Assignment #5 Answer Key*

Econ 102: Introductory Macroeconomics

May 5, 2010

Directions: Turn in the homework to your TA's box before lecture. Please legibly write your name, TA name, and section number on the front of the homework. Write your name as it appears on your ID. Late homework is not be accepted. Please show your work in a readable and organized way. Good luck!

1 Keynesian Models

1.1 Aggregate Expenditure (AE) Model, Analytical

Consider a closed economy with consumption function

$$C = \bar{C} + MPC[Y - (T - TR)] \tag{1}$$

where I is planned investment, $\bar{C}$ is autonomous consumption, MPC is the marginal propensity to consume, Y is output, T is taxes levied on households by government, and TR is the level of transfers from government to households. Use your knowledge of the aggregate expenditure model (goods market) to answer the following questions.

1.1.1 Are there any restrictions on the possible values of $\bar{C}$ and MPC? If so, list them and briefly explain why these restrictions are imposed.

$\bar{C} > 0$ (you must consume some baseline subsistence level of consumption $\bar{C}$ in order to survive and meet basic life needs (food, clothing, shelter, etc.), either borrowed or provided by the government when you personally have zero income)

$0 < MPC < 1$ (you consume some fraction of disposable income between zero and one; implicitly, this assumes that you can't borrow against current or future income in financial markets; if borrowing is allowed, it is possible that the marginal propensity to consume is greater than one for some individual in the economy)

1.1.2 Write out the aggregate expenditure equation $AE(Y)$. Is it linear in Y? If so, list the slope and y-intercept terms. Draw a graph of $AE(Y)$ versus Y.

$AE = C + I + G + NX$ *(definition)*
$AE = \bar{C} + MPC[Y - (T - TR)] + I + G$ *(consumption function; closed economy)*
$m = MPC$ *(slope)*
$b = \bar{C} + I + G - MPC(T - TR)$ *(y-intercept)*
$AE(Y) = mY + b = (MPC)(Y) + [\bar{C} + I + G - MPC(T - TR)]$ *(aggregate expenditure function)*
The aggregate expenditure function $AE(Y)$ is linear in Y.

1.1.3 Write equilibrium output level Y^* as a function of T, TR, MPC, $\bar{C}$, I, and G. (hint: write down the equilibrium condition for AE model, i.e. $AE(Y^*) = Y^*$, and solve for Y^*)

$AE(Y^*) = Y^*$ *(equilibrium condition, AE model)*
$(MPC)(Y^*) + [\bar{C} + I + G - MPC(T - TR)] = Y^*$
$Y^*(1 - MPC) = \bar{C} + I + G - MPC(T - TR)$
$Y^* = \frac{1}{1 - MPC}[\bar{C} + I + G - MPC(T - TR)]$

1.1.4 Define tax multiplier $M_T \equiv \frac{\Delta Y^*}{\Delta T}$ and government expenditure multiplier $M_G \equiv \frac{\Delta Y^*}{\Delta G}$ (in calculus terms, $M_T = \frac{\partial Y^*}{\partial T}$ and $M_G = \frac{\partial Y^*}{\partial G}$). Given your expression for Y^* from the previous part, write M_T and M_G, the fiscal policy multipliers, in terms of constants. Which multiplier is larger in magnitude? (hint: the fiscal policy multipliers are the coefficients on the T and G terms in the equation for equilibrium output Y^* that you derived previously)

Without calculus:
$\Delta Y^* = \frac{1}{1 - MPC}[-MPC(\Delta T)]$
$M_T = \frac{\Delta Y^*}{\Delta T} = -\frac{MPC}{1 - MPC}$
$\Delta Y^* = \frac{1}{1 - MPC}(\Delta G)$
$M_G = \frac{\Delta Y^*}{\Delta G} = \frac{1}{1 - MPC}$
With calculus:
$M_T = \frac{\partial Y^*}{\partial T} = -\frac{MPC}{1 - MPC}$
$M_G = \frac{\partial Y^*}{\partial G} = \frac{1}{1 - MPC}$

1.1.5   Briefly explain how the aggregate expenditure model reaches equilibrium (Y^*, C^*) from any nonzero initial level of output Y_0. *(hint: you answer should talk about how firms respond to unexpected changes in inventories under three cases: $Y_0 < Y^*$, $Y_0 = Y^*$, and $Y_0 > Y^*$; stating the goal of the firm may help)*

$Y_0 < Y^*$: excess demand, $\Delta inventories < 0$, firms scale up output to keep inventories constant to maximize profits, $Y \uparrow$ until $Y = Y^*$ and the AE model is in equilibrium.
$Y_0 = Y^*$: no excess demand or excess supply, $\Delta inventories = 0$, firms hold output constant with no change in inventories to maximize profits, $Y = Y^*$ immediately and the AE model is in equilibrium.
$Y_0 > Y^*$: excess supply, $\Delta inventories > 0$, firms scale down output to keep inventories constant to maximize profits, $Y \downarrow$ until $Y = Y^*$ and the AE model is in equilibrium.

1.1.6   <u>From this part forward</u>, assume the following: $\bar{C} = 175$, $MPC = \frac{4}{5}$, $I = 200$, $G = T - TR = 250$. If $Y = 2000$, what is planned aggregate expenditure? Given your answer, would you expect equilibrium output level Y^* to be higher or lower than $Y = 2000$?

$AE_{planned}(Y) = (MPC)(Y) + [\bar{C} + I + G - MPC(T - TR)]$
$AE_{planned}(Y = 2000) = (\frac{4}{5})(2000) + [175 + 200 + 250 - \frac{4}{5}(250)] = 1600 + 425 = 2025$
$AE_{planned}(2000) = 2025 > Y = 2000 \Rightarrow \Delta inventories < 0 \Rightarrow Y \uparrow \Rightarrow Y^* > 2000$

1.1.7   What is the equilibrium level of output Y^*? Is it consistent with your guess in the previous part?

$Y^* = \frac{1}{1 - MPC}[\bar{C} + I + G - MPC(T - TR)]$
$Y^* = \frac{1}{1 - \frac{4}{5}}[175 + 200 + 250 - \frac{4}{5}(250)] = \frac{1}{\frac{1}{5}}(425) = 5(425) = 2125$
Since $Y^* = 2125 > 2000$, our guess in the previous part was correct.

1.1.8   Now, suppose that G is reduced to 200 units with T unchanged. What is the new equilibrium output level? Calculate $\frac{\Delta Y}{\Delta G}$. Is your result consistent with the government expenditure multiplier M_G computed in part (1.1.4)?

$Y^* = \frac{1}{1 - \frac{4}{5}}[175 + 200 + 200 - \frac{4}{5}(250)] = \frac{1}{\frac{1}{5}}(375) = 5(375) = 1875$
$\Delta Y = Y_{new}^* - Y_{old}^* = 1875 - 2125 = -250$
$\Delta G = -50$
$\frac{\Delta Y}{\Delta G} = \frac{-250}{-50} = 5$
$M_G = \frac{1}{1 - MPC} = \frac{1}{1 - \frac{4}{5}} = 5$
Since $\frac{\Delta Y}{\Delta G} = M_G = 5$ in this case, our result is consistent with the previous expression for the government expenditure multiplier.

1.1.9   Now, suppose that T is reduced by 20 units with $G = 250$ and TR unchanged. What is the new equilibrium output level? Calculate $\frac{\Delta Y}{\Delta T}$. Is your result consistent with the tax multiplier M_T computed in part **(1.1.4)**?

$Y^* = \frac{1}{1-\frac{4}{5}}[175 + 200 + 250 - \frac{4}{5}(230)] = \frac{1}{\frac{1}{5}}(441) = 5(441) = 2205$

$\Delta Y = Y^*_{new} - Y^*_{old} = 2205 - 2125 = 80$

$\Delta T = -20$

$\frac{\Delta Y}{\Delta T} = \frac{80}{-20} = -4$

$M_T = -\frac{MPC}{1-MPC} = -\frac{\frac{4}{5}}{1-\frac{4}{5}} = -4$

Since $\frac{\Delta Y}{\Delta T} = M_T = -4$ in this case, our result is consistent with the previous expression for the tax multiplier.

1.2 Aggregate Expenditure Model, Computational

Let's add an explicit dimension of time to the aggregate expenditure model. Here, subscript t stands for time, so all variables are now indexed by t. $\bar{C}$ and MPC are still treated as constants; they don't vary over time. Consider a closed economy. Implement the following AE model in Excel.

Aggregate expenditure:

$$AE_t = C_t + I_t + G_t \tag{2}$$

Output:

$$Y_{t+1} = C_t + I_t + G_t \tag{3}$$

Consumption:

$$C_t = \bar{C} + MPC[Y_t - (T_t - TR_t)] \tag{4}$$

Investment:

$$I_t = 0.15Y_t \tag{5}$$

Government expenditure:

$$G_t = 0.2(Y_t - TR_t) \tag{6}$$

Taxes:

$$T_t = 0.25I_t \tag{7}$$

Transfers:

$$TR_t = 0.1(I_t - T_t) \tag{8}$$

Initial conditions (at time $t = 0$):

$$Y_0 = 200$$

Constants:

$$\bar{C} = 20$$

$$MPC = 0.65$$

You should have the following columns in your Excel file: t, Y_t, C_t, I_t, G_t, T_t, and TR_t. Each row in the Excel file should be some t (year). The first row of your Excel file should label the columns. **You do not have to print out your tables, only your graphs (when asked to do so).**

(extended hint: "Manually enter the initial conditions into the second row ($t = 0$), then write Excel formulas for consumption, investment, government spending, taxes, and transfers. Enter the formula for output next period ($t = 1$) in the third row. Notice the timing: output at time $t+1$ depends on variables from time t, the previous year. Copy your formulas for C_t, I_t, G_t, T_t, and TR_t from the second row to the third. Finally, copy your formulas for all variables from the third row down until the model runs for 500 periods (until $t = 500$).")

1.2.1 Translate the AE model formulas above into Excel formulas in a spreadsheet, and then run the model for $t = 0, 1, \ldots, 500$.

See Excel file.

1.2.2   Graph Y_t, C_t, I_t, and G_t versus t (time) in a single graph, if possible. Print your graph(s) out. What trends do you observe?

All variables converge smoothly from below to their equilibrium (steady-state) levels. See Excel file for graphs.
$Y^* = 1035$; $C^* = 675$; $I^* = 155$; $G^* = 205$.

1.2.3   Based on your answer to the previous part, which statement is correct: **(1)** $Y_0 < Y^*$; **(2)** $Y_0 = Y^*$; **(3)** $Y_0 > Y^*$?

Statement (1), $Y_0 < Y^*$, is correct since $Y_0 = 200 < Y^* = 1035$.

1.2.4   Graph T_t, TR_t, and $S_{public,t} = T_t - G_t - TR_t$ versus t (time) in a single graph, if possible. Print your graph(s) out. What trends do you observe? Describe the government budget balance over time.

See Excel file for graphs. Taxes and transfers gradually increase to steady-state, while public savings rapidly falls. The government continually runs a budget deficit from $t = 0$ to $t = 500$.
$T^* = 39$; $TR^* = 12$; $S^*_{public} = -177$.

1.2.5   Graph $S_{public,t}$, $S_{private,t} = Y_t - (T_t - TR_t) - C_t$, and $NS_t = Y_t - C_t - G_t$ versus t (time) in a single graph, if possible. Print your graph(s) out. What trends do you observe? Describe household saving behavior over time.
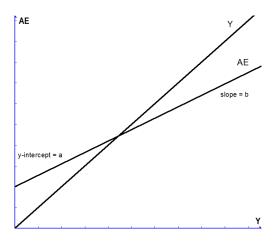
See Excel file for graphs. Public savings is negative and decreasing from $t = 0$ to $t = 500$. Private savings is positive and increasing from $t = 0$ to $t = 500$. As a result, national savings is always positive and increasing. On net, households are saving, not borrowing.
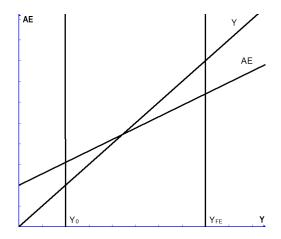*$S^*_{private} = 333$; $NS^* = 155$.*

1.2.6   Define the household savings rate as $s_t = \frac{S_{private,t}}{Y_t}(100)$ and the national savings rate as $s'_t = \frac{NS_t}{Y_t}(100)$. Graph s_t and s'_t versus t in a single graph, if possible. Print your graph(s) out. What trends do you observe?

See Excel file for graphs. Both the household and national savings rates are increasing from $t = 0$ to $t = 500$.
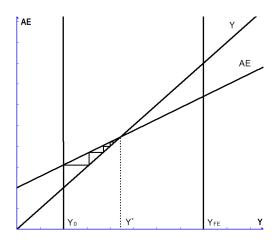*$s^* = 32.15\%$; $(s')^* = 15\%$.*

1.3 Mechanics of the Aggregate Expenditure Model

1.3.1 Assume an aggregate expenditure function of form $AE(Y) = a + bY$, where $a > 0$ and $0 < b < 1$. Draw a graph of the aggregate expenditure model that includes the $45°$ line and the aggregate expenditure function in **AE** versus **Y** space. Your graph should be labeled clearly and completely.

1.3.2 Assume that initial output level Y_0 is less than the equilibrium level of output Y^* $(Y_0 < Y^*)$. Also, let's say that the full employment level of output Y_{FE}, associated with the natural (full employment) level of unemployment, is such that $Y^* < Y_{FE}$. Reproduce your graph in the previous part, adding vertical lines denoting Y_0 and Y_{FE}.
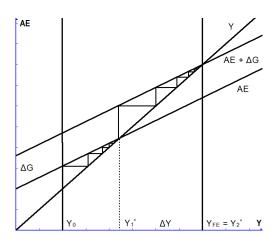
1.3.3   Briefly describe how the economy transitions from initial output Y_0 to equilibrium output Y^*. Your answer should talk about how the following economic variables are affected: (1) the level of employment; (2) the level of inventories; (3) unexpected changes in inventories; and (4) the level of output.

The equilibrium level of output Y^* is labeled on the graph as the point where the aggregate expenditure and output lines cross. This crossing point is guaranteed to exist because: (1) the y-intercept of the AE line $(a > 0)$ is greater than the y-intercept of the Y line (zero); and (b) the slope of the AE line $(0 < b < 1)$ is less than the slope of the Y line (one).

(1) The level of employment gradually increases as output goes from Y_0 to Y^*.

(2) The level of inventories decreases over the entire transition period as $\Delta inventories$ is consistently negative $(Y < AE)$.

(3) The unexpected changes in inventories are repeatedly negative due to excess demand: supply Y (output) is less than demand AE (aggregate expenditure) provided that $Y < Y^*$. Excess demand is elminated only when $Y = Y^*$ in equilibrium.

(4) The level of output gradually increases as the economy transitions from Y_0 to Y^*.

1.3.4 The government wants to attain the full employment level of output, Y_{FE}, in the short-run by adjusting the level of government expenditure G. Based on your diagram from part (1.3.2), how should the government change G to achieve their goal? On your graph, show the effects of an appropriate change in G. Label ΔG and ΔY as a result of the new fiscal policy. What is the new level of output?

The new level of output, in equilibrium, is $Y_2^* = Y_{FE}$.

1.3.5 Based on your answer to the previous part, which statement is correct: (1) $\Delta Y < \Delta G$; (2) $\Delta Y = \Delta G$; (3) $\Delta Y > \Delta G$? What is the government spending multiplier $M_G = \frac{\Delta Y}{\Delta G}$ in this case?

Statement (3) is correct; $\Delta Y > \Delta G$ (this can be seen graphically) due to the multiplier-accelerator mechanism in the aggregate expenditure model. The government spending multiplier $M_G = \frac{\Delta Y}{\Delta G} = \frac{1}{1-MPC} = \frac{1}{1-b} > 1$, where b is the slope of the aggregate expenditure function.

1.3.6 When the government implements their new fiscal policy, what happens to the economy in terms of unexpected changes in inventories and output? What if businesses anticipate the government's change in fiscal policy?

The unexpected changes in inventories during the transition are consistently negative due to excess demand $(AE > Y)$ generated by the increase in government spending. Again, output gradually increases to its new equilibrium level from Y_1^* to Y_2^*. If businesses anticipate the government's policy, they will increase output levels immediately and the transition will occur more rapidly.

2 AD / AS Model

2.1 Aggregate Demand / Aggregate Supply Model

Consider the AD / AS model with a horizontal SRAS curve and a vertical LRAS curve. The Federal Reserve implements monetary policy by adjusting the money supply (M).

Long-run aggregate supply curve (vertical):

$$Y_{LR} = 500 \tag{9}$$

Short-run aggregate supply curve (horizontal):

$$P_{SR} = 6 \tag{10}$$

Aggregate demand curve:

$$Y = \frac{4M}{P} \tag{11}$$

Money supply:

$$M = 750 \tag{12}$$

2.1.1 If the economy is initially in long-run equilibrium, what are the values of P^*_{LR} and Y^*_{LR}?

Long-run equilibrium $\Rightarrow Y_{LR} = \frac{4M}{P_{LR}} = \frac{4(750)}{P_{LR}} = \frac{3000}{P_{LR}}$

$Y^*_{LR} = 500$ (LRAS curve)

$P^*_{LR} = \frac{3000}{Y^*_{LR}} = \frac{3000}{500} = 6$ (AD curve)

Long-run equilibrium: $(P^*_{LR}, Y^*_{LR}) = (6, 500)$

2.1.2 Now suppose a supply shock moves the short-run aggregate supply curve to $P_{SR} = 4$ (still horizontal). What is the new short-run equilibrium (P^*_{SR}, Y^*_{SR})?

Short-run equilibrium $\Rightarrow Y_{SR} = \frac{4M}{P_{SR}} = \frac{4(750)}{P_{SR}} = \frac{3000}{P_{SR}}$

$P^*_{SR} = 4$ (SRAS curve)

$Y^*_{SR} = \frac{3000}{P^*_{SR}} = \frac{3000}{4} = 750$

Short-run equilibrium: $(P^*_{SR}, Y^*_{SR}) = (4, 750)$

2.1.3 If the aggregate demand and long-run aggregate supply curves are unchanged, what is the new long-run equilibrium (P^*_{LR}, Y^*_{LR}) after the supply shock?

Long-run equilibrium $\Rightarrow Y_{LR} = \frac{4M}{P_{LR}} = \frac{4(750)}{P_{LR}} = \frac{3000}{P_{LR}}$

$Y^*_{LR} = 500$ (LRAS curve)

$P^*_{LR} = \frac{3000}{Y^*_{LR}} = \frac{3000}{500} = 6$ (AD curve)

Long-run equilibrium: $(P^*_{LR}, Y^*_{LR}) = (6, 500)$

2.1.4 Suppose that after the supply shock, the Federal Reserve wants to hold output at its long-run level. What level of the money supply M' would be required to achieve this in the short-run?

Monetary policy goal: $Y_{SR}^* = Y_{LR}^* = 500$
Short-run equilibrium $\Rightarrow Y_{SR} = \frac{4M}{P_{SR}}$
$P_{SR}^* = 4$ *(SRAS curve)*
$M' = \frac{Y_{SR}^* P_{SR}^*}{4} = \frac{(500)(4)}{4} = 500$
$M' = 500 \Rightarrow$ *Short-run equilibrium:* $(P_{SR}^*, Y_{SR}^*) = (4, 500)$

2.1.5 <u>From this part forward</u>, consider the generic aggregate demand - aggregate supply model with three curves: AD, SRAS, and LRAS. Assume that the economy starts in long-run equilibrium. Additionally, assume that the short-run aggregate supply curve is horizontal in P versus Y space. In a carefully labeled graph with these three curves, diagram the effect (both short-run and long-run) of an increase in expected future corporate profits. Your graph should identify two equilibrium points, short-run equilibrium (P_{SR}^*, Y_{SR}^*) and long-run equilibrium (P_{LR}^*, Y_{LR}^*). Briefly describe how the economy transitions from short-run to long-run equilibrium. Draw arrows along the transition path.

Higher expected future corporate profits $\Rightarrow I \uparrow \Rightarrow$ *AD curve shifts to the right (positive AD shock)* $\Rightarrow P$ *unchanged,* $Y \uparrow$ *in short-run equilibrium. In the long-run, the short-run aggregate supply curve shifts up as unemployment falls, wages increase, and the cost of production (through the cost of labor) increases. Relative to short-run equilibrium,* $P \uparrow$ *and* $Y \downarrow$ *in long-run equilibrium.*
Initial conditions (in long-run equilibrium before the shock): $(P_0^*, Y_0^*) = (P_0, Y_0)$
Short-run equilibrium: $(P_{SR}^*, Y_{SR}^*) = (P_0, Y_1)$, $Y_1 > Y_0$
Long-run equilibrium: $(P_{LR}^*, Y_{LR}^*) = (P_1, Y_0)$, $P_1 > P_0$
The transition path is along the SRAS curve from (P_0, Y_1) *to* (P_1, Y_0).

2.1.6 Now assume that the short-run aggregate supply curve is upward-sloping in P versus Y space, but not vertical, with a slope near one. Continue to assume that the economy starts in long-run equilibrium. Repeat (2.1.5), but diagram the effect of an unexpected increase in the price of oil.

Unexpected increase in the price of oil $\Rightarrow$ *increased cost of production* $\Rightarrow$ *SRAS curve shifts to the left (negative SRAS shock)* $\Rightarrow P \uparrow$, $Y \downarrow$ *in short-run equilibrium. In the long-run, the short-run aggregate supply curve shifts down (to the right) as unemployment increases, wages decrease, and the cost of production (again, through labor costs) decreases. Relative to short-run equilibrium,* $P \downarrow$ *and* $Y \uparrow$ *in long-run equilibrium. In the long-run, the economy returns to its initial point in P versus Y space.*
Initial conditions (in long-run equilibrium before the shock): $(P_0^*, Y_0^*) = (P_0, Y_0)$
Short-run equilibrium: $(P_{SR}^*, Y_{SR}^*) = (P_1, Y_1)$, $P_1 > P_0$, $Y_1 < Y_0$
Long-run equilibrium: $(P_{LR}^*, Y_{LR}^*) = (P_0, Y_0)$
The transition path is along the SRAS curve from (P_1, Y_1) *to* (P_0, Y_0).