Economics 102
Fall 2015
Answers to Homework #1
Due Monday, September 21, 2015

Directions:

- The homework will be collected in a box **before** the large lecture.
- Please place your name, TA name and section number on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade.
- Late homework will not be accepted so make plans ahead of time. **Please show your work.** Good luck!

Please realize that you are essentially creating "your brand" when you submit this homework. Do you want your homework to convey that you are competent, careful, professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional. For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you do any work for someone else!

1. Math Review Question:

a. Find the solution (x,y) that satisfies both of the following equations:
Equation 1: $2x + 3y = 13$
Equation 2: $3x - 2y = 0$

Answer:
There are many ways to find this solution: here are two possible methods.

Method I: Multiplying equation 1 by 3 and equation 2 by 2 we get $6x + 9y = 39$, $6x - 4y = 0$.
Subtracting the two equations gives the following equation with respect to y: $13y = 39$. Therefore $y = 3$. Then we can find an equation with respect to x by substituting this value into Equation 1. $2x + 3(3) = 13$, or $2x + 9 = 13$. Therefore $x = 2$.The solution is therefore $(x, y) = (2, 3)$.

Method II: alternatively, you can solve equation 1 for x:
$2x = 13 – 3y$
$x = (13 – 3y)/2$
and then substitute this equation for x in the second equation:
$3[(13 – 3y)/2] – 2y = 0$
Multiply both sides of the equation by 2 to get rid of the denominator in the first term:
$3(13 – 3y) – 4y = 0$
Now, simply and solve for y:
$39 – 9 y – 4y = 0$

$39 = 13y$
$y = 3$
Use this value of y in either equation to find the value of x: $2x + 3(3) = 13$ or $x = 2$. Or, $3x – 2(3) = 0$ or $x = 2$. Thus, $(x, y) = (2, 4)$ is the solution.

b. Suppose that both Equation 1 and 2 have both shifted to the right. For each equation the shift is the same size and can be described as follows: for every initial value of the y variable the new x variable is 2 units larger than the initial x value. (Hint: you may find it helpful to draw a sketch of these two lines on separate graphs to see how the new line compares to the original line.)

i) Write down the new equations that represent these described changes.
ii) Given the new equations, find the solution (x,y) that represents the intersection of these two lines.
iii) Then, compare the initial intersection point to the new intersection point.

Answer:
i) Let (x,y) be any point that satisfies the original Equation 1, $2x+3y=13$. After the changes, let's denote any point that satisfies the new Equation 1 as (X,Y). Clearly, it follows from the condition that $X=x+2$ and $Y=y$, which can be changed to $x=X-2$ and $y=Y$. Substituting this x and y into the original Equation 1, we get $2(X-2)+3Y=13$. It can be simplified to $2X+3Y=17$.
Likewise, any point in the new Equation 2 (X,Y) satisfies $X=x+2$ and $Y=y$, or $x=X-2$ and $y=Y$. Substitute these into the old Equation 2, $3x-2y=0$, we get $3(X-2)-2Y=0$, which simplifies to $3X-2Y=6$.

ii) Now we can represent new equations as below.
Equation 1: $2x + 3y = 17$
Equation 2: $3x - 2y = 6$

Multiplying equation 1 by 3 and equation by 2 we get $6x + 9y = 51$, $6x - 4y = 12$. Subtracting the two equations gives the following equation with respect to y: $13y = 39$. Therefore $y = 3$. Then we can find an equation with respect to x by substituting $y = 3$ into Equation 1. $2x + 3(3) = 17$, or $2x + 9 = 17$. Therefore $x = 4$. The solution for the point of intersection of these two new lines is $(X, Y) = (4, 3)$.

iii) Since in both equations only the x value changes and the x value changes by the same amount for each equation, the new X value in the solution includes the same amount of change while the new Y value in the solution stays at its initial level. This result is well worth taking the time to sketch out so that you see the underlying idea!

c. Consider the two equations below where P is measured on the vertical axis and Q is measured on the horizontal axis:

$P = 8 - 2Q$ for $0 \leq Q$

$P = 2Q$ for $0 \leq Q$

Calculate the equilibrium values for (Q, P). Then, calculate the area that sits between these two equations for all positive values of Q less than or equal to the equilibrium value of Q. (Hint: This area is a triangle and you might find it helpful to draw a sketch of the two lines, their solution, and the area you are calculating.)

Answer:

Let's start by finding the solution (Q, P) or where these two lines intersect.

$8 – 2Q = 2Q$

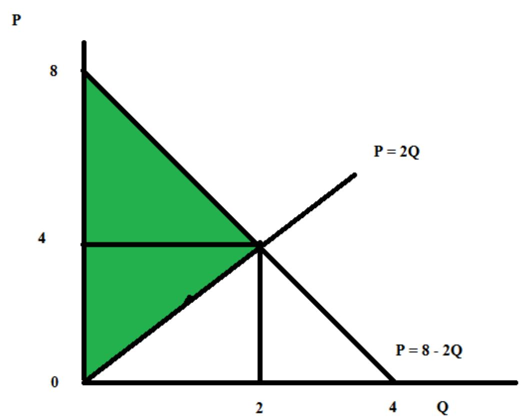
$8 = 4Q$

$Q = 2$

Use this value and either equation to find P:

$P = 8 – 2(2) = 4$ or $P = 2(2) = 4$. The two lines intersect at (Q, P) = (2, 4).

From the equations we know that the y-intercept for the first equation is at (0, 8) and for the second equation is at (0, 0). Here is a graph illustrating these two lines.



The base of the shaded triangle (in green) is 8, which is the distance between the origin and the y-intercept of P = 8 - 2Q. The height of the shaded triangle is the Q value of the intersection point. Using the formula for the area of a triangle, we have area of triangle = (1/2)(base)(height) – (1/2)bh = 0.5 * 2 * 8 = 8.

2. Math Review Question:

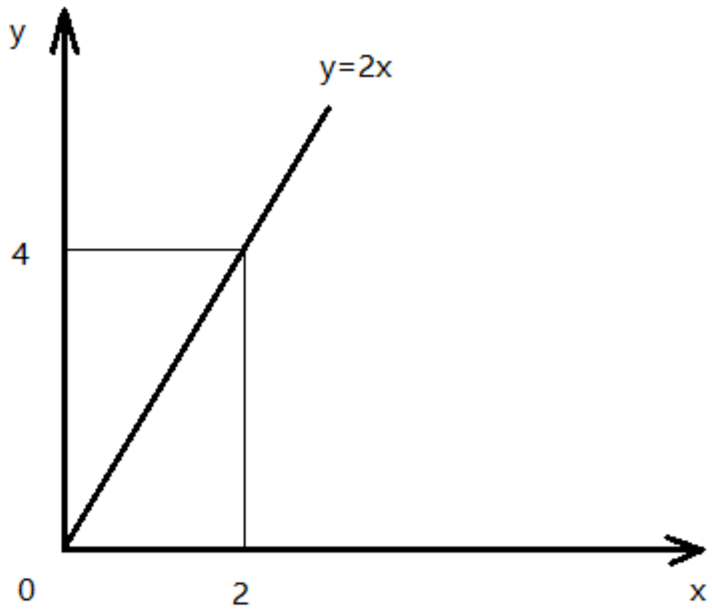
For this question assume you are working with a linear equation that takes the general form of $y = ax + b$ where y is the variable measured on the vertical axis, x is the variable measured on the horizontal axis, "a" is the slope of the equation, and "b" is the y-intercept for the equation.

a. Write the equation for the straight line that passes though the origin and and the point (2, 4). In your answer show the work you did to find this equation. Then graph this equation. In your equation, what is the value of the slope of the equation and the value of the y-intercept.

Answer:
From the information we know that this line passes through (0, 0) and (2, 4). We also know that this is a straight line due to the slope-intercept form we have been given: y = ax + b. Since the line passes through the origin, we know that the y-intercept for the line must by (0, 0). We can see that the value of "b" is zero by substituting this point into the slope-intercept form: substituting (0, 0) in this equation, we get 0 = a*0 + b = b, so b = 0. To find the value of the slope, "a", we can then substitute the point (2, 4) into this equation: we get 4 = a*2 + b. Since b = 0, 4 = 2a, a = 2.
The equation in slope-intercept form given this information is: y = 2x.

Alternatively, you can calculate the slope of this line using the two points:
Slope = (change in Y)/(change in X) = (0 – 4)/(0 – 2) = 2.

The graph:

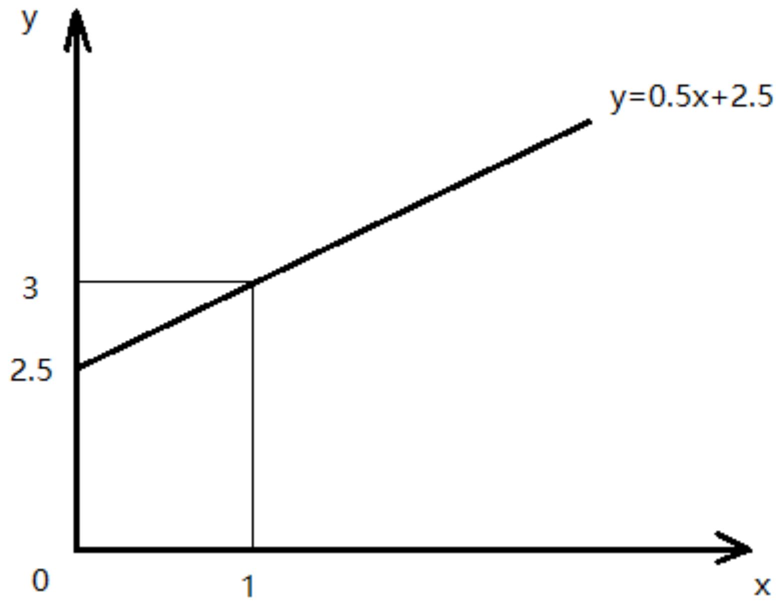


b. Write the equation for the straight line that passes through the points (x, y) = (1, 3) and (3, 4). Show the work you did to find this equation. Graph this line. What are the values of the y-intercept and slope for this line?

Answer:
From the two given points we can calculate the value of the slope:
Slope = (change in y)/(change in x) = (3 – 4)/(1 – 3) = 1/2
Then, we can write the equation as:
y = (1/2)x + b
To find the value of b we can use either point and this equation: for (1, 3) we get

y = (1/2)x + b
3 = (1/2)(1) + b
b = 2.5
For (3, 4) we get
y = (1/2)x + b
4 = (1/2)(3) + b
b = 2.5
We can therefore write the equation as: y = (1/2)x + 2.5
The slope of this equation, "a", is (1/2) and the y-intercept, "b", is 2.5.

Graph:



c. Consider the two lines you found in (a) and (b). Do these two lines intersect one another? If so, what are the coordinate values (x, y) for this point of intersection for the two lines. Show your work.

Answer:
Since the two equations have different slope values, the two lines represented by these equations are not parallel and therefore must intersect. Suppose they intersect at point (x, y), then we can use these two equations to solve for that point of intersection:

y = 2x
y = 0.5x + 2.5

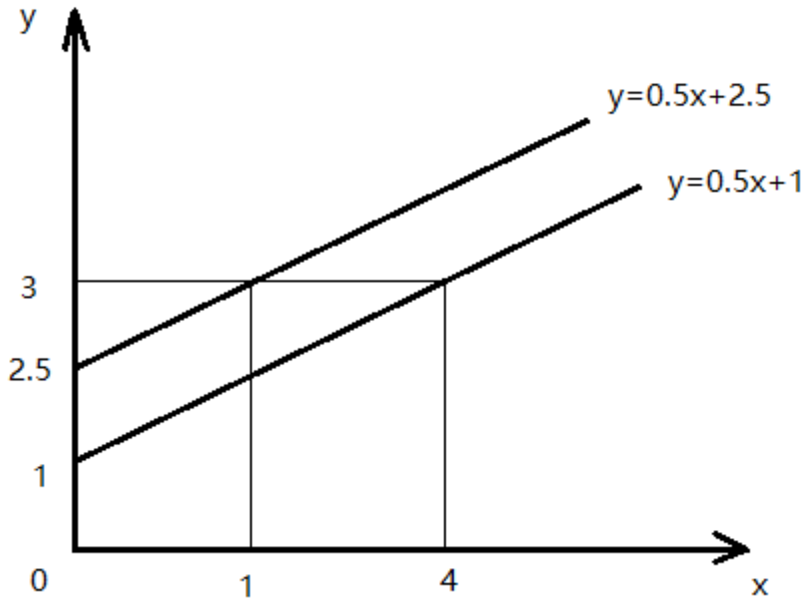
Setting these two equations equal to one another, we have:

2x = 0.5x + 2.5

Solving this equation we get x = 2.5/1.5 = 5/3
y = 2x = 2*(5/3) = 10/3
So the intersection point is (x, y) = (5/3, 10/3).

d. Suppose the line you found in (b) is shifted horizontally to the right by 3 units. Given this information, write the equation for this new line? Show your work. Then, draw a graph that represents the initial line as well as this new line.

Answer:
The function in (b) is y = 0.5x + 2.5, passing through point (1, 3).
After it shifts horizontally to the right by 3 units, the new line will pass through the point (4, 3) and be parallel to the initial line. That is, the slope of the new line does not change from the slope of the initial line, so the value of the slope remains at 0.5.
The equation of the new line still has the form y = 0.5x + b. Substitute in (4, 3) into this general form to find the y-intercept value: thus, we have 3 = 0.5*4 + b or 3 = 2 + b. So b = 1. The equation for this new line can thus be written as y = 0.5x + 1.
Graph:



e. Can you draw a single straight line that passes through all three of the following three points:

(X, Y) = (1, 3), (2, 4), and (4, 5)

Explain your answer fully.

Answer:
Draw a line segment between (1, 3) and (2, 4). The slope of the line is (4-3)/(2-1)=1.
Draw a line segment between (2, 4) and (4, 5). The slope of the line is (5-4)/(4-2)=0.5.
Since the two line segments do not have the same slope, they cannot be part of the same line. Therefore, one cannot draw a straight line though all three points.

3. Percentage change:

James took a math course that had three exams. He got a 30 out of 75 points on the first exam, and a 50 out of 75 points on the second. On the last exam, he scored 80 out of 100 points.

a. What is the percentage change in score from the first to the second exam? Show your work and provide any formulas you use in your calculation.

Answer:
The percentage change of a value can be calculated as follows:
Percentage change in value = [(New Value – Initial Value)/(Initial Value)](100%)
Percentage change in value = [(50/75 – 30/75)/(30/75)](100%)
Percentage change in value = [(20/75)/(30/75)](100%)
Percentage change in value = 66.67%

b. What is James's grade on the third exam if that score is converted to a 75 point scale? Show your work and explain your reasoning (this means include a verbal response!).

Answer:
When converting the point scale, the previous score ratio and the changed ratio should be the same. Let's denote his third grade in terms of a 75 point scale as x. Then the following equation holds.
80/100=x/75.
x can be calculated as x=75*80/100=60

c. What are James's grades on the first and second exam if those scores are converted to a 100 point scale? Show your work and explain your reasoning (this means include a verbal response!).

Answer:
By the same reasoning as in (b), the first grade x in terms of a 100 point scale satisfies the following equation.
30/75=x/100. So x can be calculated as x=100*30/75=40
Likewise, the second grade in terms of 100 point scale is 100*50/75=66.67

d. What is the percentage change in score from the first to the third exam if using a 100 point scale for both exam scores? When you use 75 point scale instead of 100 point scale, is the percentage change result different? Show your work and explain your reasoning (this means include a verbal response!).

Answer:
Intuitively you should think: "hmmm, the choice of scale should **NOT** matter because if the choice of scale did matter, it would mean that every student ought to go see each of their instructors to argue about the optimal scale to use when calculating their

particular grade." So, let's check out the details on this intuitive response.

Using the 100 point scale, the first and the third grades are 40 and 80, respectively. The percentage change is (80 - 40)/40*100% = 100%. In terms of the 75 point scale, the first and the third grades are 30 and 60, respectively. The percentage change in this case is (60 - 30)/30*100% = 100%. Both results are the same. Changing the scale does **not** affect the percentage change.

4. PPF and comparative advantage: In his famous book *The Wealth of Nations*, published in 1776, Adam Smith illustrated the benefit of international trade using the example of two countries, England and Portugal. Assume, hypothetically, that in a given year England could produce woolen cloth and wine in the following combinations:

England	Cloth (in pieces)	Wine (in bottles)
Combination 1	10	5
Combination 2	20	0

Portugal, benefitting from a more pleasant climate, could produce:

Portugal	Cloth (in pieces)	Wine (in bottles)
Combination 1	15	15
Combination 2	20	10

For this set of questions assume that both countries face constant opportunity cost and therefore both countries have linear production possibility frontiers.

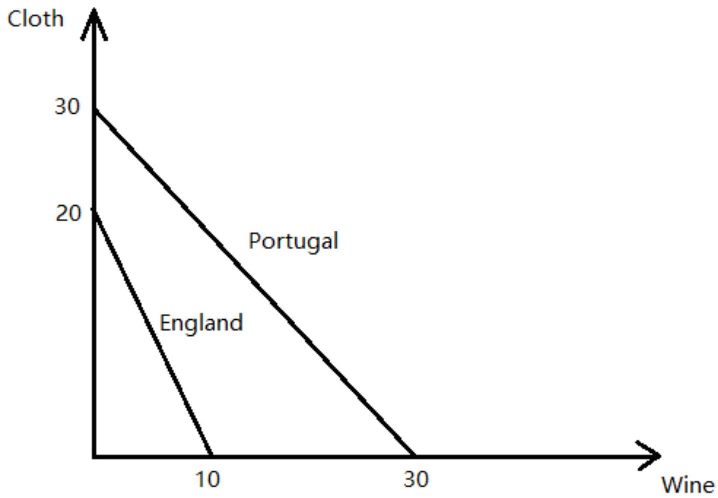
a. Given the above information, what is the opportunity cost for England to produce one additional piece of cloth in terms of bottles of wine? What is the opportunity cost for England to produce one additional bottle of wine in terms of cloth? What is the opportunity cost for Portugal to produce one additional piece of cloth in terms of bottles of wine? What is the opportunity cost for Portugal to produce an additional bottle of wine in terms of cloth?

Answer:
From combination 1 to combination 2, England produces 10 more pieces of cloth at the cost of 5 fewer bottles of wine. In other words, to produce 1 piece of cloth, England gives up 0.5 bottle of wine. Therefore, its opportunity cost of producing an additional piece of cloth in terms of wine is 5/10=0.5 bottle of wine. Its opportunity cost of producing an additional bottle of wine in terms of cloth is 10/5=2 pieces of cloth.
Similarly, from combination 1 to combination 2, Portugal produces 5 more pieces of cloth, at the cost of producing 5 fewer bottles wine. Therefore, to produce one additional unit of cloth, Portugal has to forgo one bottle of wine. Its opportunity cost

of an additional piece of cloth in terms of wine is 1 bottle of wine, and its opportunity cost of an additional bottle of wine in terms of cloth is also 1 piece of cloth.

b. Draw the Production Possibility Frontiers for both countries on a single graph. Measure cloth on the vertical axis of your graph and wine on the horizontal axis. Label each Production Possibility Frontier clearly and completely, including the horizontal and vertical intercepts for each PPF. Which country has the absolute advantage in the production of cloth? Explain your answer fully. Which country has the comparative advantage in the production of cloth? Explain your answer.

Answer:

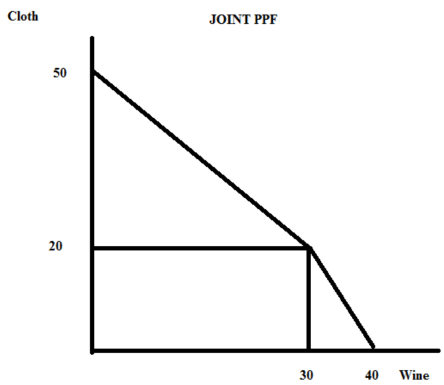


As the graph demonstrates, Portugal has an absolute advantage in production of cloth since it can absolutely produce more cloth than England: 30 pieces of cloth versus 20 pieces of cloth. However, England has a comparative advantage in the production of cloth since England's opportunity cost to produce an additional piece of cloth is 0.5 bottles of wine, and this is cheaper than Portugal's opportunity cost to produce an additional piece of cloth which is 1 bottle of wine.

c. Now, imagine the two countries engage in trade. Which country will export cloth, and which country will export wine? Draw a well labeled graph of the join production possibility frontier for these two countries if they specialize according to comparative advantage and trade with one another. In your graph, measure wine on the horizontal axis and cloth on the vertical axis. In your graph, identify the coordinate values for any "kink point". Write the equation(s) for the joint PPF. (Note: there may be more than one equation here-so provide information about which equation is right for which segment of your joint PPF.)

Answer:
Because England has the comparative advantage in producing cloth, it will concentrate in producing cloth and export it. Similarly, Portugal has the comparative

advantage in producing wine and will export wine to England. Their combined PPF is:



The combined PPF adds together the PPF for England and Portugal. When both countries produce only cloth, they produce (20+30) = 50 pieces. When both countries produce only wine, they produce (10+30) = 40 bottles. When both countries specialize, England produces 20 pieces of cloth, and Portugal produces 30 bottles of wine, so point (30, 20) is on the joint PPF.

The top segment of the joint PPF is easy to write in equation form as: $C = 50 - W$ where C is pieces of cloth and W refers to bottles of wine. This equation is true for all positive values of wine less than or equal to 30 bottles.

The lower segment of the joint PPF is harder to write in slope-intercept form. But, we can figure it out.
$C = mW + b$ where "m" is the slope of this lower segment and "b" is the y-intercept for this lower segment. By inspection of our graph we can see that the slope of this lower segment is -2. Or, you could calculate the slope using the two points (W, C) = (30, 20) and (40, 0) and get the same value.
Thus, $C = -2W + b$. Now use one of those known points to find the value of the y-intercept. Thus,
$20 = (-2)(30) + b$
$b = 80$
The lower segment of the joint PPF can be written as $C = 80 - 2W$ for all values of wine greater than or equal to 30 bottles.

d. Assume that after specialization and trade, the two countries in total will produce 25 pieces of cloth and 15 bottles of wine. Given this production combination how many pieces of cloth and how many bottles of wine does England produce? How many pieces of cloth and bottles of wine does Portugal produce? Is this production combination efficient? Explain your answer fully.

Answer:

Since England specializes in producing cloth, it would produce as much cloth as it can possibly produce. AT most England can produce 20 pieces of cloth, which means that the remaining 5 pieces must be produced by Portugal. Since England produces 20 pieces of cloth and it produces no bottles of wine, this means that Portugal must produce all 15 bottles of wine. Find point (15, 25) in the graph above. The point is inside the joint PPF. Therefore, the point is an *inefficient* level of production. Alternatively, we know that this level of production occurs within the range of the top segment of the joint PPF so we can ask the question, how much wine can be produced if 25 pieces of cloth are produced. Using the equation from (c) we have C = 50 – W and when C = 25 this implies that W = 25. When 25 pieces of cloth are produced, Portugal and England working together and specializing according to comparative advantage are capable of producing 25 bottles of wine which is substantially better than 15 bottles of wine!

e. According to a report from MIT, Portugal today (2015) is a net exporter of refined petroleum. What does this fact tell us about Portugal's productivity in terms of refined petroleum? Explain your answer fully.

Answer:

Since Portugal is a net exporter of refined petroleum, it must have a comparative advantage in producing refined petroleum, *compared to the rest of the world as a whole*.

5. **Comparative advantage**: The following table describes the amount of labor Roy and Steve each needs to produce a pencil or an eraser. Assume that both Roy and Steve have linear PPFs and that they each can devote 60 hours a week to the production of pencils and/or erasers.

	The Amount of Labor Needed to Produce One Pencil	The Amount of Labor Needed to Produce One Eraser
Roy	4 Hours of Labor	6 Hour of Labor
Steve	2 Hours of Labor	4 Hours of Labor

a. Given the above information, calculate Roy's opportunity cost of producing one pencil and one eraser in terms of the other good, respectively.

Answer:

Roy must work 4 hours to produce one pencil. Using the same amount of time, he can produce 4/6 units of eraser. Therefore 4/6 units of eraser is Roy's opportunity cost of producing one pencil in terms of erasers. Similarly, 6/4 units of pencil is his opportunity cost of producing one eraser in terms of pencils.

b. Calculate Steve's opportunity cost of producing one pencil and one eraser in terms of the other good, respectively.

Answer:
Steve must work 2 hours to produce one pencil. He can produce 2/4 units of eraser given that many hours. Therefore, 2/4 units of eraser is Steve's opportunity cost of producing one pencil in terms of erasers. Similarly, 4/2 = 2 units of pencil is his opportunity cost of producing one eraser in terms of pencils.

c. State who has the comparative advantage in the production of pencils and erasers, respectively. Explain your answer.

Answer:
Steve's opportunity cost of producing one pencil is lower than Roy's; 2/4 < 4/6. Therefore, Steve has the comparative advantage in the production of pencils. On the other hand, Roy's opportunity cost of producing one eraser is lower than Steve's; 6/4 < 2. So Roy has the comparative advantage in the production of erasers.

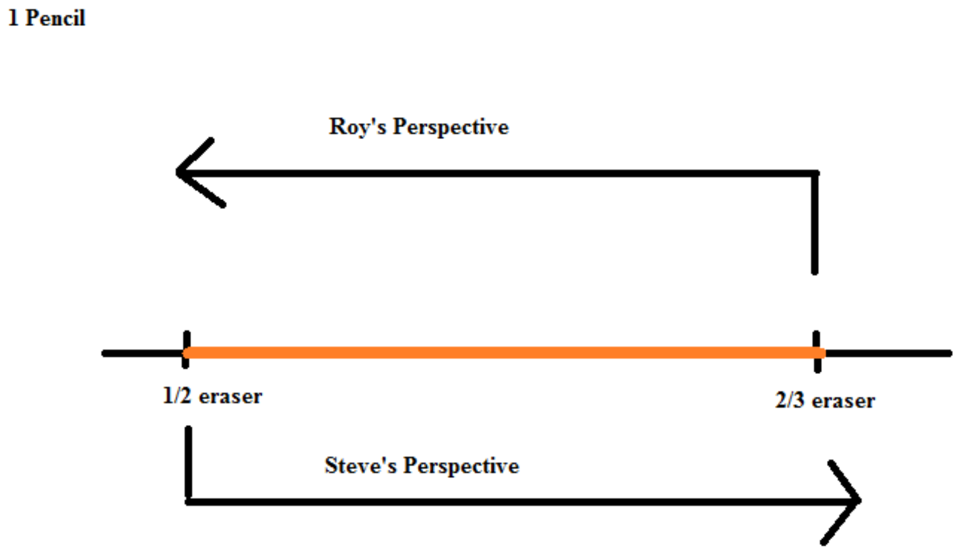
d. Now assume that Roy can devote 120 hours per week on the production of pencils and/or erasers while Steve continues to only have 60 hours per week that he can devote to this production. State who has the comparative advantage in the production of the pencils and erasers, respectively. Is there any difference between the results you found in answers (c) and (d)? Explain your answer.

Answer:
Even though Roy now has more time each week to work than Steve, the time needed to produce one unit of pencil and eraser remains the same. In other words, the given "table" has not been changed. Therefore the opportunity costs of producing one unit of pencil and eraser are the same as in the previous calculations. This implies Roy still has the comparative advantage in the production of erasers and Steve has the comparative advantage in the production of pencil. This illustrates the amount of total labor input does not affect who has the comparative advantage.

e. Given your work in this question, provide an illustration of the acceptable range of trading prices in terms of erasers for 10 pencils. Use the number line approach presented in class and make sure you designate in your diagram arrows indicating Roy's as well as Steve's perspective with regard to acceptable trading prices.

Answer:
We know that Steve has the comparative advantage in the production of pencils and that from Steve's perspective that his opportunity cost of producing one pencil is 1/2 eraser. From Roy's perspective his opportunity cost of producing one pencil is 2/3 eraser. So anytime Roy can purchase a pencil for 2/3 eraser or less he is quite happy to purchase that eraser. And, anytime Steve can sell a pencil for 1/2 eraser or more he is quite happy to make that sell. Putting this together we have the following diagram for

the range of acceptable prices for one pencil in terms of erasers.



Now, let's gross this up to 10 pencils: