, decreasing function of \( \hat{s}_\beta \) on the interior of the unit square, there exists a unique \( s^* \) such that \( R(s^*) = s^* \). It then follows from the symmetry of the reaction functions that \( s^* \) is a point of intersection of the two reaction functions.

The nondecreasing property of the reaction functions is not sufficient to rule out intersection points off the diagonal, either in the interior or on the boundary of the unit square. Consequently, asymmetric equilibria may also exist. Figure 1 illustrates a case in which there are three equilibria: the symmetric point \((s^*, s^*)\), and the boundary points, \((r, 1)\) and \((1, r)\). The boundary points represent equilibria in which one of the firms always explores in period 1 if its signal is greater than \( r \) and lets its lease expire otherwise, while the other firm always waits until period 2 to make its drilling decision. These equilibria tend to exist whenever the cost of waiting for the firm possessing the most favorable signal is small relative to the value of the information externality. This does not appear to be an unusual circumstance in oil exploration.
The symmetric equilibrium may appear to be a natural focal point given the symmetry of the model. Notice that coordination mistakes are possible in this equilibrium. Both firms may drill in period 1 and incur duplication costs, or both firms may wait in period 1 and incur needless delay costs. The boundary equilibria, however, are not uninteresting, particularly if observable asymmetries in payoff relevant variables, such as the number of leases or their purchase prices, are present. Even if such asymmetries are not present, certain norms or conventions may exist which permit the firms to coordinate their drilling decisions. This phenomenon has been observed, for example, in animal conflicts over territory, mates, or food. (See Maynard Smith and Parker (1976) for examples.)

Which type of equilibrium appears to describe actual drilling behavior on offshore leases? The length of a period corresponds to the amount of time between the date of the drilling decision and the date that the drilling outcome becomes public knowledge. This is typically several months to a year. Since firms have five years to drill, this suggests that if firms are coordinating, most leases that are explored will be drilled in the early years of the leases. By contrast, if firms are not coordinating, they have an incentive to delay drilling, and the pattern of drilling is likely to be more evenly distributed across the different lease years.

The evidence given in Hendricks et al. (1987) appears to be more consistent with an equilibrium in which firms do not coordinate. They report that, in a sample of 1,056 wildcat tracts, 234 were drilled in the first year of the lease, 138 in the second year, and approximately 100 tracts in each of the remaining three years. The relatively large number drilled in the first year is due to the fact that many of these leases were not marginal tracts. That is, firms intended to drill these leases regardless of what type of information may have been forthcoming on nearby tracts. With respect to the marginal tracts, the numbers imply that firms frequently delayed their drilling decisions.

One might argue that the above pattern reflects sequential drilling by firms which purchased several leases in a given sale and area. However, this turns out not to be true. In Table VI of their article, Hendricks et al. (1987) document the frequency ratios of tracts drilled in each lease year by firms which purchased \( m \) leases in a particular sale and area.
The pattern for firms with only one or two leases is roughly the same as the pattern given above. Thus, these firms frequently delayed drilling their tracts until the end of the lease tenure, even though no new information could have been forthcoming from their own drilling activity.

5. The efficiency results

We shall study the efficiency properties of the symmetric equilibrium by comparing the outcomes of this decision rule to those of the decision rule which would be implemented by a social planner (or a single owner of both leases) if he had access to the seismic information of both firms. In the language of mechanism design (see Holmstrom and Myerson (1983)), the latter decision rule is ex post efficient. Note that for this comparison to be of any practical importance, one has to assume that the planner can perfectly verify the information that the firms possess. In the next section we cite evidence which gives some support for this assumption. Also, we shall ignore the question of how much investment in information is socially optimal by assuming that the planner would invest the same amount as the firms.

Throughout this section, the following restriction on the return functions will be imposed:

Assumption 3. \( \int_0^\infty L(x) dH(x|s_a, s_b) < \delta \int_0^\infty W(x) dH(x|s_a, s_b) \) for all \((s_a, s_b) \in [0, 1] \times [0, 1]\).

Assumption 3 states that the value of the information externality is positive in every state of information. It implies that even when a firm is endowed with the most favorable signal, it prefers to wait rather than drill simultaneously with the other firm in period 1.

The partition which the symmetric Bayesian Nash equilibrium induces on the unit square is completely characterized by two reservation signals, one for each period. Let \( s' \) be the reservation signal in period 1. That is, if \( s_\alpha \) is larger than \( s' \), then firm \( \alpha \) drills in period 1, and if \( s_\alpha \) is less than \( s' \), it waits. Let \( s^2 \) denote the reservation signal for period 2.

Proposition 4. \( s' \) exceeds \( s^2 \).

The intuition behind Proposition 4 is as follows. Suppose firm \( \beta \) observes signal \( s^2 \). If it knew that firm \( \alpha \) would drill in period 1, then it would strictly prefer to wait (by Assumption 3). If it knew that firm \( \alpha \) did not intend to drill in period 1 (i.e., \( s_\alpha < s' \)), then by construction firm \( \beta \) would be indifferent about whether and when it drills. Since it is not certain which action firm \( \alpha \) will take in period 1, firm \( \beta \) strictly prefers to wait when its signal is \( s^2 \). Hence, \( s^2 \) is inframarginal, that is, \( s^2 < s' \).

Figure 2 illustrates the set of allocations generated by the symmetric equilibrium. There are seven regions to consider. The specific pattern of drilling activity in each region is given by the quadruple \( \{I_{a,t}, \alpha = a, b, t = 1, 2\} \). Here \( I_{a,t} = 1 \) if firm \( \alpha \) drills in period \( t \), and \( I_{a,t} = 0 \) if firm \( \alpha \) does not drill in period \( t \). If firm \( \beta \) drills in period 1 and firm \( \alpha \) does not, then \( I_{a,t} \) is set equal to \( p \), where \( p > 0 \), to indicate that firm \( \alpha \) drills in period 2 only if the returns from doing so are positive.

What is the set of allocations that we would obtain if a social planner could costlessly verify the state of information (or alternatively, if one firm owned both leases)? Recall that, given Assumption 3, it is never optimal to drill the tracts simultaneously. Let

\[
V(s_a, s_b) = \int_0^\infty [L(x) + \delta W(x)] dH(x|s_a, s_b)
\]

be the expected present value of the sequential drilling program. Then, if the firm knows \((s_a, s_b)\), it should let both leases expire if \( V(s_a, s_b) \) is negative, and adopt the sequential drilling program if \( V(s_a, s_b) \) is nonnegative.
The allocations that result from this decision rule are depicted in Figure 2. The symmetric, downward-sloping curve $VV$ represents the set of information states that solve the equation $V(s_a, s_b) = 0$. We will refer to it as the efficiency frontier. The curve $TT$ represents the locus of points that satisfy the equation $\int_0^{\infty} L(x) dH(x|s_a, s_b) = 0$. The properties of the two curves follow directly from Lemma 1. For any information state in the region above $VV$, the expected returns from the sequential drilling program are positive. If it also lies above the curve $TT$, then the expected value of a single lease is positive. For any information state in the region below $VV$, the expected returns from drilling are negative.

To compare the two decision rules, the locations of the curves $VV$ and $TT$ relative to the reservation signals $s^1$ and $s^2$ need to be determined. Let $(\bar{s}, \bar{s})$ and $(\underline{s}, \underline{s})$ denote the intersection of $VV$ with the diagonal, and the intersection of $TT$ with the diagonal, respectively.

Proposition 5. $\bar{s}$ is less than $s^1$.

Somewhat surprisingly, a similar inequality does not hold between $\bar{s}$ and $s^2$. Furthermore, without imposing additional restrictions on the distributions, no statement can be made about the relationship between $\underline{s}$ and the two reservation signals. We shall show later that the “standard” case is the one depicted in Figure 2, where $\bar{s}$ and $\underline{s}$ are less than $s^2$. The set of states in which there is too much drilling on average is marked by hatched lines slanted left. The set of states in which there is too little drilling on average is marked by hatched lines slanted right.

There are two distinct reasons why firms may overinvest in wells. The first is that firms cannot coordinate their drilling decisions. As a result, they may end up drilling their tracts simultaneously instead of sequentially. This situation is likely to occur if firms are endowed with similar signals, and these signals exceed $s^2$. The second is that firms are not fully informed about the state of information. Therefore, if one firm has a very good signal, and the other firm has a very poor and dominating signal (i.e., if $(s_a, s_b)$ lies in the region below the efficiency frontier, but one of the signals is larger than $s^2$), then at least one tract is drilled, even though efficiency requires that none be explored.

**FIGURE 2**

**Symmetric Nash Equilibrium**
Situations in which firms underinvest are also possible. They occur when one of the firms is endowed with a signal less than $s^1$ and the other firm has a signal less than $s^2$. One type of underinvestment occurs when the firm with the higher signal drills in period 2, and the outcome is favorable. Then the other firm should drill its tract in the following period, but it cannot do so since its lease expires at the end of period 2. The other type of underinvestment occurs when the signals of both firms are less than $s^2$, but the state of information lies above the efficiency frontier. In these states, no drilling occurs even though efficiency requires that at least one well should be drilled.

The first type of underinvestment is likely to be less common than the second. The government can always resell a tract whose lease has expired if the outcome on the neighboring tract is favorable. The real cost in this case is the delay in the realization of returns rather than underinvestment in wells. By contrast, if the government tried to resell a lease after both leases had expired, neither firm would be willing to pay a positive price for the tracts. In fact, even if the government tried to resell the tracts as a single unit, neither firm may be willing to pay a positive price for the lease. Each firm will evaluate the tracts conditional on its own information and on the event that the other firm’s signal is less than $s^1$. This can cause firms to become overly pessimistic relative to the actual state of information.

Notice that this situation can arise even though the private value of a lease is actually positive. Suppose that each firm obtains a signal which lies in the interval $[s, s^2]$, so that the expected return to drilling a lease conditional on $(s_1, s_2)$ is positive. The equilibrium outcome is that neither firm drills its lease. Each firm waits in period 1, since $s_1$ is less than $s^1$. Then, upon reaching period 2, each firm infers from the other firm’s inactivity in the previous period that the other firm’s signal is less than $s^1$. This lowers expectations sufficiently such that each firm concludes that its tract is not worth exploring.

The two-period structure of our model probably overstates the problem of duplication (i.e., simultaneous drilling) and understates the problem of delay. The period length in exploration is significant, but it is clearly less than 2½ years. If we added more periods to the model, holding the time horizon constant, the qualitative features of the symmetric equilibrium would remain much the same. However, the interpretation of the losses from coordination failure would change: the costs of delay would become relatively more important than the costs of duplication.

Before concluding this section, we briefly discuss the efficiency results for boundary equilibria. It is easily shown that the first-period reservation signal in a boundary equilibrium is less than $s^2$ and that the second-period reservation signal is greater than $s^2$. This leads to the conclusion that both overdrilling and underdrilling outcomes are possible in boundary equilibria. In comparison to the symmetric equilibrium, the lead firm is more likely to drill in the boundary equilibrium, and the follower firm is less likely to drill given that the lead firm chose not to drill. Furthermore, the probability that both firms drill simultaneously, either in period 1 or period 2, is zero in the boundary equilibrium. However, since the partition induced by the boundary equilibrium is neither a refinement nor a coarsening of the partition induced by the symmetric equilibrium, it is not possible to rank the probabilities of overdrilling or underdrilling in the two types of equilibria. The results will typically depend upon the properties of the underlying probability law.

A special case. Do the intersections of $VV$ and $TT$ with the diagonal usually lie above or below the point $(s^2, s^2)$? This is an important question, since, if they lie above, then underinvestment never occurs. To study this issue, we considered the following special case of our model.

Suppose that the signals provide firms with information about whether or not the tracts contain oil, but give no information about the size of the pool. That is,

---

4 However, the existence of asymmetric equilibria appears to become less likely as more periods are added.
5 See Bolton and Farrell (1988) for a more detailed discussion of this issue.
\[ g(s|x) = g_w(s) \text{ for all } x > 0, \text{ and } g(s|0) = g_d(s), \]
where subscripts \( w \) and \( d \) stand for wet and dry, respectively. We shall assume that \( g_d(s)/g_w(s) \) is decreasing in \( s \) (that is, \( g(s|x) \) satisfies the weak monotone likelihood ratio property). Note that this implies that \( G_w \) first-order stochastically dominates \( G_d \). Let \( q \) be the prior probability that the tracts contain no oil, and let \( q(s) \) be the probability of this event conditional on signal \( s \). If the tracts contain oil, then the prior distribution of \( x \) is given by \( K(x) \) with density \( k(x) \). The posterior distribution of \( x \) given signal \( s \) is
\[
H(0|s) = q(s) = q g_d(s) / [q g_d(s) + (1 - q) g_w(s)] \quad \text{if } x = 0 \\
h(x|s) = (1 - q(s)) k(x) \quad \text{otherwise.}
\]
Let \( \tilde{H} \) denote the expected present value of production profits conditional on the event that the tracts contain oil.

In this restricted model, the relationship between \( s \) and \( s^2 \) depends solely upon the firms' common belief about \( x \) before they observe the private signals.

**Proposition 6.** Suppose \((1 - q) / (\tilde{H} - C) - qC < 0. \) Then \( \tilde{s} \) is less than \( s^2 \).

Proposition 6 states that, if expected returns from drilling a single tract prior to obtaining the signal are negative, then the intersection of \( TT \) with the diagonal lies below \( s^2, s^2 \). (Notice that, since \( VV \) lies everywhere below \( TT \), the point \( (\tilde{s}, \tilde{s}) \) also lies below \( s^2, s^2 \).)

The intuition is as follows. If the firm is initially pessimistic concerning the profitability of drilling its tract, it must obtain a good signal to be convinced that the tract is worth drilling. If informed that the other firm has also obtained a good signal, less weight is given to the pessimistic prior, and expectations rise. The opposite is true if prior beliefs are overly optimistic.

We may conclude from Proposition 6 that underinvestment is empirically an important phenomenon. Most tracts do not contain any oil. Therefore, prior expectations about the probability of finding oil are usually pessimistic.

### 6. The persuasion game

Why do firms not negotiate agreements that realize the efficiency gains from trade? How does the incentive to misrepresent information complicate the negotiation process? These are subtle questions. To obtain some insights into the issues involved, we examine a bargaining model in which there is no ambiguity about which firm has the "right" to drill second in the event of disagreement. One circumstance in which this can occur is when one firm's lease expires at least one drilling period before the lease of the other firm.

If the property right to the information externality is well-defined, the incentives for bargaining are relatively straightforward, and there is a natural assignment of the roles of buyer and seller. The firm with the right to drill second wants to buy the other firm's lease as cheaply as possible, and, if its offer is refused, to persuade its rival to drill first. The firm which is at a disadvantage in the drilling game wants to learn the other firm's information so that it can demand a fair price for its lease or, failing that, make a more informed drilling decision.

More precisely, we consider the following extensive form game. Firm \( \alpha \) is assumed to have a longer lease. At the beginning of the last period in which the tract leased by firm \( \beta \) can still be drilled, firm \( \alpha \) makes an all-or-nothing offer to purchase the lease from firm \( \beta \). Along with the offer, it includes a report of its private information. Firm \( \beta \) can accept or reject the offer. If it accepts the offer, the appropriate payment is made, firm \( \beta \) reveals its information to firm \( \alpha \), and firm \( \alpha \) carries out the optimal drilling program. If firm \( \beta \) rejects the offer, bargaining ends. Firm \( \beta \) then makes a drilling decision, and firm \( \alpha \) responds optimally. If firm \( \beta \) chooses not to drill, its lease expires.

Communication between firms is modelled in the same way as in Milgrom (1981). A
report is a statement by a firm that its signal lies in a closed, nonempty subset, \( S \), of the unit interval. It must satisfy the restriction that the firm’s signal is an element of \( S \). Thus, the only way in which a firm can pretend to have signal \( s' \) when it has signal \( s \) is to report a closed set which includes both \( s \) and \( s' \). This constraint allows firm \( \alpha \) to withhold information and to bias firm \( \beta \)'s beliefs away from the truth, but it does not permit the firm to misreport its information.

We believe that this model of communication captures some essential features of exploration information. A substantial amount of the variability in firm valuations appears to be attributable to differences in methods of data analysis and experience rather than differences in the data themselves. A firm cannot be forced to disclose its interpretation of its data and the logic behind it, but must be given incentives to do so. At the same time, however, it cannot pretend to have information which it does not possess. The interpretations must be supported by the data and be consistent with geological theory.

The following anecdote from Mobil’s brief to the Subcommittee on Monopolies and Commercial Law in 1976 illustrates that information is, at least in part, verifiable if the firm chooses to reveal it. In this quote, Mobil is describing the process by which a group of firms determined the joint bid which they intended to submit on a tract.

Once agreement has been reached on tracts of mutual interest, bid levels are established on individual tracts. Each party suggests the highest amount it is willing to bid. The highest suggested bid becomes the group bid on that particular tract. At this point companies sometimes drop out of the group if they cannot support the highest suggested bid, and no further discussion of bid levels on that particular tract is held in the presence of the non-participating parties. Frequently, in attempting to reach consensus on a bid level, a party may disclose technical information it possesses in support of a higher value.

Additional anecdotal evidence on the verifiability of exploration information can be found in an article by A. Stuart which appeared in *Fortune* (1979). The subject of that article is bidding for oil leases in the Baltimore Canyon.

We proceed to a formal description of the game. A strategy for firm \( \alpha \), the buyer, consists of an offer function \( J: [0, 1] \to \mathbb{R}_+ \) and a reporting function \( Q: [0, 1] \to \mathcal{Z} \), where \( \mathcal{Z} \) denotes the set of all closed, nonempty subsets of the unit interval. A strategy of nondisclosure is one in which \( Q(s) = \{ [0, 1] \} \) and \( J(s) = p \) for all \( s \in [0, 1] \). A strategy \( (Q, J) \) which maps every signal in the unit interval into a distinct subset of \( \mathcal{Z} \times \mathbb{R}_+ \) is called a strategy of full disclosure. Firm \( \alpha \)'s beliefs about firm \( \beta \) conditional on observing signal \( s_a \) are given by \( F(\cdot | s_a) \). This distribution is obtained in a straightforward way from the underlying probability law.

A strategy for firm \( \beta \) is a function \( D: [0, 1] \times \mathcal{Z} \times \mathbb{R}_+ \to \{ 0, 1 \} \). Here \( D(s_\beta, S, p) = 0 \) means that firm \( \beta \) accepts offer \( p \) conditional on report \( S \) and signal \( s_\beta \), and \( D(s_\beta, S, p) = 1 \) means that firm \( \beta \) rejects the offer. Prior to receiving firm \( \alpha \)'s report and offer, but after it has obtained \( s_\beta \), firm \( \beta \)'s beliefs about firm \( \alpha \)'s type are described by \( F(\cdot | s_\beta) \). After observing \( (S, p) \), firm \( \beta \) updates its beliefs, using Bayes’ rule if applicable, and these beliefs will be denoted \( B(\cdot | S, p; s_\beta) \).

The equilibrium concept that we shall use is sequential equilibrium. This can be defined as follows:

**Definition.** A sequential equilibrium is a set of functions \( \{ Q, J, D, B \} \) which satisfies the following conditions:

(i) For every \( s_\beta \in [0, 1] \), and for every possible pair \((S, p)\),

\[
D(s_\beta, S, p) = \begin{cases} 
0 & \text{if } p \geq \int_0^1 E[L(x)|s_a, s_\beta]dB(s_a|S, p; s_\beta) \\
1 & \text{if } p < \int_0^1 E[L(x)|s_a, s_\beta]dB(s_a|S, p; s_\beta).
\end{cases}
\]
(ii) For every \( s_\alpha \in [0, 1] \), \( \{ Q(s_\alpha), J(s_\alpha) \} \) maximizes

\[
P(p; S; s_\alpha) = \int_{\{ D(s_\beta, s_\rho) = 0 \}} \left( \max \left[ 0, E[ L(x) + \delta W(x) | s_\alpha, s_\beta] \right] - p \right) dF(s_\beta | s_\alpha) + \int_{\{ D(s_\beta, s_\rho) = 1 \}} E[ \delta W(x) | s_\alpha, s_\rho] dF(s_\rho | s_\alpha)
\]

subject to \( S \) being closed and nonempty, and \( s_\alpha \in S \).

(iii) For every \( s_\beta \in [0, 1] \), and for every possible pair \( (S, p) \), \( B(S|S, p; s_\beta) = 1 \). If \( (S, p) \) is in the range of \( \{ Q, J \} \), then given any \( S' \subset [0, 1] \),

\[
B(S'|S, p; s_\beta) = \frac{\int_{S' \cap C(S, p)} dF(s_\alpha | s_\beta)}{\int_{C(S, p)} dF(s_\alpha | s_\beta)},
\]

where \( C(S, p) = \{ s_\alpha \in [0, 1] | Q(s_\alpha) = S, J(s_\alpha) = p \} \).

Condition (i) implies that each firm \( \beta \) always chooses its action to maximize its payoff, given its beliefs about firm \( \alpha \). This optimality criterion is required to hold for every (report, offer) pair, not just for those in the range of \( \{ Q, J \} \). Since \( p \) is nonnegative, (i) also implies that firm \( \beta \) drills its lease if and only if it rejects the offer by firm \( \alpha \). Condition (ii) is the analogous best-response condition for firm \( \alpha \). It assumes that firm \( \alpha \) responds optimally to any rejection of its offer. Condition (iii) imposes the rational expectations restrictions that firm believes that firm \( \alpha \)'s type is an element of its report, and that, along the equilibrium path, beliefs are consistent with the underlying probability law and the strategy used by firm \( \alpha \). Beliefs off the equilibrium path are not well-defined by Bayes’ rule, so there is some latitude in specifying these beliefs. Note, however, that not every specification is consistent with the definition of sequential equilibrium. (See Kreps and Wilson (1982) for details.)

It is beyond the scope of this article to give a complete characterization of the set of sequential equilibria in the bargaining game outlined above. We can, however, make the following statement:

**Proposition 7.** There is no sequential equilibrium in which firm \( \alpha \) uses a strategy of full disclosure.

The proof of this result proceeds in two steps. We first show that if firm \( \alpha \) uses a strategy of full disclosure, then the optimal offer price is zero. This follows from the adverse selection problem that firm \( \alpha \) faces in trying to buy from an informed seller. We then show that, given the zero offer, a full disclosure strategy minimizes the payoff to every firm \( \alpha \) type. A high firm \( \alpha \) type would like to be pooled with lower types, since this causes firm \( \beta \) to become overly pessimistic and induces some firm \( \beta \) types to accept an offer which is less than the actual value of the lease. Similarly, a low firm \( \alpha \) type likes to be pooled with higher types, since this causes firm \( \beta \) to become overly optimistic and causes some firm \( \beta \) types to drill when the actual value of the lease is negative. Thus, any confusion on firm \( \beta \)'s part about the value of firm \( \alpha \)'s signal increases the payoff to every firm \( \alpha \) type. This makes it impossible to specify off-equilibrium-path beliefs that support the full disclosure strategy as an equilibrium and satisfy (iii). The verifiability restriction plays a crucial role in this proof.

It is interesting to contrast our no-full-disclosure result with Milgrom's (1981) result that an informed seller of a commodity of unknown quality fully discloses any private information to an uninformed buyer. The key difference between the models is the lack of unanimity among the reporting firm types about which action by the decision maker is

---

6 This distinguishes a sequential equilibrium from a Bayesian Nash equilibrium, since the latter requires optimality only for (report, offer) pairs in the range of \( \{ Q, J \} \).
best. Low firm α types want firm β to drill, but high firm α types want firm β to accept the offer. If leases are nontransferable, however, then we obtain unanimity since every firm α type wants firm β to drill. As a result, each firm α type wants to differentiate itself from lower types since, in so doing, it increases the probability that firm β drills. Milgrom and Roberts (1986) have shown that this unanimity condition implies that, in any sequential equilibrium, the strategy of the reporting firm is one of full disclosure.

Proposition 7 establishes that every sequential equilibrium in the transferable case involves some amount of pooling. It is natural, therefore, to ask whether a strategy of nondisclosure is a sequential equilibrium. The following proposition establishes that this is indeed the case.

Proposition 8. There exists a sequential equilibrium in which firm α uses a strategy of nondisclosure and offers to buy firm β’s lease at a price of zero.7

Propositions 7 and 8 indicate that, even with verifiable disclosure of private information, bargaining with well-defined property rights does not eliminate all of the inefficiencies resulting from the decentralization of drilling decisions. If leases are nontransferable, firm α is forced to fully disclose its type, so overinvestment cannot occur. Underinvestment remains a problem, however, since unitization of ownership is not permitted. Making the leases transferable eliminates the problem of underinvestment, since firm α’s offer is only rejected in instances where firm β plans to drill. But then overinvestment may occur, since firm β may not be fully informed about the state of information when it makes its drilling decision. Consequently, firm β may reject firm α’s offer and drill its lease even though the socially efficient decision is to let both leases expire.

Since the government permits firms to sell their leases, the prediction of this bargaining model is that transfers or joint ventures should occur whenever firm β has no intention of drilling its lease. In practice, this does not appear to happen very often. One explanation is that the survey information in question is, in part, firm-specific. A firm will be reluctant to reveal information about its evaluation techniques if this gives its competitors an advantage in bidding for leases in other areas. Another explanation is that the right to drill second is usually not allocated, since in most instances, leases in the same area expire at the same time. Our conjecture is that firms are less likely to negotiate an agreement if each firm believes there may be some chance it can earn the value of the information externality in the event of disagreement.

7 We assume that firm β accepts any offer which is not less than the expected value of its lease conditional on its signal and the report made by firm α. If we assumed strict inequality, then there is no sequential equilibrium and we would have to use an epsilon-equilibrium concept. In any case, the zero price should be interpreted as a negligible offer, reflecting the fact that the opportunity cost of the tract to firm β if it intends to let the lease expire is essentially zero.

7. Some concluding remarks

The main results of this article are that the decentralization of drilling decisions on public lands causes firms to underinvest in areas which they believe are at best marginally profitable and to overinvest in areas which they believe are profitable. Furthermore, these inefficiencies are not likely to be resolved through bargaining or a resale market due to the presence of private geological information. The costs associated with these inefficiencies lower the valuations of firms, which, in turn, lower their bids, thereby lowering the revenues collected by the federal government.

What can the government do to encourage more efficient drilling decisions? The presumption here is that the private sector possesses the requisite human and physical capital for the discovery and production of oil and gas. One policy that could be pursued is to
increase the size of the lease. The major drawback of this policy, however, is that the government's share of the resource rents may fall, since the associated rise in capital requirements is likely to reduce the number of firms that are capable of competing for the lease, at least on tracts that are being sold for the first time. Another possibility is to limit the amount of nonunitized acreage which a firm can possess, thereby encouraging unitization of exploration programs. The latter policy is one which the federal government has adopted on onshore lands with some success. (See Libecap and Wiggins (1985).)

Different lease tenures should reduce the costs of coordination mistakes. The firm with the shorter lease is at a disadvantage, and recognizing this, it is more likely to drill immediately or, alternatively, to agree to a sale of its lease to the other firm. Public disclosure laws that require firms to file their seismic reports may also enhance efficiency on average. In the absence of unitization, the effects of such laws, however, are not unambiguous. There are circumstances in which, based upon its own survey, a firm would drill its tract, but if informed about the other firm's survey, would decide not to drill. Since social returns are larger than the private returns, this change in action can increase the costs of inefficiency rather than reduce them.

Appendix

The proofs of Lemma 2 and of Propositions 1, 2, 3, 4, 5, 6, 7, and 8 follow.

Proof of Lemma 2. Let \( \rho = \mu(\hat{x}|s')/\mu(\hat{x}|s) \), and recall that \( s' \) solves the equation \( \int_0^\infty \phi(x)\mu(x|s)dx = 0 \). Then,

\[
\rho \int_0^\infty \phi(x)\mu(x|s)dx = \rho \int_0^\infty \phi(x)\mu(x|s)dx - \int_0^\infty \phi(x)\mu(x|s')dx
\]

\[
= \int_0^\infty \phi(x)[\rho \mu(x|s) - \mu(x|s')]dx
\]

\[
= \int_0^\infty \phi(x)[\rho - (\mu(x|s')/\mu(x|s))]\mu(x|s)dx.
\]

The definition of the monotone likelihood ratio property (MLRP) states that, given any \( s > s' \),

\[
\rho \geq \mu(x|s')/\mu(x|s) \quad \text{as} \quad x \gtrless \hat{x}.
\]

This result, together with the fact that \( \phi(x) \) is positive for \( x > \hat{x} \), and negative for \( x < \hat{x} \), implies that \( \phi(x)(\rho - (\mu(x|s')/\mu(x|s))) > 0 \) for every \( x \neq \hat{x} \). Therefore, since \( \rho > 0 \), \( \int_0^\infty \phi(x)\mu(x|s) > 0 \) for all \( s > s' \). Q.E.D.

Proof of Proposition 1. Define

\[
\phi_1(x) = L(x) - \delta W(x)G(\Omega_\theta|x), \quad \text{and} \quad \phi_2(x) = L(x) - \delta W(x)G(\Omega_\theta|x) - \delta L(x)(1 - G(\Omega_\theta|x)).
\]

After some manipulation, \( \Phi \) can be expressed as

\[
\Phi(s_\alpha, m_\alpha) = \min \left( \int_0^\infty \phi_1(x)dH(x|s_\alpha), \int_0^\infty \phi_2(x)dH(x|s_\alpha) \right).
\]

Note that a sufficient condition for \( \phi_1 \) to satisfy the condition of Lemma 2 is that \( L(x) \geq W(x) \) for all \( x \) such that \( L(x) > 0 \). This condition holds in our model, since \( W(x) = \max \{0, L(x)\} \).

Now consider any \( s_\alpha > z \), where \( z \) is a solution to the equation \( \Phi(s, m_\theta) = 0 \), assuming one exists. Without loss of generality, assume that \( E[\phi_1(x)|s_\alpha] > E[\phi_2(x)|s_\alpha] \). Then,

\[
\Phi(s_\alpha, m_\theta) = \int_0^\infty \phi_2(x)dH(x|s_\alpha) = \int_0^\infty \phi_2(x)h(x|s_\alpha)dx + \phi_2(0)H(0|s_\alpha)
\]

\[
> \int_0^\infty \phi_2(x)h(x|z)dx + \phi_2(0)H(0|z) \geq \Phi(z, m_\theta) = 0.
\]

The strict inequality follows from Lemma 2 and from the fact that \( H(0|s_\alpha) < H(0|z) \) by the first-order stochastic dominance property of \( H \). Therefore, if \( z \) exists, it is unique, and the optimal response of firm \( \alpha \) to \( m_\theta \) is to drill in period 1 if \( s_\alpha > z \) and to wait if \( s_\alpha < z \). In this case, \( \Phi_d(m_\theta) = z \).
If $z$ does not exist, then $\Phi(s_n, m_\beta)$ is negative for all $s_n \in [0, 1]$, since, by Assumption 2(a), $\Phi(0, m_\beta) < 0$. In this case, the optimal response of firm $\alpha$ is always to wait in period 1, and $\Phi(s_n, m_\beta) = 1$. Q.E.D.

**Proof of Proposition 2.** Suppose $\delta_\beta = 1$, so that the probability of firm $\beta$ drilling in period 1 is zero. Then, firm $\alpha$ gains nothing by waiting until period 2 to drill its lease. Its optimal response is to explore in period 1 if the expected return from doing so is positive, and not to explore otherwise. It then follows from Assumption 2 that $0 < R(1) < 1$.

The continuity and nonincreasing property of $R(\cdot)$ follows directly from Assumption 1.

Suppose $\delta_\beta < 1$ and $R(\delta_\beta) < 1$. Then Proposition 1 implies that $\Phi(s_n, \delta_\beta)$ is monotonically increasing in a suitably small neighborhood of $s_n$. Hence, the implicit function theorem can be applied to obtain

$$\frac{dR(\delta_\beta)}{ds} = -\left[\frac{\partial \Phi(s_n, \delta_\beta)}{\partial \delta_\beta}\right]^{-1}.$$ 

The sign of the denominator is positive by definition of $\delta_\beta$. Differentiating $\Phi$ with respect to $\delta_\beta$ yields

$$\frac{\partial \Phi(\cdot)}{\partial \delta_\beta} = \left\{\begin{array}{ll}
\delta^{-\infty} W(x)g(\delta_\beta|x)dH(x|s_\beta) & \text{if } \int_0^\infty L(x)G(\delta_\beta|x)dH(x|s_\beta) < 0 \\
\delta^{-\infty} [W(x) - L(x)]g(\delta_\beta|x)dH(x|s_\beta) & \text{if } \int_0^\infty L(x)G(\delta_\beta|x)dH(x|s_\beta) \geq 0.
\end{array}\right.$$

Recall that $W(x) = \max[0, L(x)]$. Therefore, in each case, the derivative is positive, and $dR(\delta_\beta)/ds < 0$. Q.E.D.

**Proof of Proposition 3.** Let $r = R(1)$. Proposition 2 implies that $r < R(r)$. Continuity of $R(\cdot)$ then implies that there exists a signal, $s^* \in [r, 1]$, such that $R(s^*) = s^*$. Furthermore, $s^*$ is unique since, by Proposition 2, $R(\cdot)$ is continuous on $[0, 1]$ and strictly decreasing on the range $[r, 1]$. It then follows from the symmetry of the reaction functions that $(s^*, s^*)$ is an equilibrium. Q.E.D.

**Proof of Proposition 4.** Let $f(s_\beta|s_n) = \int_0^\infty g(s_\beta|x)dH(x|s_n)$ denote the density function of $s_\beta$ conditional on $s_n$. Then, using Bayes' rule, the difference in the equilibrium payoffs to firm $\alpha$ from leading and following when it is endowed with the signal $s_n$ can be expressed (since $m_\beta = s^1$) as

$$\Phi(s_n, s^1) = \int_0^\infty \left\{\int_0^\infty [L(x) - \delta W(x)]dH(x|s_n, \delta_\beta) f(s_\beta|s^1)ds_\beta + \int_0^\infty L(x)G(\delta_\beta|x)dH(x|s_n) f(s_\beta|s_n)ds_\beta\right\}ds_\beta$$

$$- \max\left[0, \delta \int_0^\infty \left\{\int_0^\infty [L(x) - L(x)]dH(x|s_n, \delta_\beta) f(s_\beta|s^1)ds_\beta\right\}f(s_\beta|s_\beta)ds_\beta\right].$$

Recall that $\Phi(s^1, s^1) = 0$ by definition. Since the first term on the right-hand side of the equation is strictly negative by Assumption 3, this implies that the sum of the next two terms must be positive if $s_n = s^1$. But this is possible if and only if the expected value of the lease conditional on $\{s_n = s^1, s_\beta < s^1\}$ is positive. It then follows from Lemma 1 that $s^1 > s^2$. Q.E.D.

**Proof of Proposition 5.** By definition, $V(\delta, \delta) = 0$, which implies that

$$\int_0^\infty L(x)G(\delta|x)dH(x|\delta, \delta) = -\delta \int_0^\infty W(x)dH(x|\delta, \delta) < 0.$$ 

By Proposition 4, $E[L(x)|s^1, s_\beta < s^1]$ is positive. Therefore,

$$0 < \int_0^{s^1} \int_0^{s^1} L(x)G(\delta|x)s_\beta f(s_\beta|s^1)ds_\beta \int_0^\infty L(x)G(\delta|x)s^1, s^1, s^1),$$

where the last inequality follows from Lemma 1, which implies that $E[L(x)|s^1, s_\beta]$ is an increasing function of $s_\beta$. Applying Lemma 1 once again yields the result that $s^1 > \delta$. Q.E.D.

Before proving Proposition 6, we need two preliminary results.

**Lemma 3.** $q(s, s) > q(s)$ if and only if $g_d(s) > g_w(s)$.

**Proof.** Using Bayes' rule,

$$q(s, s) = \frac{g_d(s) \cdot g_w(s) \cdot q}{[g_d(s) \cdot g_w(s) \cdot q + g_w(s) \cdot g_w(s) \cdot (1 - q)].}$$

Subtracting $q(s)$ from both sides and cross-multiplying yield the result

$$\text{sign} \{q(s, s) - q(s)\} = \text{sign} \{g_d(s) - g_w(s)\}. \quad \text{Q.E.D.}$$
Lemma 4. Let \( s' \) solve \( \int_0^\infty L(x) dH(x|s') = 0 \). Then \( \int_0^\infty L(x) dH(x|s) < \int_0^\infty L(x) dH(x|s, s) \) for all \( s > s' \) if \( (1-q)(\bar{F} - C) - qC < 0 \).

Proof. By definition, \( s' \) solves

\[
(1-q(s'))(\bar{F} - C) - q(s')C = 0.
\]

Dividing through by \( (1-q(s')) \) and substituting for \( q(s') \) yields

\[
g_a(s')/g_w(s') = (1-q(\bar{F} - C))/qC.
\]

Now, suppose \( (1-q)(\bar{F} - C) - qC < 0 \). Then \( g_a(s')/g_w(s') < 1 \). Since \( g_a(s)/g_w(s) \) is decreasing for all \( s \) by the weak MLRP property, \( g_a(s)/g_w(s) \) is less than one for all \( s > s' \). Q.E.D.

Proof of Proposition 6. First, we show that \( s^2 > s' \), where \( s' \) solves the equation \( \int_0^\infty L(x) dH(x|s') = 0 \). From the definition of \( s^2 \) and the first-order stochastic dominance relation between \( G_a \) and \( G_{a,s} \) we have that

\[
0 = E[L(x)|s^2, s_a < s^1] = -q(s^2)G_a(s^1)C + (1-q(s^2))G_{a,s}(s^1)(\bar{F} - C)
\]

\[
< G_a(s')[(1-q(s^2))(\bar{F} - C) - q(s^2)C]
\]

\[
= G_a(s')E[L(x)|s^2].
\]

Hence, \( E[L(x)|s^2] > 0 \), which, applying Lemma 1, means that \( s^2 \) is larger than \( s' \). Combining this fact with Lemma 4 then implies that

\[
0 < E[L(x)|s^2] < E[L(x)|s^2, s^2].
\]

By definition, \( E[L(x)|\bar{s}, \bar{s}] = 0 \). The result then follows from Lemma 1. Q.E.D.

Proof of Proposition 7. Suppose \( \{Q, J, C, B\} \) is a sequential equilibrium in which \( (Q, J) \) is a full-disclosure strategy. We will show first that \( J(s_a) = 0 \) for all \( s_a \in [0, 1] \) in any such equilibrium.

From Conditions (i) and (iii) of the definition of sequential equilibrium, the optimal response of firm \( B \) to \( (Q, J) \) is as follows: to accept if \( s_a \leq \delta(s_a, J(s_a)) \) and to reject otherwise, where \( \delta(s_a, J(s_a)) \) solves the equation \( J(s_a) = E[L(x)|s_a, \delta] \). (If no solution exists, \( \delta(s_a, J(s_a)) \) is set equal to one.) Now suppose firm \( s_a \) reports its type exactly and makes offer \( p \). The restriction on off-equilibrium-path conjectures implies that \( B(s_a|s_a, p; s_s) = 1 \) for all offers \( p \) and for every \( s_a \). Therefore, \( P(J(s_a), Q(s_a); s_a) \neq P(J(s_a), s_a; s_a) \). The latter payoff is a strictly decreasing function of the offer price. Consequently, if \( J(s_a) > 0 \), firm \( a \) could increase its payoff by announcing \( (s_a, 0) \), contradicting (ii). We may therefore conclude that, if \( (Q, J) \) is a full-disclosure strategy, then \( J(s_a) = 0 \) for all \( s_a \), and \( Q \) satisfies \( Q^{-1}(Q(s_a)) = s_a \).

We will now show that no such \( Q \) can exist. Without loss of generality, assume that \( Q(s) = s \) for all \( s \in [0, 1] \). Select two distinct signals, \( s^1 \) and \( s^2 \), such that the partitions associated with the reports \( s^1 \) and \( s^2 \) are different and interior. That is, \( \delta(s^1, 0) \neq \delta(s^2, 0) \), and each of these values is strictly less than one. Given Assumption 2, such signals exist. Now consider the off-equilibrium report \( S = \{s^1, s^2\} \). Each firm \( B \) type forms beliefs concerning which of the two firm \( a \) types could have made this report, and acts accordingly. Define \( A^i = \{s_a \in [0, 1] \mid D(s_a, S, 0) = 0 \wedge D(s_a, s, 0) = 1 \} \) to be the set of firm \( a \) types that drill under report \( s' \), but accept the zero offer under \( S \). Similarly, define \( B^i = \{s_p \in [0, 1] \mid D(s_p, S, 0) = 1 \wedge D(s_p, s, 0) = 0 \} \) to be the set of firm \( a \) types which accept under report \( s' \), but drill under report \( S \). Condition (ii) implies that, if \( Q \) is an equilibrium, then

\[
P(0, S; s^i) - P(0, s^i; s^i) = \int_{s^i} E[L(x)|s^i, s_a]dF(s_a|s^i) - \int_{s^i} E[L(x)|s^i, s_a]dF(s_a|s_a).
\]

By definition of \( \delta(s^i, 0) \), \( E[L(x)|s^i, s] \) is strictly positive for all \( s \in A^i \), and negative for all \( s \in B^i \). Therefore, for the above inequality to hold, \( A^i \cup B^i \) must be a set of measure zero for each \( i = 1, 2 \). But this contradicts the initial hypothesis that \( \delta(s^i, 0) \neq \delta(s^2, 0) \). Q.E.D.

Proof of Proposition 8. Suppose \( J(s_a) = 0 \), and \( Q(s_a) = [0, 1] \) for every \( s_a \in [0, 1] \). Given this (report, offer) pair, the optimal response of firm \( B \) is to drill if \( s_p \leq \delta([0, 1], 0) \), and to sell otherwise. With respect to any \( (S, p) \neq ([0, 1], 0) \), we need to distinguish three cases.

(i) Suppose first that \( (S, p) \) satisfies \( E[L(x)|s_a = \min S, s_p = \delta([0, 1], 0)] \geq p \). Define \( B \) so that it has sufficient mass at \( \min S \) that \( \delta(S, p) \) is less than \( \delta([0, 1], 0) \). This implies that, conditional on obtaining a signal \( s_a \geq \min S \), the payoff to firm \( a \) from \( (S, p) \) is less than its payoff from \( ([0, 1], 0) \). Furthermore, given any signal \( s_a < \min S \), the report \( S \) is not feasible. Thus, either firm \( a \) has no incentive to submit \( (S, p) \) or it cannot.

(ii) Suppose next that \( (S, p) \) satisfies \( E[L(x)|s_a = \max S, s_p = \delta([0, 1], 0)] < p \). In this case, define beliefs so that \( B \) has sufficient mass at min \( S \) that \( \delta(S, p) > \delta([0, 1], 0) \). This implies that the payoff to every firm \( a \) type
for which \((S, p)\) is feasible is less than its equilibrium payoff. Once again, there is no incentive to deviate to \((S, p)\).

(iii) Finally, suppose \((S, p)\) has the property that, conditional on the event \(\{s_\alpha = \min S, s_\beta = \delta([0, 1], 0)\}\), the optimal response of firm \(\beta\) is to sell, and conditional on the event \(\{s_\alpha = \max S, s_\beta = \delta([0, 1], 0)\}\), its optimal response is to drill. In that case, \(B\) needs to be chosen so that the marginal type is indifferent between drilling and selling. Hence, the decision rule of firm \(\beta\) is the same under \((S, p)\) as it is under \(([0, 1], 0)\). This gives firm \(\alpha\) no incentive to deviate to \((S, p)\). \(Q.E.D.\)

References


Information and the Coase Theorem
Joseph Farrell
Stable URL:
http://links.jstor.org/sici?sici=0895-3309%28198723%291%3A2%3C113%3AIATCT%3E2.0.CO%3B2-L

Information, Returns, and Bidding Behavior in Ocs Auctions: 1954-1969
Kenneth Hendricks; Robert H. Porter; Bryan Boudreau
Stable URL:
http://links.jstor.org/sici?sici=0022-1821%28198706%2935%3A4%3C517%3AIRABBI%3E2.0.CO%3B2-L

Sequential Equilibria
David M. Kreps; Robert Wilson
Stable URL:
http://links.jstor.org/sici?sici=0012-9682%28198207%2950%3A4%3C863%3ASE%3E2.0.CO%3B2-4

The Influence of Private Contractual Failure on Regulation: The Case of Oil Field Unitization
Gary D. Libecap; Steven N. Wiggins
Stable URL:
http://links.jstor.org/sici?sici=0022-3808%28198508%2993%3A4%3C690%3ATTOPCP%3E2.0.CO%3B2-M

Relying on the Information of Interested Parties
Paul Milgrom; John Roberts
Stable URL:
http://links.jstor.org/sici?sici=0741-6261%28198621%2917%3A1%3C18%3ARTOIO%3E2.0.CO%3B2-B

Some Implications of Land Ownership Patterns for Petroleum Policy
Edward Miller
Stable URL:
http://links.jstor.org/sici?sici=0023-7639%28197311%2949%3A4%3C414%3ASIOLOP%3E2.0.CO%3B2-S

NOTE: The reference numbering from the original has been maintained in this citation list.