Homework #10 Solution

Problem 1: More on Externality

Consider the market for education. The marginal social cost of education (MSC) and the marginal private benefit of education (MPB) are given by the following equations where Q is the number of units of education provided per year.

MSC = 10 + Q

MPB = 100 – Q

You are also told that each unit of education provides an external benefit to society of $10 per unit. This external benefit is currently not being internalized in the market.

a) Given the MSC and MPB curves, what is the current number of education units being produced by the market?

Answer:
Since the external benefits are not being currently internalized in the market this means that the market is producing where MSC = MPB. Or, 10 + Q = 100 – Q and Q is therefore equal to 45 units of education.

b) Is the current level of market production for education the socially optimal amount of education? Explain your answer.

Answer:
No, this is not the socially optimal amount of education to produce since when Q = 45 the MSC of producing the 45th unit of education is equal to $55 while the MSB of producing the 45th unit of education is equal to MPB + $10 or 100 – 45 + 10 for a MSB of $65. Since the MSB is greater than the MSC at 45 units of production this tells us that more units of education should be produced. It also indicates that there should be a deadweight loss associated with this level of production since the MSC for the last unit is not equal to the MSB of the last unit.

c) What is the value of consumer surplus (CS), the value of producer surplus (PS), and the value of the external benefits of the current level of production. Sum together (CS + PS + external benefits). Draw a diagram illustrating each of these concepts in the market for education.

Answer:
CS = (1/2)($100 per unit - $55 per unit)(45 units) = $1012.50
PS = (1/2)($55 per unit - $10 per unit)(45 units) = $1012.50
External benefits = (Benefits per unit)(# of units) = ($10 per unit)(45 units) = $450
CS + PS + external benefits = $2475

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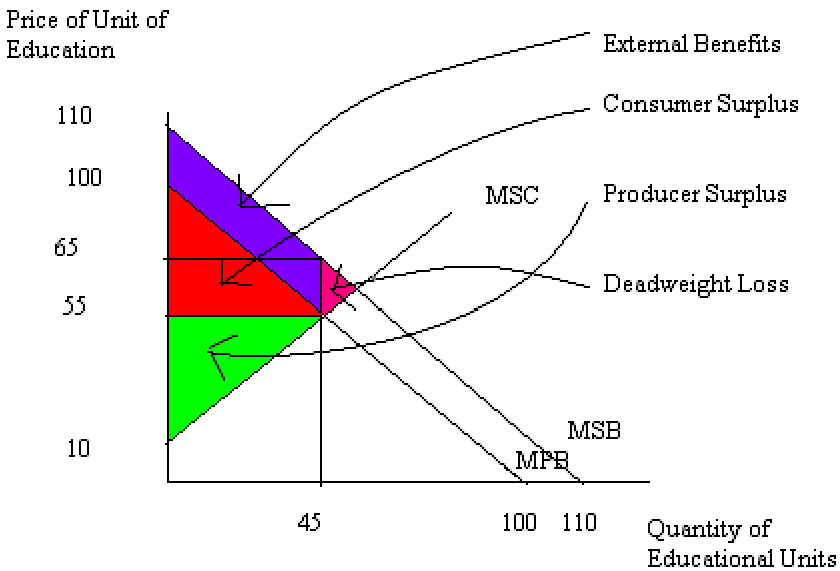

d) Given the market level of production, what is the deadweight loss in this market?

Answer:
To calculate the DWL you will need to find the Q that corresponds to where MSC = MSB. MSB = MPB + 10 or MSB = 100 – Q + 10 = 110 – Q. Thus, 10 + Q = 110 + Q or Q = 50. DWL = (1/2)($65 per unit - $55 per unit)(50 units – 45 units) = $25.

e) Suppose that the external benefit is internalized in this market when the government provides a subsidy of $10 per educational unit to consumers. What will be the socially optimal amount of education to provide given this subsidy?

Answer:
The socially optimal amount of education to provide is where MSC = MSB. From our calculation in (e) we know that the socially optimal amount of education to provide is 50 units of education.

f) Given the subsidy in (e), calculate the value of consumer surplus with the subsidy (CS'), producer surplus with the subsidy (PS'). With the subsidy there are no longer any external benefits that the market fails to account for. Sum together CS' + PS': does this total equal the sum of (CS + PS + external benefits) + DWL from parts (c) and (d)?

Answer:
CS' with the subsidy = (1/2)($110 per unit - $60 per unit)(50 units) = $1250
PS' with the subsidy = (1/2)($60 per unit - $10 per unit)(50 units) = $1250
Sum of CS' + PS' = $2500
Sum of CS + PS + external benefit + DWL = $2475 + $25 = $2500
Return to the graph: all the shaded regions are now either CS' or PS' once the subsidy is instituted in this market. There is no DWL when MSC = MSB since the socially optimal amount of the good is produced.

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Problem 2: Public Goods

Consider an economy with two consumers, Ben and Joe. There is public good in this economy in a form of tornado sirens. Ben's demand for tornado sirens is given by P = 10 - Q, and Joe's demand for tornado sirens is P = 8 - 2Q. Marginal cost for providing tornado sirens in the markets is constant, MC = 9.

a) Which two properties must be satisfied for sirens to be public goods?
Public goods must be **non-rival and non-excludable**

b) Are tornado sirens are non-rival? Explain your answer.
Yes. If one person uses it (i.e. she can hear sirens), other people's use of this good is not affected (other people can hear the sirens as well).

c) Are tornado sirens are non-exclusive?. Explain your answer.
Yes. Everyone in the area will be able to hear the siren, regardless of whether they paid for this good.
d) Is there a potential for free-rider problem?
Yes. This is implied by the fact that sirens are non-excludable, i.e. there might be people who didn't pay for the service but these people will still be able to hear the sirens. We call these people free-riders.

e) Derive market demand curve for sirens. Draw three graphs on the top of each other– first graph for Ben's demand, second graph for Joe's demand, and the third graph for market demand.

The market demand curve can be derived by VERTICAL summation (not horizontal as is the case with private goods) of the individual demand curves. At each quantity level, we see the willingness to pay of each individual and then estimate society's total willingness to pay by adding the willingness to pay of the various individuals.

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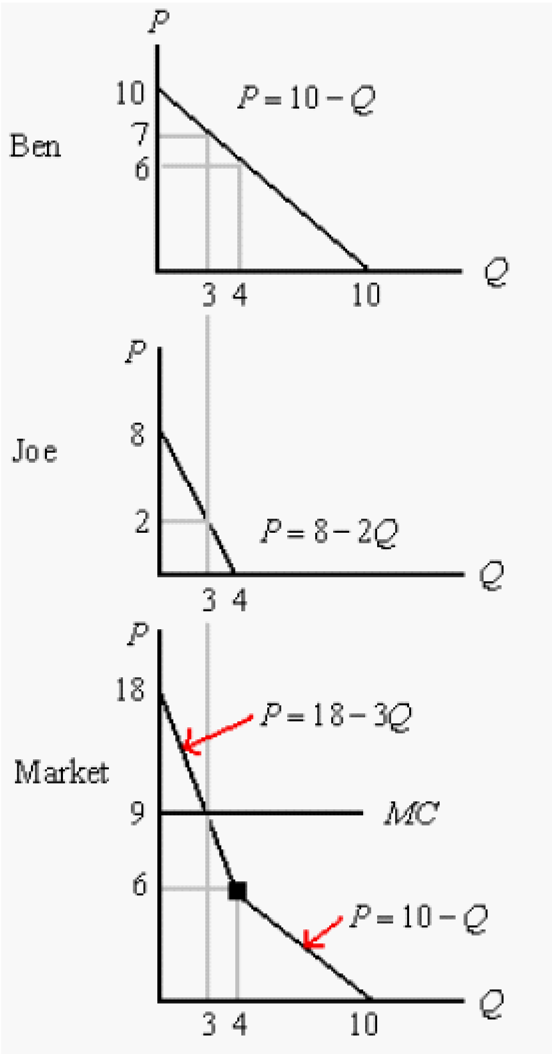

At quantity of 0 sirens, Ben is willing to pay 10 dollars, and Joe is willing to pay 8 dollars. Together, they're willing to pay 18 dollars. At quantity of 4 sirens, Joe is willing to pay 0 dollars, and Ben is willing to pay 6 dollars. To see this, just plug Q=4 into Ben's demand curve: P=10-Q=10-4=6). At quantity of 10, both consumers are aren't willing to pay anything (P=0).

Use the information calculated above to draw market demand curve. First draw the three points (10,0), (4,6), (18,0), and then draw linear curve between these points. You'll see that the market demand curve has a kink at point (4,6). Therefore, the **market demand curve has two parts: P=18-3Q if P>=6, and P=10-Q if P<=6**. Note: you can calculate derive the first part from the points (0,18) and (4,6) on the graph – you see that the y-intercept is 18, and you can calculate the slope from the two points. The second part is the same as Ben's demand curve

f) How many sirens will be provided in the market?

Draw the MC on the graph with market demand curve (MC=9). You can see that MC intersects the demand curve in the upper. MC = demand => 9=18-3Q => 9=3Q => **Q=3 (so there will be 3 sirens provided in the market)**

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g) What will be the price for these sirens?
Draw a dotted line up at Q=3 such that you see where it intersects Ben's and Joe's demand curve. Plugging Q=3 into their individual curves, you get that Ben is willing to pay $7 and Joe is willing to pay $2. **Therefore, the price in this market for providing 3 sirens will be $9.**

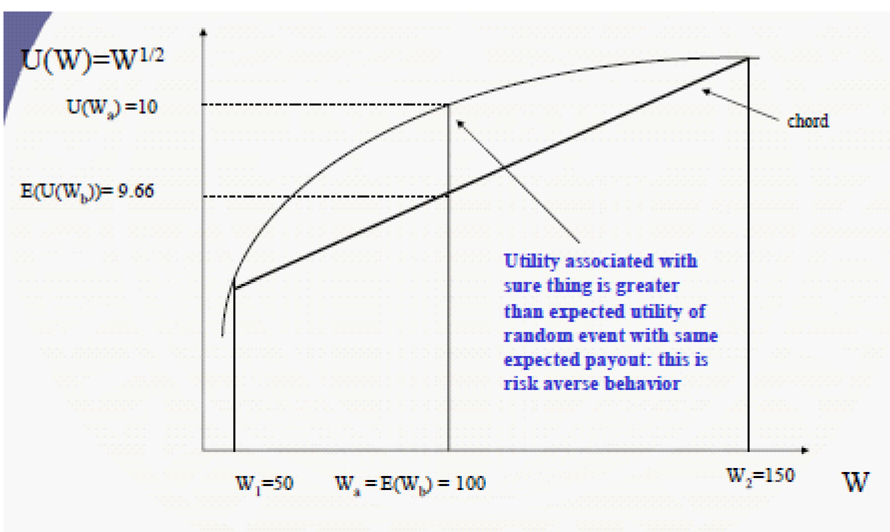
h) Is the result from previous question realistic? Discuss how government funds public goods.
No, the result is not realistic. Our example describes idealized situation in which we are given demand curves of both individuals. However, this is not the case in the real life, because in the real life, we don't know people's demand curves. In other words, people don't publicly reveal how much they are willing to pay for public service. In fact, people are often trying to become free-riders by claiming that they don't actually need the service. The government can do benefit-cost analysis and charge user fees, for example through taxes, or through entry fees.

Problem 3: Expected Utility

$U(W) = W^{1/2}$, Wa = 100 with probability 1; Wb = 50 with p1 = 0.5 and = 150 with p2 = 0.5. The expected utility for the possible two wealth situations are as follows:
E(U(Wa)) = U(Wa) (for certain wealth), E(U(Wb)) = p1 * U(W1) + p2 * U(W2)(for random wealth)

1) Compute the expected utility of certain wealth and for gamble
$E[U(Wa)] = U(Wa) = 100^{1/2} = 10$ = expected utility for sure thing Wa
$E(U(Wb)) = 50^{1/2}(0.5) + 150^{1/2}(0.5) = 7.07(0.5) + 12.25(0.5) = 9.66$ = expected utility for gamble

2) Draw the graph for expected utility



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3) Find the the certainty equivalent wealth associated with E[Wb] = 100.

$U(Wc) = Wc^{1/2} = E[U(Wb)] = p1U(W1) + p2U(W2) = 9.66$
=> Wc = (9.66)2 = 93.32
Risk premium: E[Wb] – Wc = 100 – 93.32 = 6.68

4) Draw CE on the graph