Professor Wallace   Due: September 27-28, 2012

Homework #2

Problem 1: For each of the following claims, please state if it is true, false or uncertain. Provide a brief written justification for your answer (a sentence or two will suffice).

(1) When the price of tomato sauce increases by 5%, the quantity demanded drops by 10%. Then, demand of for tomato sauce is elastic.
TRUE. When the demand of a good is elastic the variation in quantity due to a variation in price is more than 5%.

(2) Notebooks have an inelastic demand. When the price of notebooks drops by 1%, the quantity demanded demand increases by 2%.
FALSE. The change in quantity due a change in price is more than 1%, and then the good has an elastic demand.

(3) The quantity of coffee demanded drops by 5% after that the price increase by 5%. It means that the demand for coffee is unit elastic.
TRUE. In this case the demand is clearly unit-elastic.

(4) Demand a farmer's wheat is inelastic. When the price of wheat increases by 20%, the revenue of the farmer increase by 5%.
TRUE. By the revenue test if I increase the price and the revenue increase the demand is inelastic.

(5) If my income increases by 10%, then quantity of public transportation demanded drops by 5%. I can say that income elasticity of my demand for public transportation is negative and that public transportation is a normal good.
FALSE. If the income elasticity is negative the good is inferior.

(6) If the demand for cheese in France is perfectly inelastic, an increase in the price of cheese will reduce the quantity demanded.
FALSE. If the demand is inelastic the quantity demanded does not change.

Problem 2: Chuck, Michael and Hakeem are three retired basketball players. They determine that in Madison there is demand for private basketball coaching. The demand and supply curves for private basketball coaching are $Q_d = 50 - 0.5 \cdot P$ and $Q_S = P - 10$ respectively.

(1) Find the equilibrium price and quantity

We need to equalize quantity demand and supply:

$$Q_d = 50 - 0.5 \cdot P = P - 10 = Q_S$$

Then we solve out for the price
$$1.5 \cdot P = 60$$
$$P = 40$$

In the end, we plug in the equilibrium price in the demand (it would be equivalent if we do it in the supply) to get the equilibrium quantity:

$$Q = 50 - 0.5 \cdot 40$$
$$Q = 30$$

(2) What is the price elasticity of demand at the equilibrium calculated in part (1)? Is demand for basketball coaching elastic or inelastic?

The price elasticity of the demand at the equilibrium price is:

$$\varepsilon = \frac{\Delta Q / \Delta P}{Q/P} = \frac{1/2}{30/40} = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3}$$

The demand for basketball coaching is inelastic.

(3) Calculate the price elasticity of demand on the segment (arc) of the demand curve between the prices of $20 and $40

First, we need to calculate the quantity demanded at a price $20 is

$$Q = 50 - 0.5 \cdot 20$$
$$Q = 40$$

The arc elasticity in this case is:

$$\varepsilon = \frac{\dfrac{Q_1 - Q_0}{Q_1 + Q_0}}{\dfrac{2}{\dfrac{P_1 - P_0}{P_1 + P_0}}} = \frac{\dfrac{40 - 30}{40 + 30}}{\dfrac{2}{\dfrac{20 - 40}{20 + 40}}} = \frac{2}{7} \cdot \frac{4}{6} = \frac{4}{21}$$

(4) What is the price elasticity of supply calculated at the equilibrium calculated in part (2)? Is the supply of basketball coaching elastic or inelastic?

The supply elasticity at the price equilibrium price is:

$$\eta = \frac{\Delta Q / \Delta P}{Q/P} = \frac{1}{30/40} = 1 \cdot \frac{4}{3} = \frac{4}{3}$$

The supply of basketball coaching is elastic.

Problem 3: The demand and supply curves for fixed gear bikes in Hipsterland are $Q_D = 225 - 1.5 \cdot P$ and $Q_s = 1.5 \cdot P - 45$ respectively.

(1) Find the equilibrium price and quantity

We need to equalize demand and supply to find the equilibrium price:

$$Q_D = 225 - 1.5 \cdot P = 1.5 \cdot P - 45 = Q_S$$

Then we solve out for the price

$$3 \cdot P = 270$$
$$P = 90$$

In the end we plug in the equilibrium price in the demand (it would equivalent if we do it in the supply)

$$Q = 225 - 1.5 \cdot 90$$
$$Q = 90$$

(2) Find the price elastic of demand and the price elasticity of supply at the equilibrium in part (1)

The price elasticity of the demand at the equilibrium price is:

$$\varepsilon = \frac{\Delta Q / \Delta P}{Q / P} = \frac{3/2}{90/90} = \frac{3}{2} \cdot 1 = \frac{3}{2}$$

The price elasticity of the supply at the equilibrium price is:

$$\eta = \frac{\Delta Q / \Delta P}{Q / P} = \frac{3/2}{90/90} = \frac{3}{2} \cdot 1 = \frac{3}{2}$$

(3) What are the consumer surplus, producer surplus, and total welfare at the market equilibrium calculated in part (1)

As first step we calculate the inverse demand and supply, that are:

$$Q = 1.5 \cdot P_S - 45 \qquad\qquad Q = 225 - 1.5 \cdot P_D$$
$$1.5 \cdot P_S = 45 + Q \qquad\qquad 1.5 \cdot P_D = 225 - Q$$
$$P_S = 30 + \frac{2}{3} Q \qquad\qquad P_D = 150 - \frac{2}{3} Q$$

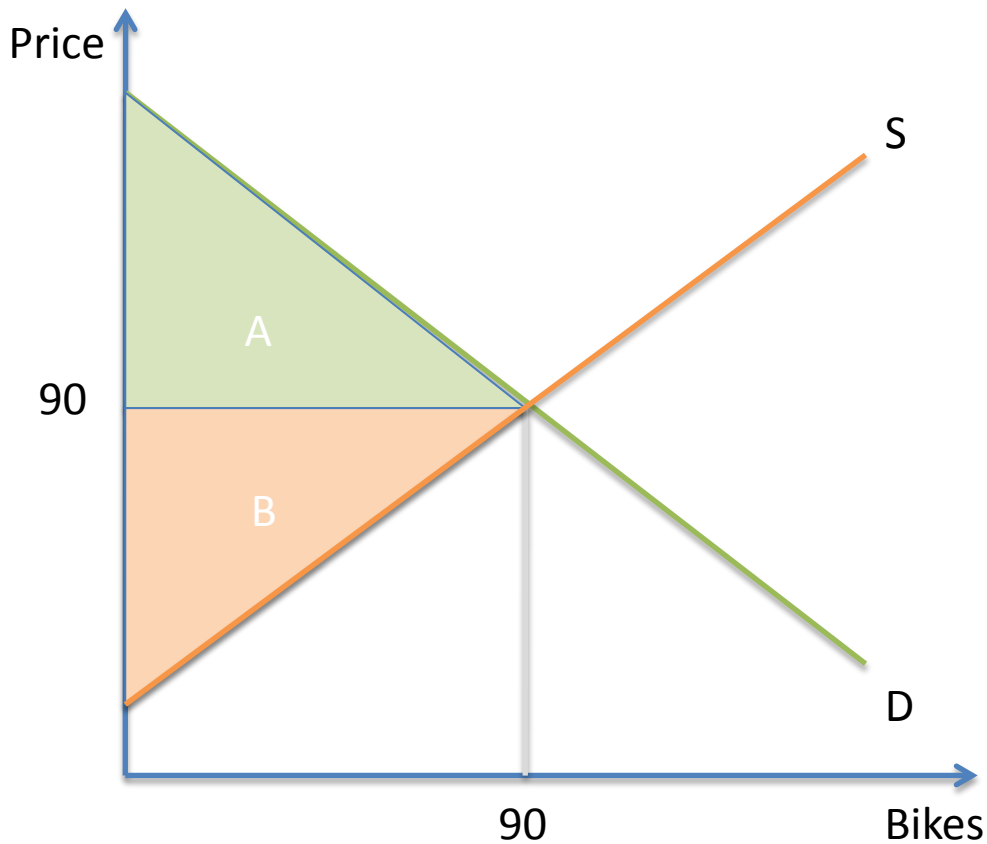The consumer surplus is the area of the triangle A:
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$$CS = \frac{(150 - 90) \cdot 90}{2} = 2700$$

The producer surplus is the area of the triangle B:

$$PS = \frac{(90 - 30) \cdot 90}{2} = 2700$$

The total surplus is the sum of consumer surplus and producer surplus, 5400.

Price

S

A

90

B

D

90   Bikes

(4) There are too many fixed gear bikes in Hipsterland and city leaders are looking for ways to reduce the quantity. They are considering policies which would reduce the number of fixed gear bikes.
   a. Suppose that municipality imposes a quota of 60 on the number of bikes. What are the producer and consumer surplus? Is there any deadweight loss? If so, how much?[1]

   Imposing a quota of 60 fix gear bikes the government would induce a change in the equilibrium price and quantity because the equilibrium quantity was above the quota.

   First, we calculate the consumer willingness to pay and producer opportunity cost for a quantity of 60:

$$60 = 225 - 1.5 \cdot P_D \qquad\qquad 60 = 1.5 \cdot P_S - 45$$
$$1.5 \cdot P_D = 165 \qquad\qquad 1.5 \cdot P_S = 105$$
$$P_D = 110 \qquad\qquad P_S = 70$$

   Second, we calculate the wedge (yellow segment on the price axes) between the two prices:

$$P_S - P_D = 110 - 70 = 40$$

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[1] We are not going to cover simple quotas in lecture, but they are covered in our text.

Now we can calculate the consumer surplus (triangle A):

$$CS = \frac{(150-110)\cdot 60}{2} = 1200$$

In this case, the producer surplus consists of two part: the triangle B and the rectangle D. The first is the surplus from the activity of producing bike. The second is the rent deriving by the limitation of the quantity in the market. Then, we calculate producer surplus:

$$PS = \frac{(70-30)\cdot 60}{2} + 40\cdot 60 = 1200 + 2400 = 3600$$

The deadweight loss in this case is the triangle C:

$$DWL = \frac{(110-70)\cdot 30}{2} = 600$$
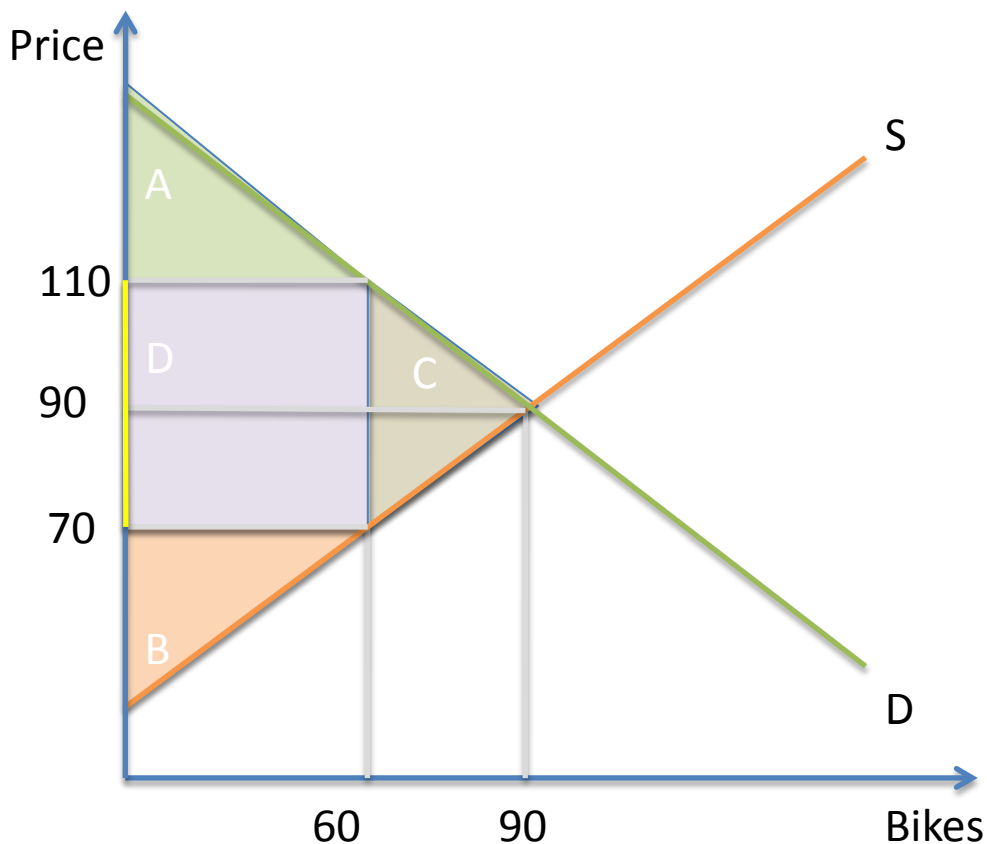
b. Suppose the municipality imposes a price floor of $110 in the market for fixed gear bikes. What are the producer and consumer surplus? Is there any deadweight loss? If so, how much?

Also here the government intervention would change the equilibrium price and quantity, because the original equilibrium price was 90, below the floor set by the government.

First, we calculate the quantity demanded at that price floor:

$$Q_D = 225 - 1.5 \cdot 110$$
$$Q_D = 60$$

Then, we want to calculate also the opportunity cost of producing that amount for the producer is:

$$60 = 1.5 \cdot P_S - 45$$
$$1.5 \cdot P_S = 105$$
$$P_S = 70$$

Now we can calculate the consumer surplus (triangle A):

$$CS = \frac{(150 - 110) \cdot 60}{2} = 1200$$

In this case, the producer surplus consists of two parts: the triangle B and the rectangle D. The first is the surplus from the activity of producing bike. The second is the rent deriving by the price floor. Then, we calculate producer surplus:

$$PS = \frac{(70 - 30) \cdot 60}{2} + 40 \cdot 60 = 1200 + 2400 = 3600$$

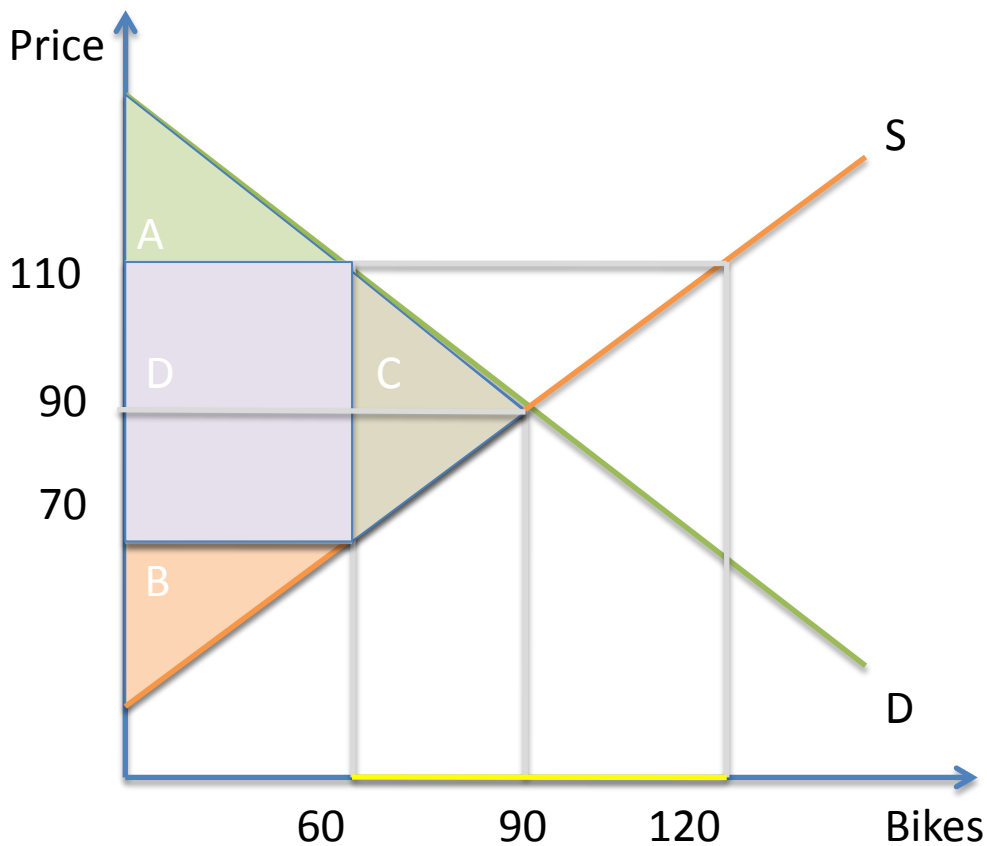The Deadweight loss in this case is the triangle C:

$$DWL = \frac{(110 - 70) \cdot 60}{2} = 600$$

In the end, we can also calculate the surplus in the market (yellow segment),

$$Q_S = 1.5 \cdot 110 - 45$$
$$Q_S = 120$$

Then, we take the difference between the quantity demanded and the quantity supplied at that price, 60.

c. Suppose the municipality decided levy a unit tax $40 on retailers for each fixed gear bike sold. Calculate the new equilibrium price and quantity. What is the incidence of the tax on consumers? On producers? What are the producer and consumer surplus? What is the revenue of the tax? Is there any deadweight loss? If so, how much?

We need to find the new equilibrium price with the tax:

$$Q_D = 225 - 1.5 \cdot P = 1.5 \cdot (P - 40) - 45 = Q_S$$
$$225 - 1.5 \cdot P = 1.5 \cdot P - 105$$
$$3 \cdot P = 330$$
$$P = 110$$

At this equilibrium price the equilibrium quantity is:

$$Q = 225 - 1.5 \cdot 110 = 60$$

We want also calculate the opportunity cost for the producer to produce 60 bikes, this is:

$$1.5 \cdot (P_S - 40) - 45 = 60$$
$$1.5 \cdot P_S = 105$$
$$P_S = 70$$

The tax incidence on the consumer is:

$$TI_C = \frac{\varepsilon}{\varepsilon + \eta} = \frac{\dfrac{3/2}{60/110}}{\dfrac{3/2}{60/110} + \dfrac{3/2}{60/110}} = \frac{3}{2} \cdot \frac{2}{6} = \frac{1}{2}$$

The tax incidence on the producer is:

$$TI_P = \frac{\eta}{\varepsilon + \eta} = \frac{\dfrac{3/2}{60/110}}{\dfrac{3/2}{60/110} + \dfrac{3/2}{60/110}} = \frac{3}{2} \cdot \frac{2}{6} = \frac{1}{2}$$

It means that producers and consumers share equally the cost of the new tax

The consumer surplus is the triangle A:

$$CS = \frac{(150 - 110) \cdot 60}{2} = 1200$$
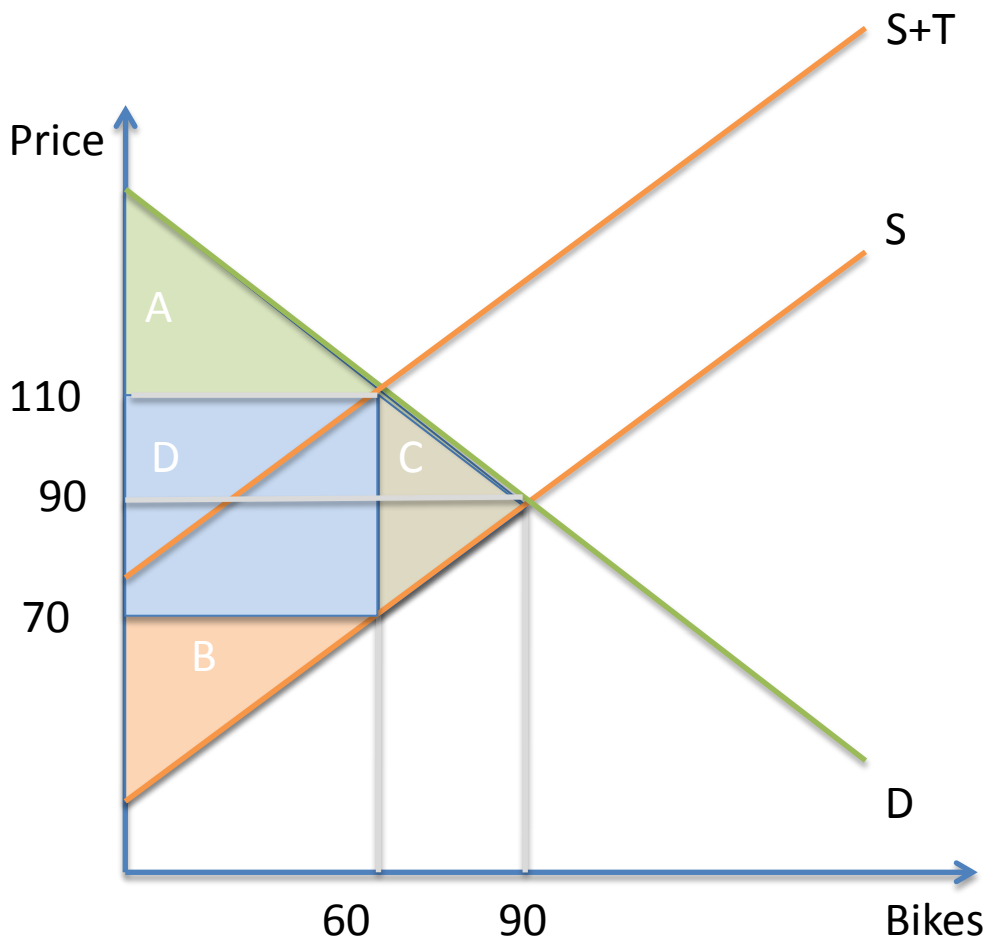
The producer surplus is the triangle B:

$$PS = \frac{(70 - 30) \cdot 60}{2} = 1200$$

The revenue for the government is the rectangle D:

$$TAX = 60 \cdot 40 = 2400$$

The Deadweight loss in this case is the triangle C:

$$DWL = \frac{(110 - 70) \cdot 60}{2} = 600$$

S+T

Price

S

A

110

D

C

90

70

B

D

60   90   Bikes

d. Compare the policy options in parts a., b., and c., on the basis of the reduction in fixed gear bikes, revenue, and dead weight loss. Which option would you recommend and why?

We can summarize the impact of the three policies using the following table:

	CONSUMER SURPLUS	PRODUDUCER SURPLUS	TAX REVENUE	DEADWEIGHT LOSS
QUOTA	1200	3600	0	600
PRICE FLOOR	1200	3600	0	600
TAX	1200	1200	2400	600

The three policies induce the same deadweight loss of 600. The consumer surplus also is the same across the policies, 1200. What changes is the producer surplus. With the first two policies, quota and price floor, it is 3600. With the third one, the unit tax, it is 1200. Conversely the tax revenue is null under the first two policies and it is 2400 with tax. Considering that the government gets higher revenue in the third option, I might recommend they implement the tax.