## Economics 102

Summer 2017
Answers to Homework \#1

## Due 6/1/17

Directions: The homework will be collected in a box before the lecture. Please place your name, TA name and section number on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Please remember the section number for the section you are registered, because you will need that number when you submit exams and homework. Late homework will not be accepted so make plans ahead of time. Please show your work. Good luck!

Please remember to

- Staple your homework before submitting it.
- Do work that is at a professional level: you are creating your "brand" when you submit this homework!
- Not submit messy, illegible, sloppy work.

1. This set of questions will help you review some basic algebra, the slope-intercept form, finding a solution given two linear equations, and finding a new equation based upon an initial equation that has undergone a change. Each question below is independent of the other questions in the set.
a. You are given two pairs of coordinates that have a linear relationship. The two pairs of coordinates are $(x, y)=(20,40)$ and $(30,10)$. You are asked to find the equation for the line that these two points lie on.
b. You are given two pairs of coordinates that have a linear relationship. The two pairs of coordinates are $(x, y)=(7,14)$ and $(14,28)$. You are asked to find the equation for the line that these two points lie on.
c. You are given two equations:

Equation 1: w = $50+2 z$
Equation 2: $w=60-3 z$
Find the ( $\mathrm{z}, \mathrm{w}$ ) solution that represents the intersection of these two lines.
d. You are given two equations where $P$ is the variable measured on the $y$-axis (this is like our renaming $y$ to be "P") and $Q$ is the variable measured on the $x$-axis (this is like our renaming $x$ to be " Q "):

Equation 1: $P=48-2 Q$
Equation 2: $P=24+4 Q$
i. Find the $(Q, P)$ solution that represents the intersection of the given lines.

Now, you are also told that equation 1 has changed and now the $Q$ value is 24 units smaller at every P value than it was initially.
ii. Write the equation that represents the new Equation1'.
iii. Given the new Equation 1' and Equation 2, find the (Q', P') solution that represents the intersection of these two lines.

## Answer:

a. Start by finding the slope of the equation using the two points: slope $=$ (change in $y) /($ change in $x)=(40-10) /(20-30)=[(30) /(-10)]=-3$. Then, use the slope-intercept form, $\mathrm{y}=\mathrm{mx}+\mathrm{b}$, to find the equation for the line. Thus, $\mathrm{y}=(-3) \mathrm{x}+\mathrm{b}$. Then, plugging in one of the given point-in this case, let's use $(20,40)$ we get $40=(-3)(20)+b$ or $b=100$. The equation is therefore $\mathrm{y}=(-3) \mathrm{x}+100$.
b. Start by finding the slope of the equation using the two points: slope $=$ (change in $y) /($ change in $x)=(28-14) /(14-7)=14 / 7=2$. Then, use the slope-intercept form, $\mathrm{y}=\mathrm{mx}$ $+b$, to find the equation for the line. Thus, $y=(2) x+b$. Then, plugging in one of the given points-in this case, let's use $(7,14)$ we get $14=(2)(7)+b$ or $b=0$. The equation is therefore $y=(2) x$.
c. To find where these two lines intersect set the two equations equal to one another:
$50+2 \mathrm{z}=60-3 \mathrm{z}$
$5 z=10$
$\mathrm{z}=2$
$\mathrm{w}=50+2 \mathrm{z}=50+2(2)=54$
Or, $w=60-3 z=60-3(2)=54$
$(\mathrm{z}, \mathrm{w})=(2,54)$
d.
i. To find the point of intersection of the two given lines $(\mathrm{Q}, \mathrm{P})$ we need to set the two equations equal to one another.
$48-2 Q=24+4 Q$
$24=6 \mathrm{Q}$
Q = 4
$P=48-2 Q=48-2(4)=40$
Or, $\mathrm{P}=24+4 \mathrm{Q}=24+4(4)=24+16=40$
$(\mathrm{Q}, \mathrm{P})=(4,40)$
ii. We know that $(0,48)$ was on the original line represented by Equation 1; the new Equation 1' would contain the point $(24,48)$ since the $Q$ value at every $P$ value has increased by 24 units. The slope of Equation 1 ' is the same as the slope of Equation 1. Thus, $\mathrm{P}^{\prime}=\mathrm{b}^{\prime}-2 Q^{\prime}$ where $\mathrm{b}^{\prime}$ is the $y$-intercept of the new Equation $1^{\prime}$. Use the point $(24,48)$ to find the value of $b^{\prime}$. Thus, $48=b^{\prime}-2(24)$ or $b^{\prime}=96$. The equation for Equation $1^{\prime}$ is $P=-2 Q+96$. iii. To find where Equation $1^{\prime}$ and Equation 2 intersect set the two equations equal to one another:
96-2Q' = $24+4 Q^{\prime}$
$6 Q^{\prime}=72$
$Q^{\prime}=12$
$\mathrm{P}^{\prime}=96-2 \mathrm{Q}^{\prime}=96-2(12)=92-24=72$
Or, $\mathrm{P}^{\prime}=24+4 \mathrm{q}^{\prime}=24+4(12)=24+48=72$
( $Q^{\prime}, P^{\prime}$ ) $=(12,72)$
2. The price of money is called the interest rate. Suppose that when the interest rate is $6 \%$, the demand for money is $\$ 14,000$ and when the interest rate is $10 \%$ the demand for money
is $\$ 10,000$. Assume the relationship between the quantity of money demanded $(\mathrm{Q})$ and the interest rate (r) is linear.
a. Draw a graph representing the above information. In your graph measure $Q$ on the horizontal axis and $r$ on the vertical axis.
b. Write an equation for this relationship in slope-intercept form.

Answer:
a.

Interest Rate, $r$
b. We know that the two points $(Q, r)=(14,000,6)$ and $(10,000,10)$ both sit on this line. We also know the slope intercept form $\mathrm{y}=\mathrm{mx}+\mathrm{b}$ and we can rewrite this general formula using our two variables as $r=m Q+b$ since the interest rate, $r$, is the variable measured on the vertical axis and the quantity of money demanded, Q , is the variable measured on the horizontal axis. We then need to calculate the slope of this line using the two given points:
$m=(6-10) /(14,000-10,000)=(-4 / 4000)=(-1 / 1000)$
$r=b+(-1 / 1000) Q$
To find the value of " b ", substitute in the coordinates of a point that you know is on this
line: $(\mathrm{Q}, \mathrm{r})=(10,000,10)$
$10=b+(-1 / 1000)(10,000)$
b $=20$
$r=(-1 / 1000) Q+20$ is the equation for this relationship.
3. Veronica's wealth on June 30, 2014 was $\$ 30,000$. On June 30,2015 her wealth was equal to $\$ 36,000$. On June 20,2016 her wealth was $\$ 30,000$. Use this information to answer this next set of questions. For this set of questions assume there was no inflation during this three year period of time.
a. What was the percentage change in Veronica's wealth between June 30, 2014 and June 30, 2015?
b. What was the percentage change in Veronica's wealth between June 30, 2014 and June 30, 2016?
c. What was the percentage change in Veronica's wealth between June 30, 2015 and June 20, 2016?
d. Given that in both (a) and (c) you are measuring percentage changes and the numbers in both examples use $\$ 30,000$ and $\$ 36,000$, do you get the same answers? Explain your answer.

Answers:
a. Percentage change in Veronica's wealth from June 30, 2014 to June 30, $2015=[(\mathrm{New}$ wealth-Initial wealth)/(Initial wealth) $[*(100 \%)]=[(36,000-30,000) /(30,000)] * 100 \%=$ $(.2)(100 \%)=20 \%$. Veronica's wealth increased by $20 \%$ between June 30, 2014 and June 30, 2015.
b. Percentage change in Veronica's wealth from June 30, 2014 to June 30, $2016=[(\mathrm{New}$ wealth-Initial wealth)/(Initial wealth) $[*(100 \%)]=[(30,000-30,000) /(30,000)]^{*} 100 \%=$ $(0)(100 \%)=0 \%$. Veronica's wealth did not change between June 30, 2014 and June 30, 2016.
c. Percentage change in Veronica' wealth from June 30, 2015 to June 30, $2016=[(\mathrm{New}$ wealth-Initial wealth $) /($ Initial wealth $)\left[{ }^{*}(100 \%)\right]=[(30,000-36,000) /(36,000)]^{*} 100 \%=(-$ $6000 / 36,000)(100 \%)=(-1 / 6)(100 \%)=-16.67 \%$. Veronica's wealth decreased by $16.67 \%$ between June 30, 2015 and June 30, 2016.
d. Even though both (a) and (c) are measuring percentage changes and they both use $\$ 30,000$ and $\$ 36,000$ you do not get the same answers. That is because the base value, or the initial value, is different in the two questions. The choice of base matters in measuring percentage changes.
4. Marco, Hans, and Ruby produce brownies (B) and cookies (C). They all have linear production possibility frontiers. Marco knows that he can produce $(C, B)=(20,30)$ and $(30$, 15). Hans know that the maximum amount of brownies he can produce is 40 and the maximum amount of cookies he can produce is 10 . Ruby knows that she must give up two cookies for every brownie she produces. Ruby is currently producing 16 brownies and 8 cookies.
a. Using the above information answer the following questions assuming that each individual is producing on their production possibility frontier.
i. When Marco produces 51 brownies, his cookie production must equal $\qquad$ .
ii. When Marco produces 36 cookies, his brownie production must equal $\qquad$
iii. When Hans produces 3 cookies, his brownie production must equal $\qquad$ _.
iv. When Hans produces 14 brownies, his cookies production must equal $\qquad$ . v. The equation for Ruby's PPF can be written as $\qquad$ .
b. For Marco, the opportunity cost of producing an additional 4 brownies is equal to
c. For Hans, the opportunity cost of producing an additional 4 cookies is equal to $\qquad$ .
d. Who has the comparative advantage in producing brownies? Who has the comparative advantage in producing cookies? Explain your answer.
e. Construct Marco, Hans, and Ruby's joint PPF measuring cookies (C) on the horizontal axis and brownies (B) on the vertical axis.
f. Write the equation and the range for each segment of the joint PPF.
g. Use the number line approach to illustrate the acceptable range of trading prices in terms of cookies for one brownie for Marco, Hans and Ruby. Be thoughtful here! Label your answer clearly and completely.

Answers:
a. Start by constructing the individual PPFs.

i. The equation for Marco's PPF can be written as $B=60-(3 / 2) C$. So if Marco is producing $B=51$, then $C$ must equal 6 cookies.
ii. The equation for Marco's PPF can be written as $B=60-(3 / 2) C$. If Marco is producing $\mathrm{C}=36$, then B must equal 6 brownies.
iii. The equation for Han's PPF can be written as $B=40-4 C$. If Hans is producing $C=$ 3, then B must equal 28 brownies.
iv. The equation for Han's PPF can be written as $B=40-4 C$. If Hans is producing $B=$ 14 , then C must equal 6.5 cookies.
v . We know that Ruby's opportunity cost of producing 1 brownie is 2 cookies. We can use this information to find Ruby's opportunity cost of producing 1 cookie: Ruby's opportunity cost of producing 1 cookie will be $1 / 2$ brownie. Since we are measuring brownies on the vertical axis, this tells us that the slope of the PPF is ($1 / 2)$. We also know that the point $(C, B)=(8,16)$ sits on Ruby's linear PPF. We can write the equation for her PPF now since we have its slope and a point: $\mathrm{B}=20$ (1/2)C.
b. The opportunity cost of Marco producing an additional 4 brownies is equal to $8 / 3$ cookies.
c. For Hans the opportunity cost of producing an additional 4 cookies is equal to 16 brownies.
d. Start by looking at the slopes of the three individual PPFs. The slope of Marco's PPF is $3 / 2$ when brownies are measured on the vertical axis. The slope of Hans' PPF is -4 . The slope of Ruby's PPF is $-1 / 2$. Since Ruby has the lowest opportunity cost of producing 1 cookies she has the comparative advantage in producing cookies. Since Hans has the greatest opportunity cost of producing cookies this means that he must have the comparative advantage in producing brownies.
e. Here is the joint PPF:

f. The top segment of the joint PPF is $B=120-(1 / 2) C$ for $100 \leq B \leq 120$.

The middle segment takes more work. We can look at the graph and see that the slope of the segment is $-3 / 2$. We then use one of our points that we know is on this segment to solve for the $y$-intercept of this equation. We know $(C, B)=(40,100)$ and $(80,40)$ both sit on this segment. Thus, $B=(-3 / 2) C+b$ and using one of our points: $100=(-3 / 2)(40)+b$ and thus, $b$ $=160$. The equation for the middle segment is therefore $B=160-(3 / 2) C$ for $40 \leq B \leq 100$.

The final segment uses the same process. The slope of this segment is -4 . And we know that $(C, B)=(80,40)$ and $(90,0)$ both sit on this segment. Thus, $B=(-4) C+b$ and using one of our points: $0=(-4)(90)+b$ and thus, $b=360$. The equation for the lowest segment of the joint PPF is therefore $B=360-4 C$ for $40 \leq B \leq 0$.
g. Here is the number line illustrating the range of trading prices for 1 brownie: Trade takes place when price is above the production cost to the seller but below the production cost to the buyer.


