

Directions: The homework will be collected in a box **before** the lecture. Please place **your name** on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Late homework will not be accepted so make plans ahead of time. **Please show your work.** Good luck!

Please realize that you are essentially creating “your brand” when you submit this homework. Do you want your homework to convey that you are competent, careful, and professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional. For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you do any work for someone else!

Monopoly

1. Suppose Charter is a monopoly firm in the market for high speed internet in Madison. The total cost and the marginal cost functions for this firm are given by:

$$TC = 0.125q^2 + 15q + 1000$$

$$MC = 0.25q + 15$$

Where q is the number of user accounts.

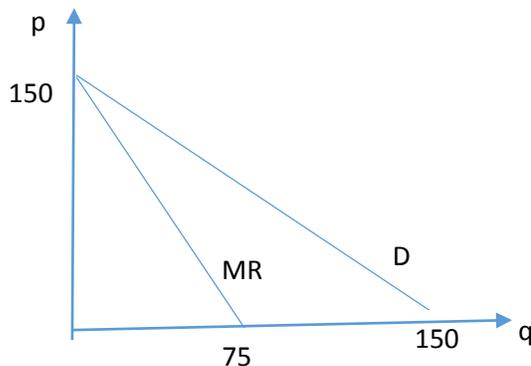
The demand for high speed internet in Madison is given by:

$$p = 150 - q$$

Where p is the price in dollars per account.

- a. Find the equation for marginal revenue for Charter. Draw a graph representing this monopolist's demand curve and its marginal revenue curve.

MR=150-2q (twice the slope of the linear demand function)



- b. Suppose that Charter acts like a perfectly competitive firm. What quantity and price would Charter charge under these assumptions? What are the values of consumer and producer

surplus? If Charter acted like a perfectly competitive firm, what would its economic profits equal? Draw a graph illustrating this situation. In your graph identify the price, quantity, area of consumer surplus, and area of producer surplus.

In a competitive market: $P = MC$ so Charter will want to equate its demand curve to its MC curve.

$$150 - q = 0.25q + 15$$

$$\Rightarrow q = 108$$

$$\Rightarrow p = \$42$$

$$\text{Consumer surplus: } (150-42) \cdot 108 / 2 = \$5832$$

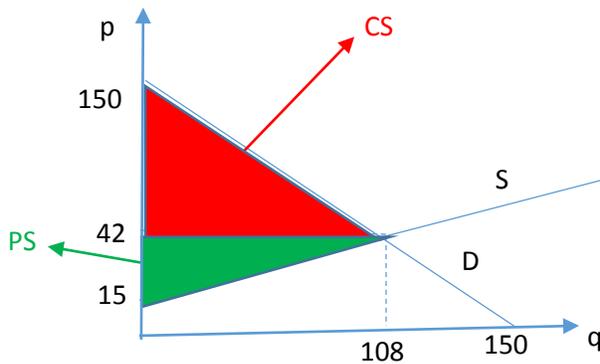
$$\text{Producer surplus: } (42-15) \cdot 108 / 2 = \$1458$$

$$\text{Profit: } \pi = TR - TC = 42 \cdot 108 - (0.125 \cdot 108^2 + 15 \cdot 108 + 1000) = \$458$$

$$\Rightarrow \pi = 4536 - (1458 + 1620 + 1000) = \$458$$

Or:

$$\pi = \text{producer surplus} - \text{Fixed costs} = 1458 - 1000 = \$458$$



- c. Suppose that Charter uses its monopoly power and acts as a monopolist. What quantity and price will it charge under this assumption? What will be its profit? Compute the values of consumer and producer surplus. What is the value of deadweight loss if Charter acts as a monopolist? Draw a graph illustrating this situation. In your graph identify the price, quantity, area of consumer surplus, area of producer surplus, and area of deadweight loss.

Monopoly: $MC = MR$ to find the quantity and then go to the demand curve to get the price for that quantity.

$$150 - 2q = 0.25q + 15$$

$$\Rightarrow q = 60$$

$$\Rightarrow p = \$90$$

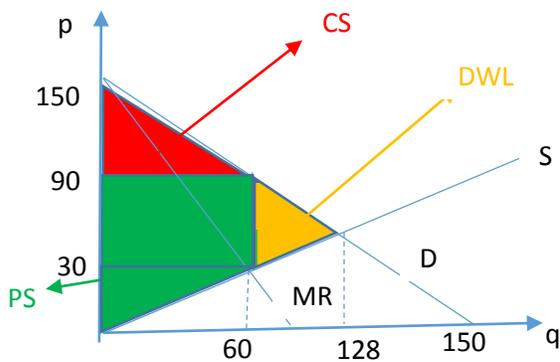
Consumer surplus: $(150 - 90) * 60 / 2 = \$1800$

Producer surplus: $[(90-15)+(90-30)] * 60 / 2 = \4050

Profits: $\pi = 4050 - 1000 = \$3050$ (or by using $TR - TC$)

Deadweight loss: $(90-30) * (108-60) / 2 = \1440

(Another way to find the deadweight loss is by looking at the change in the total surplus. i.e. total surplus was \$7290 in part (b) and in part (c) it became \$5850, which means it has decreased by \$1440!)



Price Discrimination

2. Metro Transit is the only bus transportation provider in the city of Madison. Its total cost and marginal cost equations are given by:

$$TC = 0.2q + 10000$$

$$MC = 0.2$$

Where q is the number of rides.

We can divide the population of Madison into two groups. The first group (Group 1) will include students, elderly people and people with special needs. The second group (Group 2) will include all the rest of the population. The demand functions of the two groups are given by:

$$\text{Group 1: } p = 3 - 0.001q_1$$

$$\text{Group 2: } p = 6 - 0.001q_2$$

- a. Suppose that Metro Transit is a competitive firm and takes the market determined price as the price it will charge.

- (1) Find the number of rides that each group would choose given this assumption. What will be the price of a ride?
- (2) Find the values of producer and consumer surplus.
- (3) Find the profit of Metro Transit.
- (4) Find the deadweight loss given this pricing decision.
- (5) Provide a graph illustrating your answer.

(1) First we must find the market demand, and then the intersection point with the marginal cost curve.

Market demand:

For $6 > p > 3$ only group 2 is demanding a positive quantity, so

$$Q_{market}^D = q_2^D = 6000 - 1000p \quad \text{for } 3 < p < 6$$

For $3 > p$ both groups are demanding positive quantities, so

$$Q_{market}^D = q_1^D + q_2^D = 9000 - 2000p \quad \text{for } 0 < p < 3$$

Or I can rewrite it:

$$p = \begin{cases} 6 - 0.001Q & \text{for } 0 < Q < 3000 \\ 4.5 - 0.0005Q & \text{for } 3000 < Q < 6000 \end{cases}$$

(Note that I can write the intervals as a function of Q , and that's by plugging $p = 6$, $p = 3$ and $p = 0$ in the appropriate market demand function!)

Now we start by guessing that the intersection of MC is with first part of the market demand:

$$MC = p$$

$$0.2 = 6 - 0.001Q$$

$$\Rightarrow Q = 5800$$

Which means, we guessed the wrong interval because we got $Q = 5800 > 3000$!

Now we check the second interval:

$$MC = p$$

$$0.2 = 4.5 - 0.0005Q$$

$$\Rightarrow Q = 8600$$

$$\Rightarrow p = 0.2$$

To find the quantity of each group, we plug the equilibrium price back in the demand function of each group:

$$q_1 = 3000 - 0.2 \cdot 1000 = 2800$$

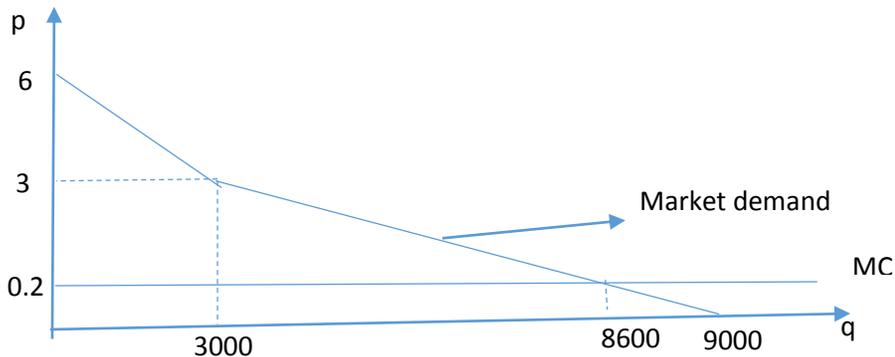
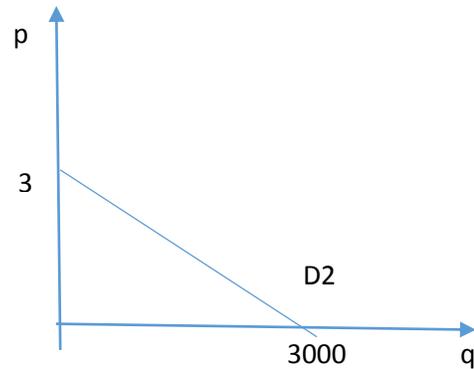
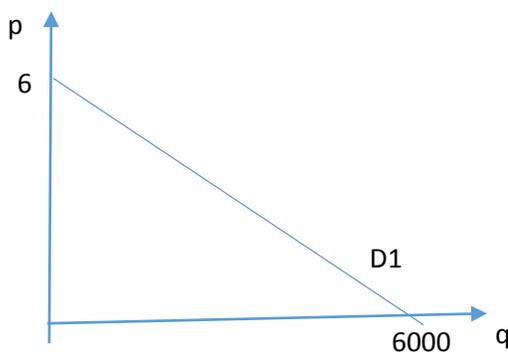
$$q_2 = 6000 - 0.2 \cdot 1000 = 5800$$

(2) Producer surplus = \$0

$$\text{Consumer surplus} = CS1 + CS2 = (6-0.2)*5800/2 + (3-0.2)*2800/2 = 16820 + 3920 = \$20740$$

(3) Metro Transit profit = producer surplus - fixed costs = 0 - 10000 = -\$10000

(4) Deadweight loss = \$0



b. Suppose that Metro Transit is allowed to behave as a perfect price discriminating (1st degree) monopoly and it can charge different prices for different units and different people..

(1) Find the number of rides that each group will choose if the firm acts as a perfect price discriminator.

(2) Find the values of producer and consumer surplus.

(3) Find the profit of Metro Transit.

(4) Find the deadweight loss when the firm acts as a perfect price discriminator.

(1) The monopoly here can simply extract all consumer surplus by charging different prices until the consumers' willingness to pay (the demand) is equal to its marginal cost, namely until $p = MC$.

So for group 1:

$$p = MC$$

$$3 - 0.001q_1 = 0.2$$

$$q_1 = 2800$$

For group 2:

$$p = MC$$

$$6 - 0.001q_2 = 0.2$$

$$q_2 = 5800$$

(2) Producer surplus = $(6 - 0.2) \cdot 5800 / 2 + (3 - 0.2) \cdot 2800 / 2 = \20740

Consumer surplus = \$0

(3) Profit = producer surplus - fixed costs = $20740 - 10000 = \$10740$

(4) Deadweight loss = \$0

- c. Suppose now that Metro Transit is allowed to behave as a monopoly but it has to charge all its customers the same price (a single price monopolist).
- (1) Find the number of rides that each group will choose to consume given the firm acts as a single price monopolist. Find the price the firm will charge.
 - (2) Find the values of producer and consumer surplus if the firm acts as a single price monopolist.
 - (3) Find the profit of Metro Transit when the firm acts as a single price monopolist.
 - (4) Find the deadweight loss when the firm acts as a single price monopolist.

- (1) We are looking at a standard monopoly here, so we need find the MR of the market demand (that we found in part (a)!)

$$MR = \begin{cases} 6 - 0.002Q & \text{for } 0 < Q < 3000 \\ 4.5 - 0.001Q & \text{for } 3000 < Q \end{cases}$$

We need to find the quantity for which $MR = MC$.

Important! We must check both intervals since MR is not continuous and MC might intersect with MR twice. (see the graph)

First interval:

$$6 - 0.002Q = 0.2$$

$$Q = 2900$$

And we can find the price by plugging back the quantity in the market demand (first interval):

$$P = 6 - 0.001Q = \$3.1$$

The profit of the monopoly if it chooses this price:

$$\pi = TR - TC = 2900 \cdot 3.1 - (0.2 \cdot 2900 + 10000) = -\$1590$$

Second interval:

$$4.5 - 0.001Q = 0.2$$

$$Q = 4300$$

And we can find the price by plugging back the quantity in the market demand (second interval):

$$P = 4.5 - 0.0005Q = \$2.35$$

$$\pi = TR - TC = 4300 \cdot 2.35 - (0.2 \cdot 4300 + 10000) = -\$755$$

The monopoly would choose the second option because it generates higher profits.

We can find the quantity for each group by plugging the price into the demand functions:

$$q_1 = 3000 - 2.35 \cdot 1000 = 650$$

$$q_2 = 6000 - 2.35 \cdot 1000 = 3650$$

(2) Producer surplus = profit + fixed costs = $-755 + 10000 = \$9245$

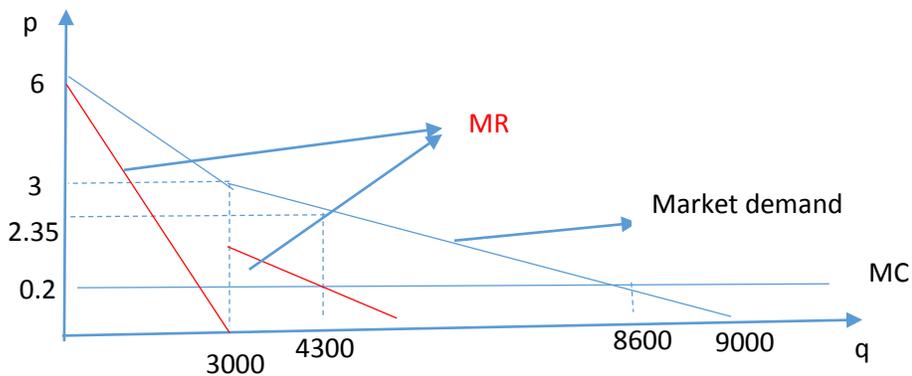
Or you can find it by using the graph.

Consumer surplus = $CS_1 + CS_2 = (6 - 2.35) \cdot 3650 / 2 + (3 - 2.35) \cdot 650 / 2 = 6661.25 + 211.25 = \6872.5

(3) Profit = $-\$755$

(4) Deadweight loss = $(2.35 - 0.2) \cdot (8600 - 4300) / 2 = \4622.5

Or you can find it by looking at the change in the total surplus.



d. Suppose now that Metro Transit is allowed to behave as a monopoly and it can discriminate between the groups and charge each group a different price (3rd degree price discrimination).

- (1) Find the number of rides that each group would choose if the firm acts as a third degree price discriminator. Find the price each group will pay if the firm acts as a third degree price discriminator.
- (2) Find the values of producer and consumer surplus if the firm acts as a third degree price discriminator.
- (3) Find the profit of Metro Transit if the firm acts as a third degree price discriminator.

(4) Find the deadweight loss when the firm acts as a third degree price discriminator.

- You can solve this problem the same way we solved problem 3 in handout #13, and that's because MC is constant. But if MC wasn't constant, you have to follow the steps below.

(1) To find the equilibrium in this case we should take the following steps:

Step 1 – Find the MR of each group

$$MR_1 = 3 - 0.002q_1$$

$$MR_2 = 6 - 0.002q_2$$

Step 2- Find the “aggregate” MR (same way as aggregate demand)

For $p > 3$, the aggregate MR is equal to MR_2 , so

$$MR = MR_2 = 6 - 0.002Q$$

For $p < 3$, we must find the aggregate MR by looking at the horizontal sum of MR_1 and MR_2 . i.e.

$$MR_1 = 3 - 0.002q_1 \Rightarrow q_1 = 1500 - 500MR_1$$

$$MR_2 = 6 - 0.002q_2 \Rightarrow q_2 = 3000 - 500MR_2$$

$$\Rightarrow Q = q_1 + q_2 = 1500 - 500MR + 3000 - 500MR = 4500 - 1000MR$$

$$\Rightarrow MR = 4.5 - 0.001Q$$

So the aggregate MR:

$$MR \begin{cases} 6 - 0.002Q & \text{for } Q < 1500 \\ 4.5 - 0.001Q & \text{for } Q > 1500 \end{cases}$$

NOTE! The intervals are different from the ones we found for the MR of the aggregate demand!

The kink point here is $Q=1500$ and $MR=3$

Step 3- solve $MR=MC$

Here we don't need to worry so much about the intervals because we know that MC will intersect with MR only once.

Let's guess that the intersection is in the second interval:

$$MC = MR$$

$$0.2 = 4.5 - 0.001Q$$

$$Q = 4300 \text{ (right interval! } 4300 > 1500)$$

$$MR = 4.5 - 0.001 \cdot 4300 = 0.2$$

If you were to guess that they intersect in the first interval, you would have gotten:

$$MC = MR$$

$$0.2 = 6 - 0.002Q$$

$$Q = 2900 \text{ (wrong interval! } 2900 > 1500)$$

Step 4- use $MR_1 = MR(4300)$ and $MR_2 = MR(4300)$ to find the number of rides for each group

$$MR_1 = MR(4300)$$

$$3 - 0.002q_1 = 0.2$$

$$q_1 = 1400$$

$$MR_2 = MR(4300)$$

$$6 - 0.002q_2 = 0.2$$

$$q_2 = 2900$$

Here you can check yourself! $Q = q_1 + q_2 = 1400 + 2900 = 4300$! Exactly what we found when we did the calculation.

Step 5 – Plug in q_1 and q_2 back in the demand function of each group to find the prices that each group will pay:

$$p_1 = 3 - 0.001 \cdot 1400 = \$1.6$$

$$p_2 = 6 - 0.001 \cdot 2900 = \$3.1$$

(2) Producer surplus = $(1.6 - 0.2) \cdot 1400 + (3.1 - 0.2) \cdot 2900 = \10370

Consumer surplus = $CS_1 + CS_2 = (3 - 1.6) \cdot 1400 / 2 + (6 - 3.1) \cdot 2900 / 2 = 980 + 4205 = \5185

(3) Profits = $1.6 \cdot 1400 + 3.1 \cdot 2900 - (0.2 \cdot 4300 + 10000) = \370

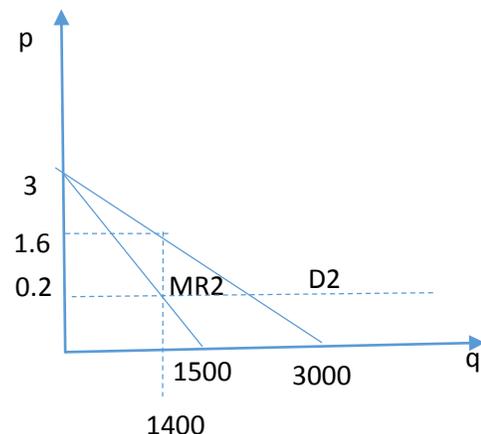
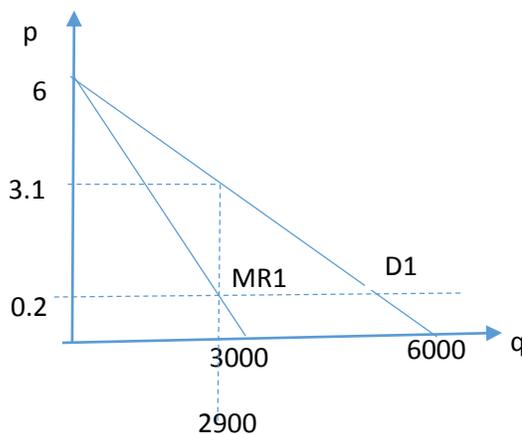
(4) Deadweight loss = Change in total surplus

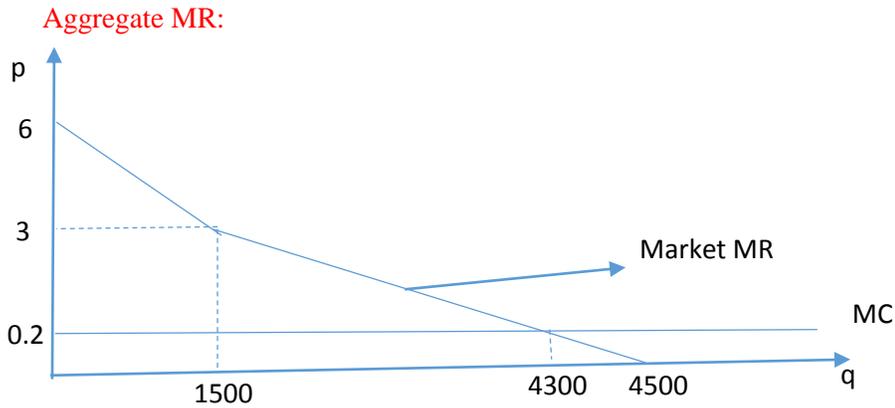
Total surplus under perfect competition = $CS_1 + CS_2 + PS = 16820 + 3920 + 0 = \20740

Total surplus with 3rd degree price discrimination = $CS_1 + CS_2 + PS = \$15555$

So the deadweight loss = $20740 - 15555 = \$5185$

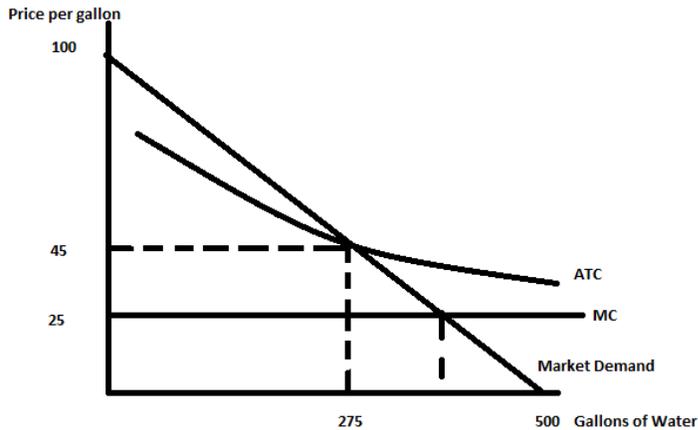
The deadweight loss with 3rd degree price discrimination is higher than with standard monopoly.





Natural Monopoly

3. Madison water utility is the only provider of tap water in Madison. The market demand curve, the average total cost curve and the marginal cost curve for water per day are given in the graph below:



- a. Write an equation for the market demand curve given the above information and assuming that the market demand curve is linear.

$$P = 100 - (1/5)Q$$

- b. Suppose that this monopoly acts as a single price monopolist. What price will it charge and what quantity will it produce? Explain how you found this answer. Calculate the value of consumer surplus, producer surplus, and deadweight loss if this firm acts as a single price monopolist. Provide a graph to illustrate your answers: this graph should indicate the area of consumer surplus, producer surplus, and deadweight loss. From your graph determine whether this monopoly earns positive, negative, or zero economic profit.

To find the price and quantity the monopolist will choose we need the MR curve. We can easily get that from the market demand curve:

$$\text{Market Demand: } P = 100 - (1/5)Q$$

$MR = 100 - (2/5)Q$ since the MR for a single price monopolist will have the same y-intercept as the demand curve and twice the slope of the demand curve.

Set $MR = MC$ to find the profit maximizing quantity:

$$100 - (2/5)Q = 25$$

$$75 = (2/5)Q$$

$$Q = 187.5 \text{ gallons}$$

To find the profit maximizing price, use $Q = 187.5$ and your demand curve:

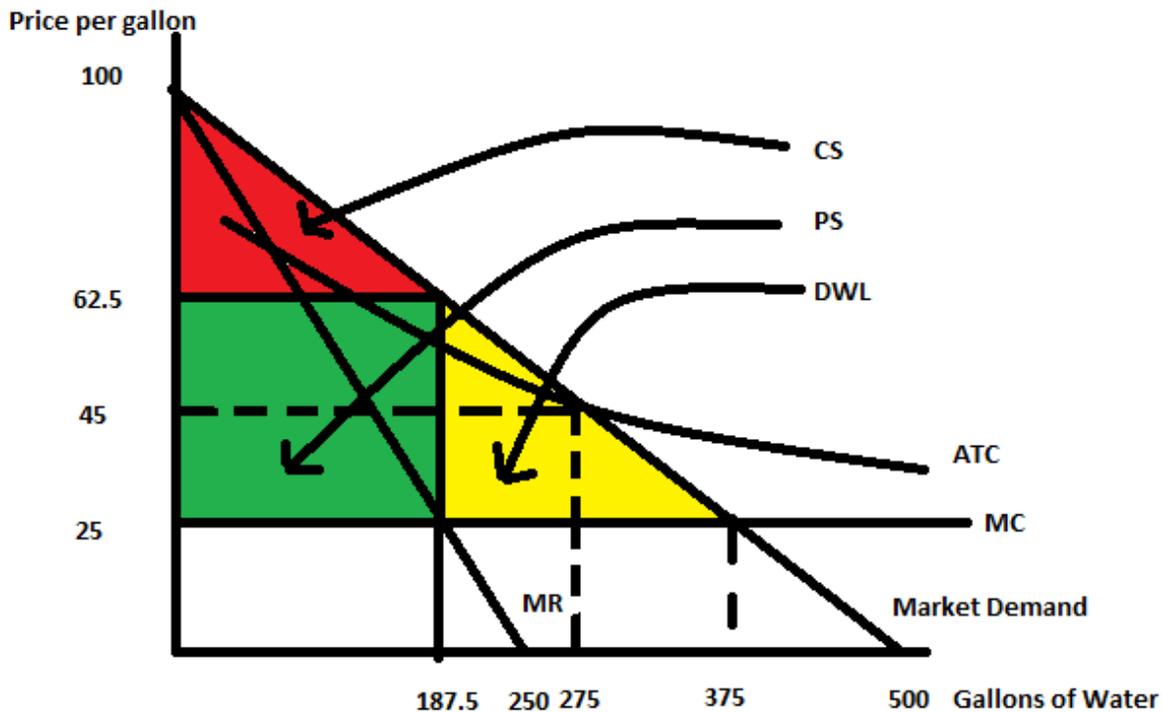
$$P = 100 - (1/5)(187.5) = \$62.50 \text{ per gallon}$$

$$CS = (1/2)(100 - 62.5)(187.5) = \$3515.625$$

$$PS = (62.5 - 25)(187.5) = \$7031.25$$

$$DWL = (1/2)(62.5 - 25)(375 - 187.5) = \$3515.625$$

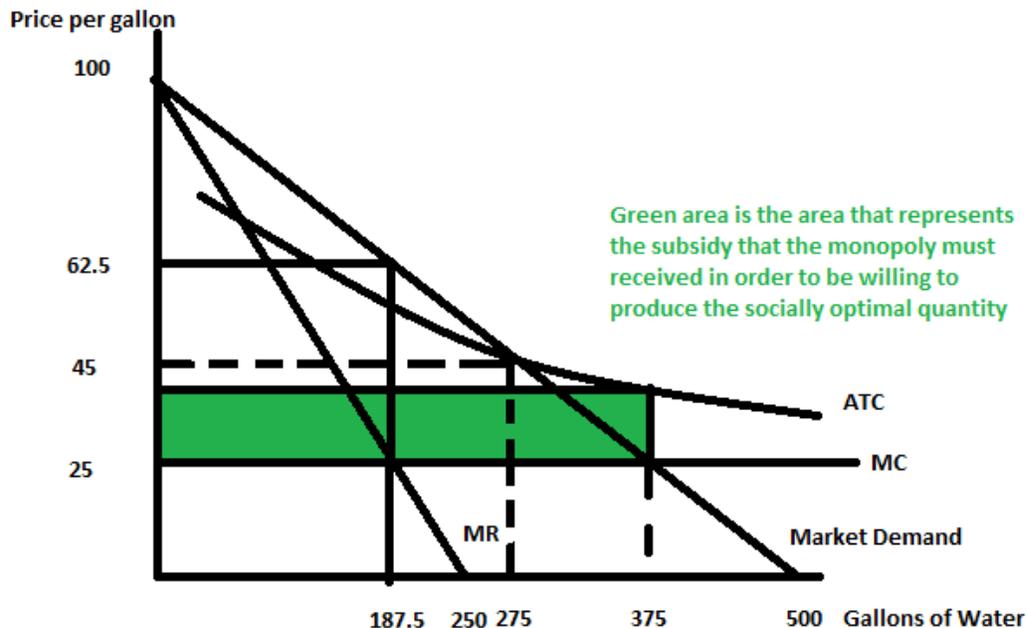
Since P is greater than ATC when $Q = 187.5$ gallons, this monopoly will earn positive economic profits when it acts as a single price monopolist.



c. Suppose that the government in Madison dislikes this monopoly restricting output and charging such high prices. The government also does not like the fact that the monopoly is earning a positive economic profit. If the government decides to regulate this firm so that it produces the socially optimal amount of the good, how many units will the firm produce and what price will consumers be willing to pay for this amount? Will the firm be willing to provide the socially optimal amount of the good? Explain your answer.

The socially optimal amount of the good is where the price of the last unit is equal to the marginal cost of producing the last unit. That is, the socially optimal quantity of the good is where $P = MC$ for the last

unit. This will occur at that quantity where the MC curve intersects the Market Demand curve or when $Q = 375$ gallons. Consumers will be willing to pay \$25 per gallon. The regulated monopoly will be unwilling to supply this amount at this price since it will earn negative economic profits. The government if it wants this socially optimal outcome will have to pay the regulated monopoly a subsidy. The graph below illustrates the amount of the subsidy required in order to get this firm to produce 375 gallons and sell it to consumers at \$25 per gallon.



d. Suppose that the government in Madison dislikes this monopoly restricting output and charging such high prices but knows that the public would never agree to paying a subsidy to the monopoly. The government also does not like the fact that the monopoly is earning a positive economic profit. If the government decides to regulate this firm so that it earns zero economic profit, how many units will the firm produce and what price will consumers be willing to pay for this amount? Calculate the deadweight loss from the implementation of this type of regulation. Calculate the value of consumer surplus and producer surplus with this type of regulation. Is the consumer better off with regulation or better off with an unregulated market? Explain your answer. Provide a graph to illustrate your answer.

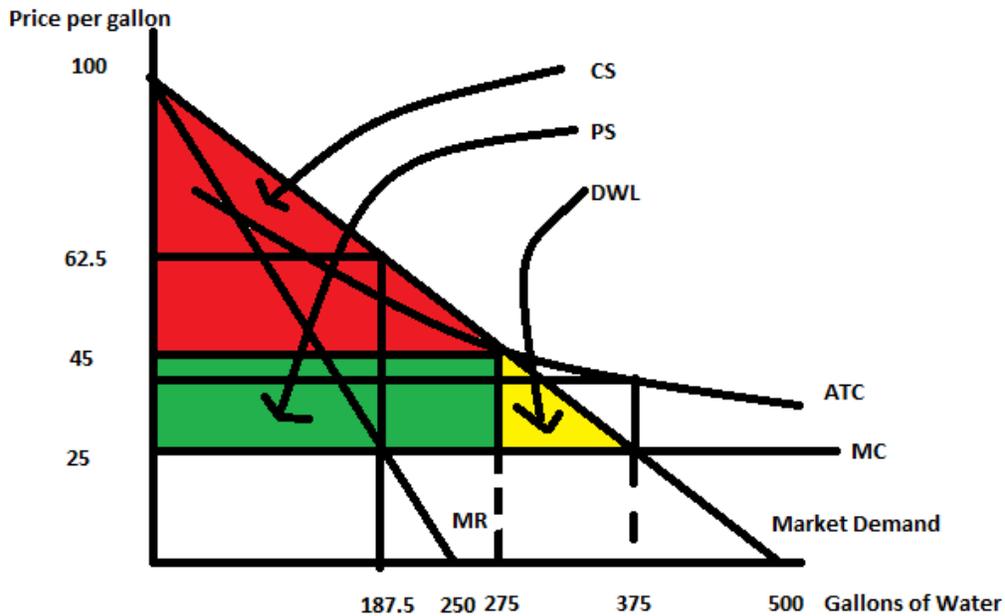
If regulated so that it earns zero economic profit, we know that the firm must be producing a quantity where the $P = ATC$. This will occur at a quantity of 275 gallons and a price of \$45 per gallon. At this quantity and price the firm's total revenue will equal its total cost and hence, its profits will be equal to zero.

$$DWL = (1/2)(45 - 25)(375 - 275) = \$1000$$

$$CS = (1/2)(100 - 45)(275) = \$7562.50$$

$$PS = (45 - 25)(275) = \$5500$$

The consumer prefers the regulated monopoly since they get more of the good at a lower price. Their consumer surplus is greater with the regulated monopoly than it is with the unregulated monopoly.



Game Theory

4. Suppose that in the market for Falafel (Arabic food) only two firms (firm A and firm B) operate in the Madison area. Each firm can choose a price per meal: the meal can be priced at \$6, \$8, or \$10. The market demand is given by: $p = 16 - 2q$. Consumers in Madison will always buy from the firm that offers the lowest price. Assume that if the firms choose the same price, then each firm will get 50% of the market.

The cost functions for each firm is given by the following equations:

$$TC_A = q_A + 15$$

$$TC_B = 3q_B$$

- a. What are the strategies for each firm?

Each firm has 3 strategies: they can price their meals at $p=\$6$, $p=\$8$, or $p=\$10$.

- b. Complete the following table:

Firm B Firm A	$P_B = 6$	$P_B = 8$	$P_B = 10$
$P_A = 6$	$\pi_A = ? \quad \pi_B = ?$	$\pi_A = ? \quad \pi_B = ?$	$\pi_A = ? \quad \pi_B = ?$
$P_A = 8$	$\pi_A = ? \quad \pi_B = ?$	$\pi_A = ? \quad \pi_B = ?$	$\pi_A = ? \quad \pi_B = ?$
$P_A = 10$	$\pi_A = ? \quad \pi_B = ?$	$\pi_A = ? \quad \pi_B = ?$	$\pi_A = ? \quad \pi_B = ?$

I will show the work for two cases (you should do the rest by yourself)

- $P_A = 6$ and $P_B = 6$

If both firms choose to charge \$6, then the price in this market will be \$6.

Given this price, the quantity demanded is:

$$Q = 8 - \frac{p}{2} = 8 - \frac{6}{2} = 5$$

And therefore, each firm will provide 2.5

The profits of the firms:

$$\pi_A = TR_A - TC_A = p^* \cdot q_A - (q_A + 15) = 6 \cdot 2.5 - (2.5 + 15) = -2.5$$

$$\pi_B = TR_B - TC_B = p^* \cdot q_B - (3q_B) = 6 \cdot 2.5 - (3 \cdot 2.5) = 7.5$$

- $P_A = 10$ and $P_B = 8$

In this case, the price in the market will be \$8, and firm B will take all the market.

Quantity demanded when $p = \$8$:

$$Q = 8 - \frac{p}{2} = 8 - \frac{8}{2} = 4$$

$$q_A = 0 \text{ and } q_B = 4$$

The profits of the firms:

$$\pi_A = TR_A - TC_A = p^* \cdot q_A - (q_A + 15) = 8 \cdot 0 - (0 + 15) = -15$$

$$\pi_B = TR_B - TC_B = p^* \cdot q_B - (3q_B) = 8 \cdot 4 - (3 \cdot 4) = 20$$

Firm B Firm A	$P_B = 6$	$P_B = 8$	$P_B = 10$
$P_A = 6$	$\pi_A = -2.5 \quad \pi_B = 7.5$	$\pi_A = 10 \quad \pi_B = 0$	$\pi_A = 10 \quad \pi_B = 0$
$P_A = 8$	$\pi_A = -15 \quad \pi_B = 15$	$\pi_A = -1 \quad \pi_B = 10$	$\pi_A = 13 \quad \pi_B = 0$
$P_A = 10$	$\pi_A = -15 \quad \pi_B = 15$	$\pi_A = -15 \quad \pi_B = 20$	$\pi_A = -1.5 \quad \pi_B = 10.5$

- c. For each firm determine if it has a dominant strategy and, if so, determine what that strategy is. Is there a strategy that firm A or firm B definitely will not engage in?

Neither firm has a dominant strategy. That is, neither has a strategy they will adopt independent of what the other firm does.

To see this, consider first firm A's analysis. We can do this by thinking of the table as three separate columns: a column where firm A assumes that firm B is holding $P = \$6$ as its strategy, a column where firm A assumes that firm B is holding $P = \$8$ as its strategy, and a column where firm A assumes that firm B is holding $P = \$10$ as its strategy. In the image below an "X" represents firm A's optimal strategy given the assumption about firm B's pricing strategy.

For Firm A:	Firm B prices at $P = \$6$	Firm B prices at $P = \$8$	Firm B prices at $P = \$10$
$P = \$6$	X	X	
$P = \$8$			X
$P = \$10$			

From the figure we can see that firm A's decision as to what strategy it prefers is dependent on what strategy firm B is taking.

But, from the table we can also see that firm A never prefers $P = \$10$ as its pricing strategy.

Now, let's consider firm B's analysis. We can do this by thinking of the table as three separate rows: a row where firm B assumes that firm A is holding $P = \$6$ as its strategy, a row where firm B assumes that firm A is holding $P = \$8$ as its strategy, and a row where firm B assumes that firm A is holding $P = \$10$ as its strategy. In the image below a "Y" represents firm B's optimal strategy given the assumption about firm A's pricing strategy.

From Firm B's Perspective:

	P = \$6	P = \$8	P = \$10
Firm A prices at P = \$6	Y		
Firm A prices at P = \$8	Y		
Firm A prices at P = \$10	Y		

From the figure we can see that firm B's decision as to what strategy it prefers is dependent on what strategy firm A is taking.

But, from the table we can also see that firm B never prefers $P = \$10$ as its pricing strategy.

- d. Given your analysis of the payoff matrix, make a prediction about the pricing strategy you expect to see firm A and firm B pursue in this market. Explain your reasoning.

We know that neither firm will pursue the strategy of $P = \$10$. If Firm A realizes that firm B will not pursue the strategy of $P = \$10$, then firm A's optimal strategy is to set price at $P = \$6$ since firm A will earn its greatest profits (true, they are negative when firm B charges $P = \$6$, but they are at their highest level given the other pricing alternatives) with this pricing strategy no matter whether firm B selects $P = \$6$ or $P = \$8$.

If firm B realizes that firm A will not pursue the strategy of $P = \$10$, then firm B's optimal strategy is to set price at $P = \$6$ since firm B will earn its greatest profits with this pricing strategy no matter whether firm A selects $P = \$6$ or $P = \$8$.

Both firms will end up with $P = \$6$.

- e. What price should each firm charge if they want to maximize the total profit?

It's clear from the table that the pair of prices that will lead to the maximum total profit is:

$$(P_A = 8, P_B = 10)$$

And the total profit is equal to \$13.

- f. How much firm A is willing to pay firm B in order to convince firm B to use the strategy that will lead to the optimal allocation? Will firm B accept the offer?

Firm A is willing to pay up to \$15.5 in order to convince firm B to choose $P_B = 10$. That is the change in A's profit if they reach an agreement to change their strategies from:

$$(P_A = 6, P_B = 6) \quad \text{to} \quad (P_A = 8, P_B = 10)$$

From a loss of \$2.5 to a profit of \$13.

Firm B will agree to choose $P_B = 10$, if it gets paid at least \$7.5. That's the change in B's profit if they reach an agreement to change their strategies from:

$$(P_A = 6, P_B = 6) \quad \text{to} \quad (P_A = 8, P_B = 10)$$

From profit of \$7.5 to profit zero.

Which means they can reach agreement, and instead of choosing the $P = \$6$ for firm A and $P = \$6$ for firm B strategies, they will choose:

$$(P_A = 8, P_B = 10)$$

And firm A will pay firm B between \$7.5 and \$15.5.

Externalities

5. The market demand for tobacco in Madison is given by:

$$p = 40 - 2q$$

And the market supply is given by:

$$p = 2q$$

Where q is the number of pouches of tobacco and p is the price in dollars per pouch of tobacco.

Suppose that smoking imposes negative externalities on the population of Madison. Assume that each pouch of tobacco costs the population of Madison \$4.

- a. What is the price and quantity in this market for tobacco assuming that this is a competitive market and that the externality cost is not incorporated in the market by either the producers or the consumers of the product?

To find the competitive equilibrium set the Marginal private benefit equal to the Marginal private cost. That is, set the demand curve equal to the supply curve:

$$\Rightarrow 40 - 2q = 2q$$

$$q = 10$$

$$p = 20$$

- b. Given the above information, find the marginal social benefit and marginal social cost functions for this market.

If not specified, usually you can incorporate the externality in the marginal social benefit or in the marginal social cost. (But don't double count the externality)

We can choose to look at the externality as an additional cost, in this case:

Marginal social benefit is equal to marginal private benefit, $MSB = MPB = 40 - 2q$

Marginal social cost is equal to marginal private cost plus the \$4 cost imposed on the society by smoking, $MSC = MPC + 4 = 2q + 4$.

(If you wanted to model this on the demand side you could view the externality as a loss in the marginal benefit: then you would have $MSB = MPB - 4 = 36 - 2q$, and $MSC = MPC = 2q$. Either way of modeling this externality will work and provide you with the same final answer.)

- c. What is the socially optimal number of tobacco pouches in this market? That is, if the externality was internalized in the market, what would be the price of tobacco and how many units would be consumed?

To find optimal number, we use:

$$\begin{aligned}MSB &= MSC \\40 - 2q &= 2q + 4 \\q &= 9 \\p &= \$22\end{aligned}$$

- d. Suppose the government sees that without intervention, the competitive equilibrium isn't socially optimal in this market, and wants to use an excise tax or a subsidy per unit to try and lead the market to the socially optimal equilibrium. What excise tax or subsidy per unit should the government choose to implement to help this market reach the social optimum? What price will the consumers pay for the product if this government program is implemented? What price will producers receive for the product if this government program is implemented? Calculate the tax revenue for the government or the cost to the government of the imposed program.

The government knows that the consumers and the producers are not taking into account the negative externality that tobacco products impose on the society and therefore they are choosing to produce and consume too much of the good (10 units of the good is more than the socially optimal amount of the good of 9 units). The government should of course impose an excise tax (not a subsidy) to discourage the use of tobacco.

To find the optimal level of the tax, we use:

$$\begin{aligned}MPB &= MPC + t \\40 - 2q &= 2q + t\end{aligned}$$

And since we want $q = 9$, we plug that into the equation above:

$$\begin{aligned}40 - 2 \cdot 9 &= 2 \cdot 9 + t \\t &= \$4\end{aligned}$$

The government should impose a \$4 tax per unit of the good.

In this case, by plugging in $q = 9$ back into the original demand and supply functions, the price that consumers will pay is \$22, and the price that producers will get is \$18.

And the government tax revenue = $t \cdot q = 4 \cdot 9 = \36 .

- e. What is the deadweight loss if the government decides not to intervene in this market? That is, what is the deadweight loss that Madison incurs when this externality is not accounted for in the market?

The easiest way to calculate the deadweight loss is to look the change in the total surplus:

With government intervention:

Total surplus = consumer surplus + producer surplus + government tax revenue - negative externality

$$\text{Total surplus} = (40 - 22) \cdot 9/2 + (18 - 0) \cdot 9/2 + 4 \cdot 9 - 4 \cdot 9 = \$162$$

Without government intervention:

Total surplus = consumer surplus + producer surplus - negative externality

$$\text{Total surplus} = (40 - 20) \cdot 10/2 + (20 - 0) \cdot 10/2 - 4 \cdot 10 = \$160$$

$$\text{So the deadweight loss} = \text{DWL} = 162 - 160 = \$2$$

Public good

6. Suppose that 3 student associations are interested in placing more free little libraries around campus. The three associations have different values for these little libraries and they are willing to pay money to make that happen. The willingness to pay for each association is given by the following demand functions:

$$\text{Association 1: } p_1 = 200 - 2Q$$

$$\text{Association 2: } p_2 = 100 - 0.5Q$$

$$\text{Association 3: } p_3 = 142 - Q$$

The cost of placing a new free library is: \$50. That is, the MC of an additional free library is simply $\text{MC} = \$50$.

- a. Suppose this market is treated as a competitive market. How many libraries will each association want to have on campus? How much will each association contribute or pay for each little library? Are there any free riders in this market? Explain your answer.

If this market is treated as a competitive market, then each association will basically ask itself "How many libraries do we want if the price is \$50 per library?"

$$\text{Association 1 wants } Q = 100 - \frac{p_1}{2} = 100 - \frac{50}{2} = 75 \text{ units}$$

$$\text{Association 2 wants } Q = 200 - 2p_2 = 200 - 2 \cdot 50 = 100 \text{ units}$$

$$\text{Association 3 wants } Q = 142 - p_3 = 142 - 50 = 92 \text{ units}$$

Associations 1 and 3 know that even if they don't pay anything, association 2 will have no choice but to pay for all the 100 units it wants at the price of \$50 per library. And associations 1 and 3 will enjoy the new free libraries without having to pay anything: that is, they will be free riders.

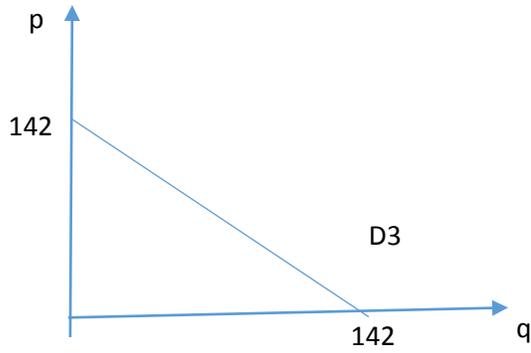
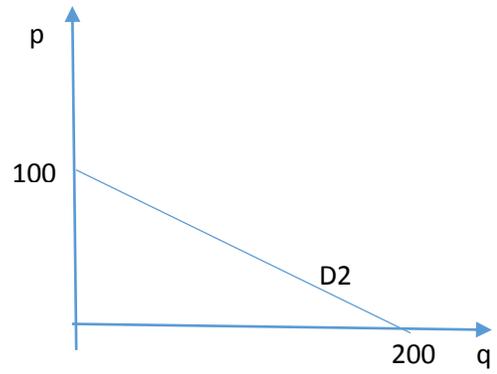
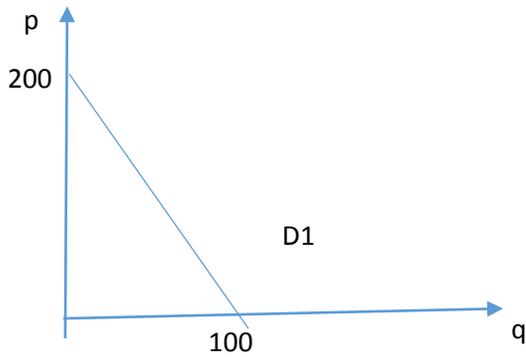
So in the competitive equilibrium:

$$Q = 100, p_1 = p_3 = 0, p_2 = \$50$$

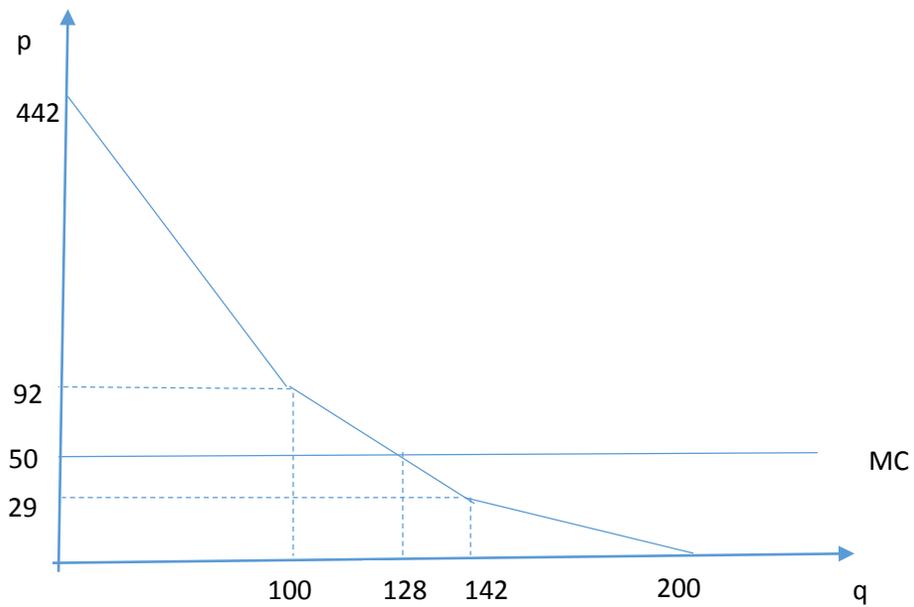
Which means association 2 will finance all 100 units.

- b. Find the aggregate demand! (Or the marginal social benefit. Note that we are talking about a public good!!!)

Public good! So we sum the demand functions vertically (we are summing the willingness to pay, using the fact that each unit of the public good can be used by the three associations).



Aggregate Demand:



For $Q < 100$, all three associations are willing to pay something, so:

$$p = p_1 + p_2 + p_3 = 200 - 2Q + 100 - 0.5Q + 142 - Q = 442 - 3.5Q$$

For $100 < Q < 142$, firms 2 and 3 are willing to pay something, so

$$p = p_2 + p_3 = 100 - 0.5Q + 142 - Q = 242 - 1.5Q$$

For $Q > 142$, only firm 2 is willing to pay something, so

$$p = p_2 = 100 - 0.5Q$$

- c. What is the socially optimal number of new free libraries? How much should each association contribute per little library in order to get the socially optimal number of free libraries?

You can check all intervals, but as you can see in the graph, $p = 50$ intersects with the second interval, and therefore:

$$\begin{aligned} 50 &= 242 - 1.5Q \\ Q &= 128 \end{aligned}$$

To find how much each association needs to pay per unit, we plug $Q = 128$ back into each firm's demand function:

$$p_1 = 200 - 2Q = 200 - 256 = -56 \Rightarrow p_1 = 0$$

We know that already since we used the second interval!

$$p_2 = 100 - 0.5Q = 100 - 64 = \$36$$

$$p_3 = 142 - Q = 142 - 128 = \$14$$