

**Directions:**

- The homework will be collected in a box **before** the lecture.
- Please place **your name, TA name, and section number** on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade.
- Late homework will **not** be accepted so make plans ahead of time.
- **Show your work.** Good luck!

**Please realize that you are essentially creating “your brand” when you submit this homework. Do you want your homework to convey that you are competent, careful, and professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional? For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you submit any work for someone else.**

Part I: Perfect Competition

1. You have just graduated, and your first job is as a manager at Widget Co., a company that makes widgets. Your engineer gives you the following table of costs, but, for some reason, he has forgotten to fill in most of it. The per unit cost of labor (the wage) is \$10, and the per unit cost of capital is \$50.

Q	K	L	MPL	APL	FC	VC	TC	AFC	AVC	ATC	MC
0	10	0	-	-				-	-	-	-
	10	1									\$1
	10			12.5				\$20			
35	10	3									
	10					\$40				\$13.50	
	10						\$550				\$10

a) Using your knowledge of producer theory, fill in the missing information from the table. (You may need a calculator for some of the entries.) Remember that:

MPL = marginal product of labor = [(change in total product)/(change in total labor)]

APL = the average product of labor = [(total product)/(total labor)]

AFC = average fixed cost = FC/Q

AVC = average variable cost = VC/Q

ATC = average total cost = TC/Q

MC = marginal cost = [(change in total cost)/(change in output)]

**Start by filling in the easy entries: Cost of capital is \$50 per unit, so fixed costs are \$500 all the**

way down. Now we systematically move down the table one row at a time. In the first row, no labor is used so  $VC = 0$ , and  $TC = FC = \$500$ .

Moving down one row, we are now using 1 unit of labor, so  $VC = \$10$  and  $TC = \$510$ . We know that  $MC = \$1$  on this line, and that tells us that the  $(\text{change in total cost})/(\text{change in output}) = 1$ . The change in total cost is  $510 - 500$  or  $\$10$ . We can then solve this equation to find the change in output. The change in output must be 10 units. Or, since  $MC = \$1$ , and we increased total spending by  $\$10$ , we can infer that  $Q = 10$ . Since labor increased by 1 and quantity increased by 10,  $MPL = 10$ . Using this, we can derive the various average costs, by dividing by quantity.

Moving down again, in the third row we are given  $AFC = 20$ . Since  $AFC = FC/Q$ , we can solve for  $Q$ , finding  $Q = 25$ . Since  $APL = Q/L$ , we have  $L = 2$ . Since labor increased by 1 and quantity increased by 15, we can see  $MPL = 15$ . Since we increased spending by another  $\$10$  and increased production by 15, we have  $MC = 10/15 = \$0.67$ . We can then fill in average costs as usual.

In the fourth row, we're given  $Q = 35$  and  $L = 3$ .  $MPL$  thus is 10 since quantity increased by 10 in response to increasing labor by 1 unit.  $APL$  is simply  $35/3 = 11.67$ . Spending increased by  $\$10$ , and the quantity increased by 10, so  $MC = \$1$ . The remaining costs can be filled in directly using the standard formulae.

In the fifth row, we are given  $VC = \$40$ , so using the cost of labor, we have  $L = 4$ . With these, we can compute all the relevant total costs. Since we now know  $TC = \$530$ , and  $ATC = \$13.50$ , we must have  $Q = 40$ . From there we can compute the remaining average costs and marginal cost in the usual way.

Finally in the last row, total cost is  $\$550$ , so  $L = 5$ . Since  $MC = \$10$ , and the cost of the last unit of labor was  $\$10$ ,  $Q$  must have only increased by 1, so  $Q = 41$ . With that the remaining columns can be filled from the usual formulae.

Q	K	L	MPL	APL	FC	VC	TC	AFC	AVC	ATC	MC
0	10	0	-	-	\$500	\$0	\$500	-	-	-	-
10	10	1	10	10	\$500	\$10	\$510	\$50	\$1	\$51	\$1
25	10	2	15	12.5	\$500	\$20	\$520	\$20	\$0.80	\$20.80	\$0.67
35	10	3	10	11.67	\$500	\$30	\$530	\$14.29	\$0.86	\$15.14	\$1
40	10	4	5	10	\$500	\$40	\$540	\$12.50	\$1	\$13.50	\$2
41	10	5	1	8.20	\$500	\$50	\$550	\$12.19	\$1.21	\$13.41	\$10

b) Suppose the market price of widgets is  $\$1$ . Using your answers to the previous part, what level of production would you recommend in the short run? *Rigorously justify your answer.*

You know that at an optimum, you should set price equal to marginal cost. Notice from the table, there are 2 production levels that have a marginal cost of  $\$1$ . Let's consider the first option of  $Q = 10$  first. Suppose you produce  $Q = 10$ . Notice that marginal cost is *falling* at that quantity, so if you were to increase production, the marginal units would cost *less* than  $\$1$ , and can be sold at market price for  $\$1$ . So it would be profitable to produce more. Thus the optimal level of

production would be at  $Q = 35$ . Lastly, in the short run, we just need to ensure we are covering variable costs (so the cost of labor). At  $Q = 35$ ,  $AVC = \$0.86$  which is less than the price, so producing  $Q = 35$  covers the cost of labor and producing 35 units is better than immediately shutting down.

c) Suppose you do not expect any major price increases in the foreseeable future. From this table, should you be concerned about your long-run job prospects at Widget Co.? Why?

Supposing the production levels in the table represent all possible production levels, you should be worried about your job prospects because at all levels of production,  $MC$  is below  $ATC$ . Thus, at any price that would induce a level of production from the table, ( $P \leq \$10$ ), the price would be below  $ATC$ , so Widget Co., would earn losses. Although, producing in the short run would be optimal for high enough prices, eventually, Widget Co. will sell its capital and exit the industry.

2. Suppose the market for cheese curds is perfectly competitive. The demand curve for cheese curds is given by

$$Q = 600 - 50P$$

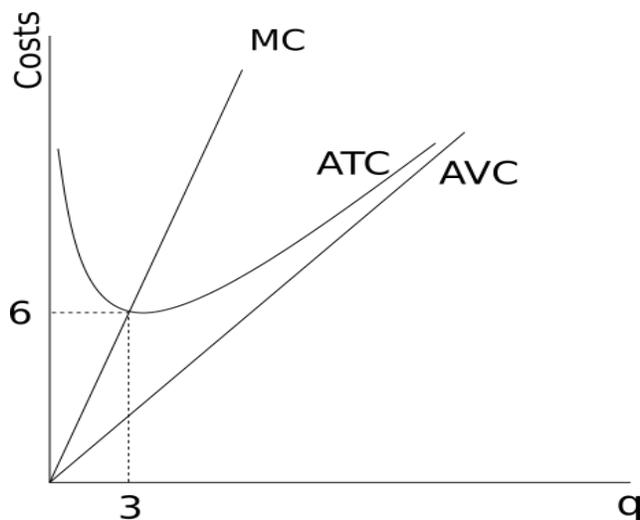
Suppliers of cheese curds are all identical and have the following cost structure:

$$\text{Marginal cost for the representative firm: } MC = 2q$$

$$\text{Total cost for the representative firm: } TC = q^2 + 9$$

a) Find the equations for the fixed costs, variable costs, average variable costs, and average total costs for a representative firm. Plot marginal costs, average variable costs, and average total costs for a representative firm on the same graph. Hint: don't be worried if your  $AVC$  curve looks a bit strange: we want to keep the math simple here!

From the total cost equation, we can see fixed costs are simply  $FC = \$9$  and variable costs are  $VC = q^2$ . Using the regular formulae, we have  $AVC = q$  and  $ATC = q + 9/q$ . Plotting these we have



b) For what prices would a typical cheese curd producer produce, even in the short run? For what prices would a typical producer produce in the long run?

In the short-run, a typical producer would produce for any positive price, since marginal costs are always above average variable costs. In the long-run, a producer will only stay in business if the price is high enough to cover average total costs at the optimal level of production. This occurs provided the price is at least \$6.

c) Suppose there are currently 50 producers of cheese curds in the market. Give an equation for the total (short-run) supply of cheese curds in this market. Using this information, find the market equilibrium. What are the short-run profits of a typical producer? Do you expect to see entry or exit in this market in the long run?

From the previous part, we know producers will stay open in the short run for any positive price, so the short run supply curve for an individual firm is simply the same as its marginal cost curve. So the individual firm supply curve is given by  $P = 2q$ . Rearranging, we have the quantity supplied by an individual firm is  $q = (1/2)P$ . Since there are 50 identical firms, we have the total market supply,  $Q$ , is given by  $Q = 25P$ . Putting these two together, we have market supply given by  $Q = 25P$ .

Now, we can set this equal to the demand curve given at the start of the problem and solve for the equilibrium. Following the usual process, we have  $P = \$8$ ,  $Q = 200$ . From this we have each individual producer makes  $q = 4$ . At this level production, each firm earns profits:

$$\text{Profits} = \text{TR} - \text{TC}$$

$$\text{TR} = (\$8 \text{ per unit})(4 \text{ units}) = \$32$$

$$\text{TC} = q^2 + 9 = (4)(4) + 9 = 25$$

$$\text{Profits} = 32 - 25 = \$7$$

Given that producers earn strictly positive profits, we would expect to see entry in this market in the long run.

d) What is the long-run equilibrium price in this market? How many cheese-curd producers will be in the market in the long run?

From the cost structure of the individual firms, we know that the break-even price is \$6. This is the price where  $MC = ATC = MR$  for the representative firm in the industry. To get this price follow this reasoning:

$$\text{ATC} = \text{MC} \text{ at the breakeven point}$$

$$q + (9/q) = 2q$$

$$9/q = q$$

$$3 = q$$

So the profit maximizing output when the representative firm produces at the long run equilibrium is 3 units ( $q = 3$ ). What price is associated with this quantity? Use the MC curve or the ATC curve and this quantity to find that price:

$$\text{MC} = 2q$$

$$MC = P = 2(3) = \$6 \text{ per unit}$$

When the price is \$6 per unit and the representative firm produces 3 units, the firm will earn 0 economic profit: the firm will breakeven which is a necessary condition to be at the long run equilibrium. This must be the long-run price since lower prices would induce exit, and higher prices would induce entry. Turning to the market demand curve, by plugging in \$6, long-run total quantity (the market quantity) must be  $Q = 300$ . Since each firm produces  $q = 3$ , we must have 100 firms in the market in the long run in order to meet this demand.

## Part II: Monopoly

3. Suppose there is a monopolist in the market for pain reliever pills. The monopolist's marginal cost equation is given by  $MC = 10 + 2Q$  where  $Q$  is the quantity of pills supplied. The monopoly firm's total cost is  $TC = 10Q + Q^2$ . (Note that there is no fixed cost.) The market demand curve for pain reliever pills is given by  $P = 70 - Q$  where  $P$  is the price per pill.

a) What is the monopolist price and quantity in the pain reliever pills market? Show how you found your answer. Find the monopolist's total revenue and profit. Calculate the value of consumer surplus in the market.

The marginal revenue curve for a monopolist with a linear demand curve will have the same y-intercept as the demand curve and a slope that is twice the demand curve's slope. Thus,  $MR = 70 - 2Q$ .

The profit maximizing rule for a monopolist is to produce that quantity where  $MR = MC$  and then charge the price from the demand curve associated with this quantity. Thus,

$$MR = MC$$

$$70 - 2Q = 10 + 2Q$$

$$60 = 4Q$$

$$Q = 15 \text{ units of pills.}$$

$$P = 70 - 15 = \$55 \text{ per unit of pills.}$$

$$\text{Total Revenue} = P \cdot Q = \$825.$$

$$\text{Profit} = (P - ATC) \cdot Q$$

$$ATC = 10 + Q = \$25$$

$$\text{Profit} = (55 - 25) \cdot 15 = \$450.$$

The consumer surplus is the yellow shaded area.

$$\text{Consumer surplus: } (70 - 55) \cdot 15 / 2 = \$112.5$$

b) Suppose the monopoly acted as a competitive firm. Find the price and quantity it would sell. Calculate the value of consumer surplus and the value of producer surplus.

In a competitive market:  $P = MC$ . Thus, to find the price and quantity in this market if it were a competitive market we need to equate the MC and demand equations.

$$P = 70 - Q \text{ and } MC = 10 + 2Q$$

$$70 - Q = 10 + 2Q$$

$$3Q = 60$$

$$Q = 20 \text{ units of pills}$$

$$P = \$50.$$

Consumer surplus:  $(70 - 50) * 20 / 2 = \$200$ . (yellow shaded area plus red shaded area)

Producer surplus:  $(50 - 10) * 20 / 2 = \$400$ . (blue shaded area)

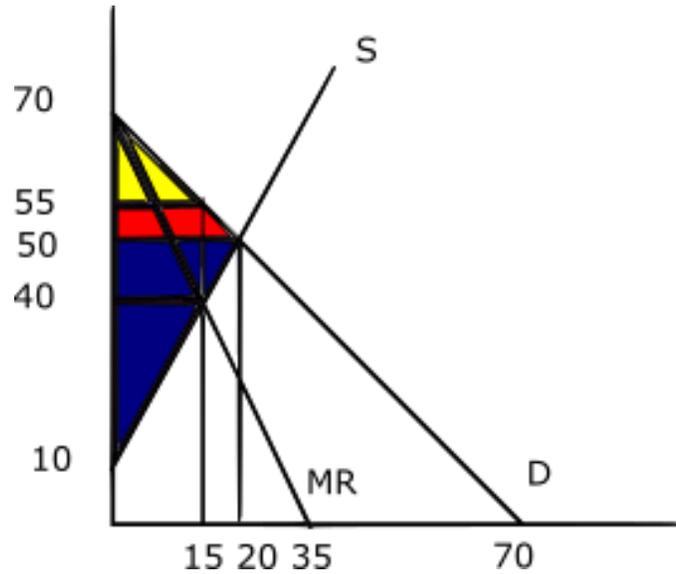
Profit = Producer surplus - fixed cost =  $400 - 0 = \$400$

OR

Profit =  $TR - TC = 50 * 20 - (10 * 20 + 20^2) = 1000 - 600 = \$400$ .

c) Suppose this monopolist implements first-degree price discrimination to increase the monopolist's profits. Calculate the profit when this monopolist practices first degree price discrimination.

When monopolist practices first degree price discrimination, it captures total surplus as its profit. Profit =  $200 + 400 = \$600$ .



### Part III: Price Discrimination

4. Consider a monopolist that sells flight tickets to two groups of buyers: students and non-students. The monopolist is unable to distinguish whether a particular buyer is a student or not unless the buyer's ID is checked. The monopolist knows the following information where  $P$  is the price per flight ticket,  $Q_s$  is the quantity of flights demanded by students and  $Q_{ns}$  is the quantity of flights demanded by non-students. In this problem assume that there are no fixed costs.

Demand for flight tickets from students:  $P = 550 - 2Q_s$

Demand for flight tickets from non-students:  $P = 1050 - (1/2) Q_{ns}$

Marginal Cost:  $MC = \$150$

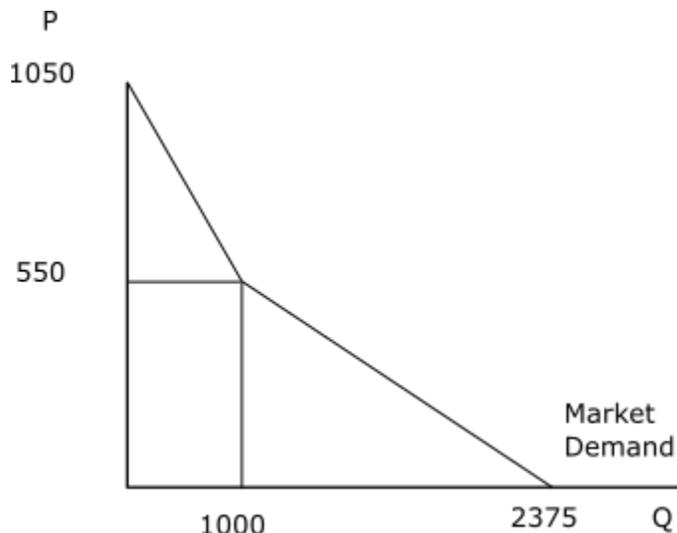
a) Suppose by law, one's student ID cannot be checked when booking a flight. Find the market demand curve for this monopolist. This market demand curve will be the horizontal summation of the demand curves from the two groups. Provide all equations and ranges needed to describe the market demand curve. Draw the market demand curve.

$$P = 1050 - (1/2)Q \text{ if } P \geq 550$$

When  $P < 550$ , adding the two demand curves horizontally

$$Q = (550/2 - 0.5P) + (2100 - 2P) = 2375 - 2.5P.$$

$$P = 950 - 0.4Q.$$



b) What is the monopolist price and quantity if the monopolist charges a single price for each flight? What will be the profit for the monopolist? (Assume there is no fixed cost.) Show how you found your answer.

Once you have the market demand curve, you can find the MR curves:

$$MR = 1050 - Q \text{ if } P \geq 550$$

$$MR = 950 - 0.8Q \text{ if } P < 550$$

Now, set  $MR = MC$

(Case I:  $P \geq 550$ )

$$1050 - Q = 150$$

$$Q = 900, P = 1050 - (1/2)Q = 600$$

$$\text{Profit} = (600-150)*900 = \$405,000.$$

(Case II:  $P < 550$ )

$$950 - 0.8Q = 150$$

$$0.8Q = 800$$

$$Q = 1000, P = 950 - 0.8*1000 = 150$$

$$\text{Profit} = (150-150)*1000 = \$0.$$

Hence, the monopolist produces  $Q = 900$  at  $P = \$600$ .

c) Now suppose the government allows the monopolist airline to check the buyer's (student) ID card to determine whether a particular buyer is a student or not. The monopolist now can practice third degree price discrimination. How many flight tickets will this monopolist supply to the market for students and the market for non-students? What prices will students and non-students pay? What will be the profits from each market separately? What would be the total profit? Show how you found your answers to each of these questions.

For students:  $MR = 550 - 4Q$  and  $MC = 150$ . Equating these two equations,

$$550 - 4Q = 150, Q = 100, P = 550 - 2*100 = \$350$$

$$\text{Profit from students} = (350-150)*100 = \$20,000$$

For non-students:  $MR = 1050 - Q$  and  $MC = 150$ .

$$1050 - Q = 150, Q = 900, P = 1050 - (1/2)*900 = \$600.$$

$$\text{Profit from non-student buyers} = (600-150)*900 = 450*900 = \$405,000.$$

$$\text{Total Profit from being a third degree price discriminator} = 20,000 + 405,000 = \$425,000.$$

d) Referring to the answers from b) and c), is this firm better off from practicing third degree price discrimination?

The monopoly earns a higher profit under third-degree price discrimination. With third degree price discrimination the monopolist sells the flight tickets at two different prices, which increases the total profit of the monopolist.

Part IV: Game Theory

5. In the battle of the sexes, a couple argues over what to do over the weekend. Both know that they want to spend the weekend together, but they cannot agree over what they want to do. The man prefers to go to the gym, whereas the woman wants to go to yoga. <sup>1</sup>

Since the couple wants to spend time together, if they go their separate ways, they will receive no utility. If they go either to the gym or to yoga, both will receive some utility from the fact that they're together, but only one of them will enjoy the specific activity. The payoff matrix is listed below where the numbers in each cell represent (utility to the man, utility to the woman):

		WOMAN	
		Gym	Yoga
MAN	Gym	(2,1)	(0,0)
	Yoga	(0,0)	(1,2)

a) Is the choice of the couple choosing different activities an equilibrium given the above information? Explain your answer.

No, because if the woman chooses yoga, the man would also choose yoga. If the man chooses gym, the woman would choose gym. Hence, (gym, yoga), (yoga, gym) cannot be an equilibrium.

b) Is there any strictly dominant strategy for the man?

No, because if the woman chooses the gym, the man would choose the gym but if the woman chooses yoga, the man would choose yoga. Hence, neither the choice of the gym or the choice of yoga is a dominant strategy for the man.

c) Is there any strictly dominant strategy for the woman?

No, because if the man chooses the gym, the woman would choose the gym but if the man chooses yoga, the woman would choose yoga. Hence, neither choosing the gym or choosing yoga is a dominant strategy for the woman.

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<sup>1</sup> This is a modified version of the classic battle of the sexes example.

d) What's your prediction for the equilibrium outcome of this game?

I would predict either (gym, gym) or (yoga, yoga). If the woman chooses yoga, the man would choose yoga. If the man chooses gym, the woman would choose the gym.

#### Part IV: Externality

6. Suppose the market demand (the marginal private benefit curve) for vaccinations for the population on an island is given by the equation below where P is the price per vaccination and Q is the number of vaccinations:

$$\text{Market Demand: } P = 100 - 4Q$$

The market supply (the marginal private cost curve) is given by:

$$\text{Market Supply: } P = Q$$

Suppose that vaccination imposes positive externalities on the population of the island. Assume that each person getting vaccinated benefits the population of the island by \$20.

a. What is the price and quantity in this market for vaccines assuming that it is a competitive market and that the external benefit is not internalized in the market by either the producers or the consumers of the vaccines in this island?

$$MPB = MPC$$

$$100 - 4Q = Q$$

$$100 = 5Q$$

$$Q = 20$$

$$P = 100 - 4 \cdot 20 = 100 - 80 = \$20$$

b. Given the above information, find the marginal social benefit and marginal social cost functions for this market.

$$MSB = MPB + \text{External Benefit} = 100 - 4Q + 20 = 120 - 4Q$$

$$MSC = MPC = Q$$

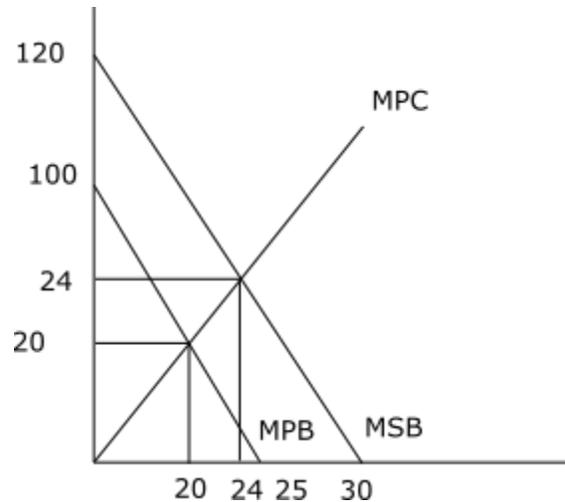
c. What is the socially optimal number of people getting vaccinated in this market? That is, if the externality was internalized in this market, what would be the price of vaccines and how many people would be vaccinated? Illustrate your answer with a graph that depicts the MPB, MSB, and MPC curves in a graph as well as the socially optimal price and quantity.

$$MSB = MSC$$

$$120 - 4Q = Q$$

$$5Q = 120$$

$$Q = 24 \quad P = \$24.$$



d. Suppose the government would like to intervene in the market to correct this externality. What excise tax or subsidy per unit should the government implement in this market to enable this market to reach the socially optimal outcome? What price will the population pay if the tax/subsidy is implemented? What price will producers receive for the vaccine? Calculate the tax revenue for the government if the government imposes a tax or the cost to the government if the government imposes a subsidy.

The government should subsidize to encourage the vaccination of its population.

To find the optimal level of subsidy, we use:

$$MPB + \text{subsidy} = MPC$$

$$100 - 4Q + \text{subsidy} = Q$$

Since we want  $Q = 24$ ,

$$\text{Subsidy} = 5Q - 100 = \$20.$$

Plugging,  $Q = 24$  back to the original demand and supply curves, the price that consumers will pay is  $100 - 4 \cdot 24 = \$4$  (or simply  $\$24 - \$20$ ) and the price that the producers will get is  $\$24$ . The cost to the government is  $\$20 \cdot 24 = \$480$ .

e. What is the deadweight loss if the government decides not to intervene in this market?

The deadweight loss is  $(1/2) \cdot (\text{size of externality}) \cdot (\text{socially optimal quantity} - \text{privately chosen quantity}) = 0.5 \cdot 20 \cdot (24 - 20) = 0.5 \cdot 20 \cdot 4 = \$40$ .

## Part V: Public Good

7. Suppose that three student organizations are interested in holding TA review session for the upcoming Econ101 Final. Each student organization has different values for these TA review sessions. The willingness to pay for each organization is given by the following demand curves where  $P$  is the price per hour of TA-led review sessions and  $Q$  is the number of hours of TA-led review sessions:

$$\text{Student Organization A: } P_A = 40 - 5Q$$

$$\text{Student Organization B: } P_B = 30 - Q$$

$$\text{Student Organization C: } P_C = 70 - 2Q$$

The cost of providing one hour of a TA-led review session is \$20. Hence, the MC of an additional hour of a TA-led review session is  $MC = \$20$ .

a. Suppose this market is treated as a competitive market. How many hours of review sessions will each organization want to have? How much will each organization contribute or pay for an hour of TA-led review session? Are there any free riders in this market? Explain your answer.

$$\text{Student Organization A wants } Q = 8 - (1/5) P_A = 8 - (1/5)*20 = 4$$

$$\text{Student Organization B wants } Q = 30 - P_B = 30 - 20 = 10$$

$$\text{Student Organization C wants } Q = 35 - (1/2) P_C = 35 - (1/2)*20 = 25$$

Student Organization A and B know that even if they do not pay anything, Student Organization C will have no choice but to pay for all the 25 hours of TA review sessions. Student Organization A and B will enjoy the free-ride provided by Student Organization C: they will not offer any review sessions since Student Organization C is providing more review sessions than either A or B want.

$$\text{Hence, } Q = 25, P_A = P_B = 0 \text{ and } P_C = \$20$$

b. Find the aggregate demand for this good if we recognize that it is a public good.

We sum the demand functions vertically.

For  $Q < 8$ ,

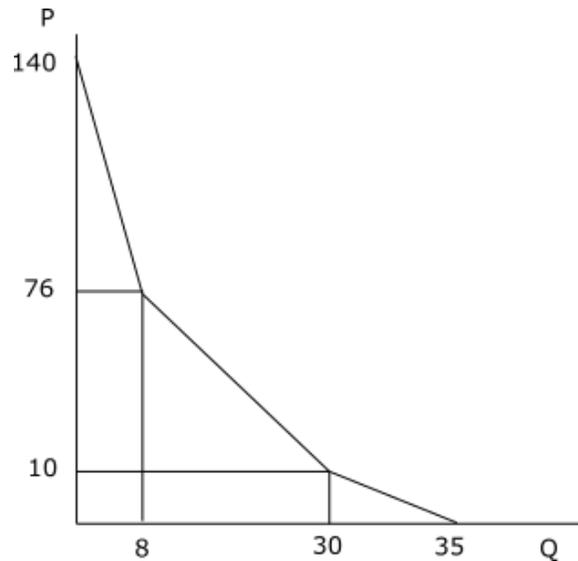
$$P = (40 - 5Q) + (30 - Q) + (70 - 2Q) = 140 - 8Q$$

For  $8 \leq Q < 30$ ,

$$P = (30 - Q) + (70 - 2Q) = 100 - 3Q$$

For  $Q \geq 30$ ,

$$P = 70 - 2Q$$



c. What is the socially optimal number of hours of TA-led review sessions? How much should each organization contribute per hour in order to get the socially optimal number of hours of TA-led review sessions? Your answer may be left in fractions and does not need to equal a whole number of hours.

You can check all intervals, but from graph,  $P = 20$  intersects the second interval:

$$20 = 100 - 3Q$$

$$3Q = 80$$

$$Q = 80/3$$

$$P_A = 0 \text{ (since we used the second interval!)}$$

$$P_B = 30 - 80/3 = \$ 10/3$$

$$P_C = 70 - 2 * (80/3) = (210 - 160)/3 = \$50/3$$